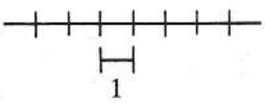
NAME ______________________ DATE ______________

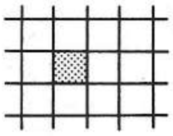
11–1 Areas of Rectangles

Objectives: Understand the area postulates. Know and use the formula for the areas of rectangles.

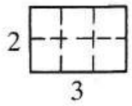
Area postulates allow us to define areas of figures as positive numbers. Remember that area is always measured in square units, such as square centimeters (cm^2), square meters (m^2), and square feet (ft^2).



length: 1 unit



Area: 1 square unit



$A = 6$ square units

The area of a square is the square of the length of a side. $A = s^2$

The area of a rectangle equals the product of its base and height. $A = bh$

Example 1

a. Find the area of a rectangle with base 5 cm and height 4 cm.

b. The area of a rectangle is 36 cm^2. If its height is 9 cm, find the base.

Solution

a. $A = bh$
$A = 5 \cdot 4 = 20$ The area is 20 cm^2.

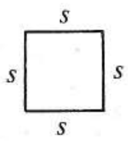
b. $A = bh$
$36 = 9 \cdot b$
$4 = b$ The base is 4 cm.

Exercises 1–8 refer to rectangles. Complete the table.

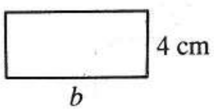
	1.	2.	3.	4.	5.	6.	7.	8.
b	8 cm	15 cm	?	12 ft	$5\sqrt{3}$	$\sqrt{6}$	$3x$	$y + 1$
h	7 cm	?	24 cm	?	$4\sqrt{3}$	$\sqrt{3}$	$x - 1$	$y - 2$
A	?	60 cm^2	120 cm^2	144 ft^2	?	?	?	?

Example 2

a. The area of a square is 9 cm^2. Find its perimeter.



b. The perimeter of a rectangle is 20 cm. If its height is 4 cm, find its area.



Solution

a. $A = s^2$ $p = 4s$
$9 = s^2$ $= 4 \cdot 3$
$3 = s$ $= 12$
The perimeter is 12 cm.

b. $p = 2(b + h)$ $A = bh$
$20 = 2(b + 4)$ $A = 6 \cdot 4$
$10 = b + 4$ $= 24$
$6 = b$ The area is 24 cm^2.

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11–1 Areas of Rectangles *(continued)*

Solve.

9. Find the side of a square if its area is 64 cm^2.
10. Find the perimeter of a square if its area is 144 cm^2.
11. Find the area of a square if its perimeter is 40 cm.
12. Find the area of a rectangle if its perimeter is 30 and its base is 8.

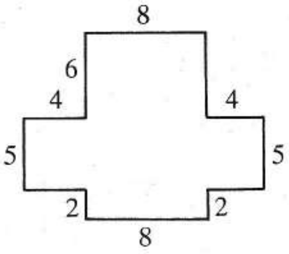
Exercises 13–19 refer to rectangles. Complete the table.

	13.	14.	15.	16.	17.	18.	19.
b	12 cm	22 cm	?	?	$3\sqrt{2}$	$2x$	$y + 5$
h	4 cm	?	9 cm	18 ft	$5\sqrt{2}$	?	$y - 5$
A	?	?	?	72 ft^2	?	?	?
p	?	52 cm	28 cm	?	?	$7x$	?

Area Congruence Postulate	**If two figures are congruent, then they have the same area.**
Area Addition Postulate	**The area of a region is the sum of the areas of its non-overlapping parts.**

Example 3

Consecutive sides of the figure are perpendicular. Find its area.



Solution

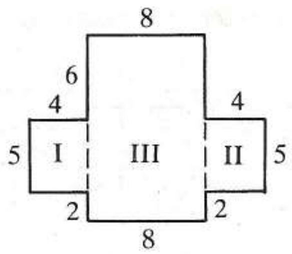
Divide the figure into rectangles. One way of dividing the figure is shown.

Area of rectangle I $= 5 \cdot 4 = 20$

Area of rectangle II $= 5 \cdot 4 = 20$

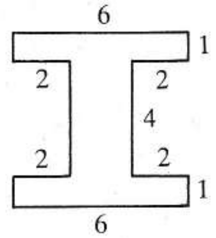
Area of rectangle III $= 8(6 + 5 + 2)$
$= 8 \cdot 13 = 104$

Area of the figure $=$ I + II + III
$= 20 + 20 + 104$
$= 144$

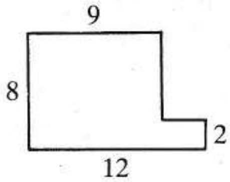


Consecutive sides of the figures are perpendicular. Find the area of each figure.

20.



21.



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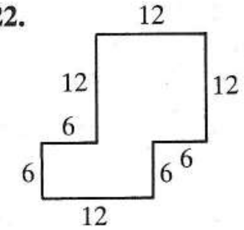
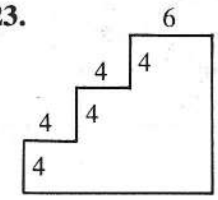


23.



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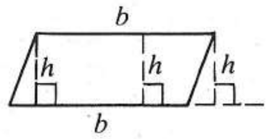
11–2 Areas of Parallelograms, Triangles, and Rhombuses

Objective: Know and use the formulas for the areas of parallelograms, triangles, and rhombuses.

Any side of a parallelogram may be considered a **base**, b. An **altitude** is any segment perpendicular to the line containing a base from any point on the opposite side. The length of an altitude is called the **height**, h, of the parallelogram.

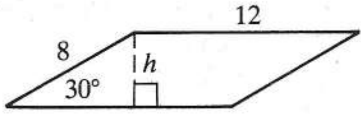


The area of a parallelogram equals the product of a base and the height to that base. $A = bh$

Example 1

a. Find the area of a parallelogram with base $5\sqrt{3}$ and corresponding height $4\sqrt{3}$.

b. Find the area of the parallelogram shown.



Solution

a. $A = bh$
$= 5\sqrt{3} \cdot 4\sqrt{3} = 60$

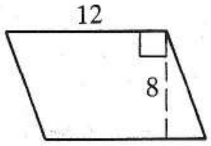
b. Since h is the shorter leg of a 30°-60°-90° triangle:
$h = \frac{1}{2} \cdot \text{hypotenuse} = \frac{1}{2} \cdot 8 = 4$
$A = bh = 12 \cdot 4 = 48$

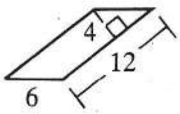
Solve.

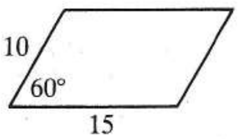
1. Find the area of a parallelogram with base 6 cm and corresponding height 7 cm.

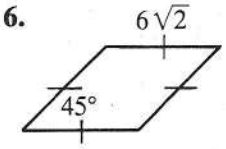
2. Find the area of a parallelogram with base $6\sqrt{2}$ and corresponding height $10\sqrt{2}$.

Find the area of each parallelogram.

3. 

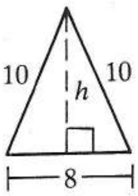
4. 

5. 

6. 

The area of a triangle equals half the product of a base and the height to that base. $A = \frac{1}{2}bh$

Example 2 Find the area of the triangle.



Solution

The altitude to the base of an isosceles triangle bisects the base. Use the Pythagorean Theorem.

$4^2 + h^2 = 10^2$
$h^2 = 100 - 16 = 84$
$h = 2\sqrt{21}$
$A = \frac{1}{2}bh = \frac{1}{2} \cdot 8 \cdot 2\sqrt{21} = 8\sqrt{21}$

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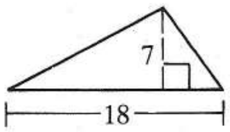
11–2 Areas of Parallelograms, Triangles, and Rhombuses *(continued)*

Find the area of each figure.

7.



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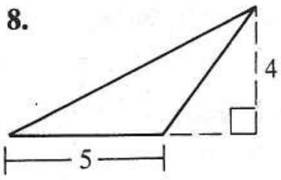

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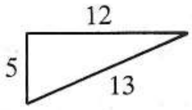

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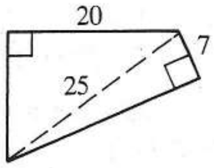


11. Find the area of an isosceles triangle with sides 30, 30, and 24.

12. Find the area of an isosceles triangle with base 16 and perimeter 52.

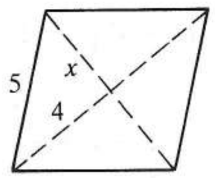
13. Find the area of an equilateral triangle with sides 12 cm.

14. Find the area of an equilateral triangle with height $6\sqrt{3}$.

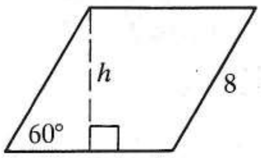
> **The area of a rhombus equals half the product of its diagonals.** $A = \frac{1}{2}d_1d_2$

Example 3 Find the area of each rhombus.

a.



b.



Solution

a. Remember the diagonals of a rhombus are ⊥ bisectors of each other.
$5^2 = 4^2 + x^2$, so $x = 3$.

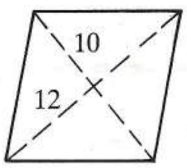
$A = \frac{1}{2}d_1d_2$
$= \frac{1}{2}(2 \cdot 4)(2 \cdot 3)$
$= 24$

b. A rhombus is a ▱, so you can use the formula $A = bh$ to find its area. h is the longer leg of a 30°-60°-90° triangle with hypotenuse 8.

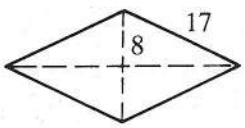
$h = 8 \cdot \frac{1}{2} \cdot \sqrt{3} = 4\sqrt{3}$
$A = bh$
$= 8 \cdot 4\sqrt{3} = 32\sqrt{3}$

Find the area of each rhombus.

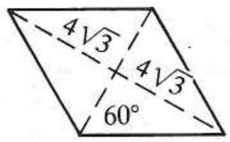
15.



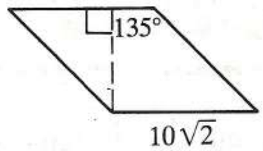
16.



17.



18.



19. Find the area of a rhombus with diagonals 8 m and 20 m.

20. Find the area of a rhombus with perimeter 52 and one diagonal 10.

21. Find the area of a rhombus with perimeter 100 and one diagonal 14.

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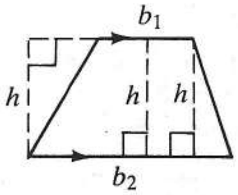
11–3 Areas of Trapezoids

Objective: Know and use the formulas for the areas of trapezoids.

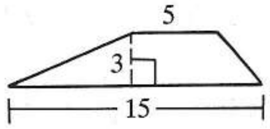
In a trapezoid, the bases are the parallel sides. An altitude of a trapezoid is defined in the same way as an altitude of a parallelogram. The altitude of a trapezoid is any segment perpendicular to a line containing one base from a point on the opposite base. In a trapezoid, all altitudes have the same length, called the height, h.



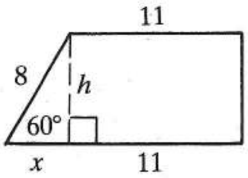
The area of a trapezoid equals half the product of the height and the sum of the bases. $A = \frac{1}{2}h(b_1 + b_2)$

Example 1 Find the area of each trapezoid.

a.



b.



Solution

a. $A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2} \cdot 3 \cdot (5 + 15)$

$= 30$

b. x is the shorter leg of a 30°-60°-90° triangle, so $x = \frac{1}{2} \cdot 8 = 4$. The longer base of the trapezoid is $x + 11 = 15$.
The height is the longer leg of a 30°-60°-90° triangle, so $h = 4\sqrt{3}$.

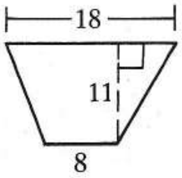
$A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2} \cdot 4\sqrt{3} \cdot (15 + 11)$

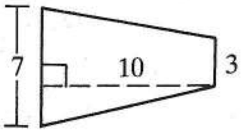
$= 52\sqrt{3}$

Find the area of each trapezoid.

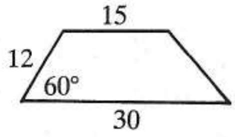
1.



2.



3.



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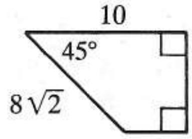
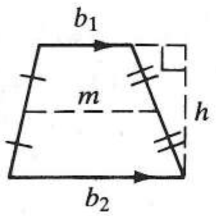


In the figure, m represents the length of the median of the trapezoid. You may remember that the median of a trapezoid is the segment connecting the midpoints of the legs. The formula for the length of the median is $\frac{1}{2}(b_1 + b_2)$. Notice how $\frac{1}{2}(b_1 + b_2)$ is used in the area formula. You can say that the area of a trapezoid = height × median ($A = hm$).



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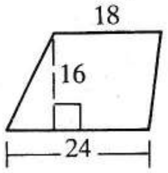
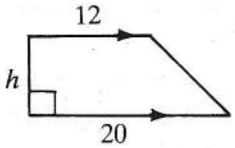
11–3 Areas of Trapezoids *(continued)*

Example 2

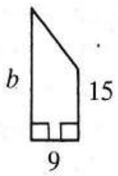
a. Find the length of the median and the area of the trapezoid.



b. If the area of the trapezoid is 128, find the height.



c. If the area of the trapezoid is 189, find the other base.



Solution

a. $m = \frac{1}{2}(b_1 + b_2)$

$= \frac{1}{2}(18 + 24) = 21$

$A = hm$

$= (16)(21) = 336$

b. $A = \frac{1}{2}h(b_1 + b_2)$

$128 = \frac{1}{2}h(12 + 20)$

$128 = 16h$

$8 = h$

c. $A = \frac{1}{2}h(b_1 + b_2)$

$189 = \frac{1}{2}(9)(15 + b)$

$378 = 9(15 + b)$

$42 = 15 + b$

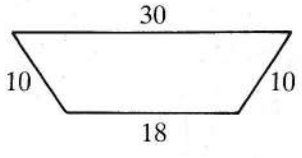
$27 = b$

Exercises 5–11 refer to trapezoids. Complete the table.

	5.	6.	7.	8.	9.	10.	11.
b_1	15	25	8	42	?	?	$14x$
b_2	13	10	?	14	9	8	?
h	5	?	4	?	5	$6\sqrt{3}$	$7x$
A	?	140	46	336	?	$33\sqrt{3}$	$70x^2$
m	?	?	?	?	7	?	?

Example 3

Find the area of the isosceles trapezoid.



Solution

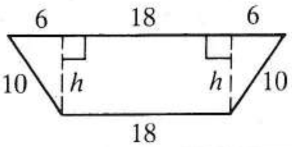
When you draw the altitudes shown, you get a rectangle and two congruent right triangles.

$6^2 + h^2 = 10^2$

Using Pythagorean triples, $h = 8$.

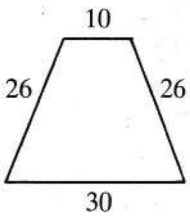
$A = \frac{1}{2}h(b_1 + b_2)$

$= \frac{1}{2} \cdot 8 \cdot (30 + 18) = 192$

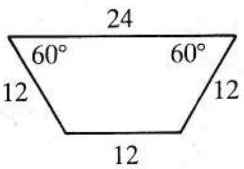


Find the area of each isosceles trapezoid.

12.



13.



14. Find the area of an isosceles trapezoid with legs 25 cm and bases 16 cm and 30 cm.

15. Find the area of an isosceles trapezoid with 60° base angles and bases 9 and 13.

16. Find the area of a trapezoid with 45° base angles and bases 17 and 23.

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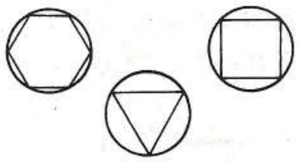
11–4 Areas of Regular Polygons

Objective: Know and use the formula for the areas of regular polygons.

A regular polygon is both equilateral and equiangular. Any regular polygon can be inscribed in a circle. Therefore, many of the terms associated with circles are also used with regular polygons.



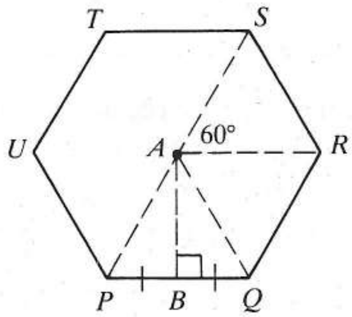
The **center** of a regular polygon is the center of the circumscribed circle.

The **radius** of a regular polygon is the distance from the center to a vertex. $\overline{AP}$ and $\overline{AR}$ are radii.

A **central angle** is an angle formed by two radii drawn to consecutive vertices. $\angle PAQ$ and $\angle SAR$ are central angles; $\angle SAQ$ is not a central angle.

The **measure of a central angle** of a regular polygon with n sides is $\frac{360}{n}$. For example, the measure of each central angle in regular hexagon $PQRSTU$ is $\frac{360}{6} = 60$.

The **apothem** of a regular polygon is the distance from the center to a side. AB is an apothem. $\overline{AB}$ is the $\perp$ bisector of side $\overline{PQ}$.

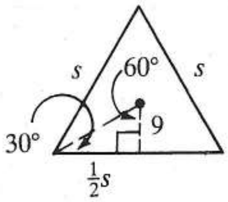


If you know the apothem and perimeter of a regular polygon, the following theorem allows you to find the area of the polygon.

The area of a regular polygon is equal to half the product of the apothem and the perimeter. $A = \frac{1}{2}ap$

Example 1

Find the perimeter and area of a regular triangle with apothem 9.



Solution

When the radius and apothem of an equilateral triangle are drawn, a 30°-60°-90° triangle is formed.

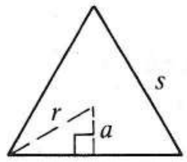
If $a = 9$, then $\frac{1}{2}s = 9\sqrt{3}$ and $s = 18\sqrt{3}$, so the perimeter $p = 3s = 54\sqrt{3}$.

$A = \frac{1}{2}ap$

$= \frac{1}{2}(9)(54\sqrt{3}) = 243\sqrt{3}$

Complete the table for regular triangles.

	r	a	s	p	A
1.	10	?	?	?	?
2.	?	6	?	?	?
3.	?	?	8	?	?
4.	?	?	?	15	?
5.	?	?	?	$12\sqrt{3}$	?



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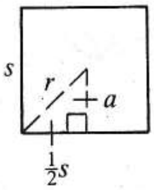
11–4 Areas of Regular Polygons *(continued)*

Example 2

Find the apothem and radius of a regular quadrilateral with area 100.



Solution

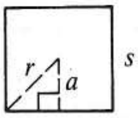
When the radius and apothem of a square are drawn, a 45°-45°-90° triangle is formed. If the area is 100, $A = s^2 = 100$ and $s = 10$,

so $a = \frac{1}{2}s = 5$

and $r = a\sqrt{2} = 5\sqrt{2}$.

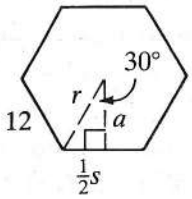
Complete the table for regular quadrilaterals.

	r	*a*	*s*	*A*
6.	?	?	?	144
7.	$10\sqrt{2}$	?	?	?
8.	?	7	?	?
9.	?	?	8	?



Example 3

Find the area of a regular hexagon with side 12.



Solution

A hexagon has 6 sides, so
$p = 6s = 6(12) = 72$.
When an apothem and radius are drawn, a 30°-60°-90° triangle is formed.

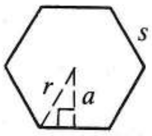
$a = \frac{1}{2}s\sqrt{3} = \frac{1}{2} \cdot 12 \cdot \sqrt{3} = 6\sqrt{3}$

$A = \frac{1}{2}ap$

$= \frac{1}{2} \cdot 6\sqrt{3} \cdot 72 = 216\sqrt{3}$

Complete the table for regular hexagons.

	r	*a*	*s*	*p*	*A*
10.	4	?	?	?	?
11.	?	15	?	?	?
12.	?	?	6	?	?
13.	?	?	?	$18\sqrt{3}$	?



Find the area of each polygon.

14. An equilateral triangle with radius $6\sqrt{3}$

15. A regular hexagon with perimeter 48 cm

16. A square with radius 24

17. A regular hexagon with apothem $12\sqrt{3}$

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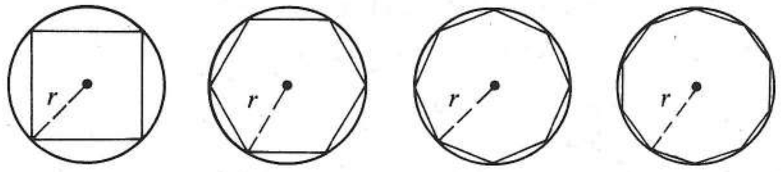
11–5 Circumferences and Areas of Circles

Objective: Know and use the formulas for circumferences and areas of circles.

The perimeter of a polygon is defined as the sum of the lengths of the segments making up its sides. Since a circle is not made up of line segments, the perimeter of a circle must be defined differently.

Consider a sequence of regular polygons inscribed in the circle. Four such polygons are shown below, but imagine more and more polygons having more and more sides.



4 sides — 6 sides — 8 sides — 10 sides

Now consider the perimeters of the polygons. As the drawings suggest, these perimeters give us a sequence of numbers that get closer to a limiting number. This limiting number is defined to be the perimeter, or **circumference**, of the circle.

The **area** of a circle is defined in a similar way. The areas of the inscribed regular polygons get closer and closer to a limiting number, and this limit is defined to be the area of the circle.

In work with circles, the ratio of the circumference to the diameter $\left(\frac{C}{d}\right)$ is important. This ratio is constant for all circles, and is denoted by the Greek letter π (*pi*).

Circumference of a circle with radius *r*	$C = 2\pi r$
Circumference of a circle with diameter *d*	$C = \pi d$
Area of a circle with radius *r*	$A = \pi r^2$

Example 1

a. Find the circumference and area of a circle with radius $8\sqrt{3}$.

b. Find the circumference of a circle if the area is 100π.

Solution

a. $C = 2\pi r$
$= 2\pi \cdot 8\sqrt{3} = 16\pi\sqrt{3}$
$A = \pi r^2$
$= \pi(8\sqrt{3})^2 = 192\pi$

b. $A = \pi r^2$
$100\pi = \pi r^2$
$10 = r$

$C = 2\pi r$
$= 2\pi \cdot 10$
$= 20\pi$

Complete the table.

	1.	2.	3.	4.	5.	6.	7.	8.
Radius	5	8	$3\sqrt{2}$	?	?	?	?	?
Circumference	?	?	?	12π	100π	?	?	?
Area	?	?	?	?	?	16π	121π	64π

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11–5 Circumferences and Areas of Circles *(continued)*

When doing calculations with areas and circumferences of circles, leave your answer in terms of π unless told to use an approximation. Because π is an irrational number there isn't any decimal or fraction that expresses π exactly. Some common approximations of π are 3.14 and $\frac{22}{7}$.

Example 2

a. A bicycle wheel has a diameter of 60 cm. How far will it travel if it makes 50 revolutions? Use $\pi \approx 3.14$.

b. A circular swimming pool has radius 14 ft. What is the area of the swimming pool? Use $\pi \approx \frac{22}{7}$.

Solution

a. The wheel will travel 50 times its circumference.

$C = \pi d$

$\approx 3.14 \cdot 60 = 188.4$

distance $= 50 \cdot C$

$\approx 50 \cdot 188.4 = 9420$

It will travel about 9420 cm, or 94.2 m.

b. $A = \pi r^2$

$\approx \left(\frac{22}{7}\right)(14^2)$

$= 616$

The area of the pool is about 616 ft^2.

Solve.

9. Find the circumference and area of each circle to the nearest tenth. Use $\pi \approx 3.14$.

a. $r = 12$ **b.** $r = 1.5$ **c.** $d = 400$

10. Find the circumference and area of each circle. Use $\pi \approx \frac{22}{7}$.

a. $r = 49$ **b.** $r = 21$ **c.** $d = 70$

11. Find the area of a circular rug with radius 0.8 m to the nearest square meter (m^2). Use $\pi \approx 3.14$.

12. Find the radius of a pizza pan if its circumference is 12.56 ft. Use $\pi \approx 3.14$.

13. Find the diameter of a pipe if the area of a cross-section is 50.24 cm^2. Use $\pi \approx 3.14$.

14. Marge wants to enclose her circular flower garden with fencing that costs \$2.95 a foot. If her garden has a radius of $3\frac{1}{2}$ ft, how much will the fencing cost? Use $\pi \approx \frac{22}{7}$.

15. The diameter of a bicycle wheel is 28 in. Use $\pi \approx \frac{22}{7}$.

a. How far does the bicycle travel with each revolution of the wheel?

b. How many revolutions will it take to go 1 mi (63,360 in.)?

16. A truck travels 1 mi (5280 ft) for every 490 revolutions of its wheels. What is the diameter of its wheels in feet? Use $\pi \approx \frac{22}{7}$.

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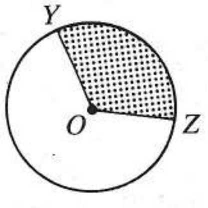
11–6 Arc Lengths and Areas of Sectors

Objective: Know and use the formulas for arc lengths and the areas of sectors of circles.

There are two different numbers that describe the size of an arc. One is its measure, $m\overset{\frown}{YZ}$. The other is the **arc length,** the length of the piece of the circumference that is $\overset{\frown}{YZ}$. It is a fraction of the whole circumference.

A **sector of a circle** is a region bounded by two radii and an arc of the circle. The shaded region in the circle at the right is called sector YOZ. The unshaded region is also a sector. The area of a sector is a fraction of the area of the whole circle.

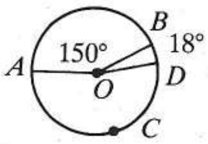


If $m\overset{\frown}{YZ} = x$, then the length of $\overset{\frown}{YZ} = \frac{x}{360} \cdot 2\pi r$, or, informally, the length of the arc equals the fraction times the circumference of the circle.

If $m\overset{\frown}{YZ} = x$, then the area of sector $YOZ = \frac{x}{360} \cdot \pi r^2$, or, informally, the area of the sector equals the fraction times the area of the circle.

Example 1

In $\odot O$ with radius 6 and $m\angle AOB = 150$, find the lengths of $\overset{\frown}{AB}$ and $\overset{\frown}{ACD}$.



Solution

For minor $\overset{\frown}{AB}$:

Fraction: $\frac{150}{360} = \frac{5}{12}$

Circumference: $2\pi r = 2\pi(6) = 12\pi$

Multiply: arc length $= \frac{5}{12} \cdot 12\pi = 5\pi$

For major $\overset{\frown}{ACD}$:

$m\overset{\frown}{ACD} = 360 - 168 = 192$

Fraction: $\frac{192}{360} = \frac{8}{15}$

Circumference: $2\pi r = 2\pi \cdot 6 = 12\pi$

Multiply: arc length $= \frac{8}{15} \cdot 12\pi = \frac{32}{5}\pi$

Sector AOB is described by giving $m\angle AOB$ and the radius of $\odot O$. Make a sketch and find the length of $\overset{\frown}{AB}$.

	1.	2.	3.	4.	5.
$m\angle AOB$	90	240	300	120	108
radius	6	12	12	2.4	$10\sqrt{2}$

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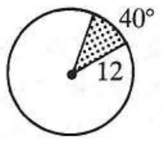
11–6 Arc Lengths and Areas of Sectors *(continued)*

Example 2

Find the area of the shaded sector.



Solution

Fraction: $\frac{40}{360} = \frac{1}{9}$

Area of circle: $\pi r^2 = \pi \cdot 12^2 = 144\pi$

Multiply: area of sector $= \frac{1}{9} \cdot 144\pi = 16\pi$

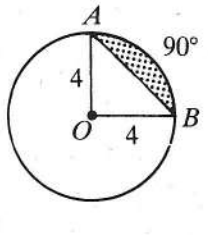
Sector *AOB* is described by giving $m\angle AOB$ and the radius of $\odot O$. Make a sketch and find the area of sector *AOB*.

	6.	7.	8.	9.	10.
$m\angle AOB$	30	240	330	75	108
radius	6	9	12	2.4	$5\sqrt{3}$

11. The area of sector *AOB* is 48π and $m\angle AOB = 270$. Find the radius of $\odot O$.
12. The area of sector *AOB* is $\frac{9}{4}\pi$ and $m\angle AOB = 40$. Find the radius of $\odot O$.

Example 3

Find the area of the shaded region.



Solution

First, find the area of sector *AOB*.

$\frac{90}{360} = \frac{1}{4}$ $\qquad \pi r^2 = 16\pi$

area of sector $AOB = \frac{1}{4} \cdot 16\pi = 4\pi$

Then, find the area of $\triangle AOB$.

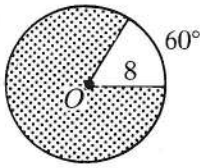
$A = \frac{1}{2}bh = \frac{1}{2} \cdot 4 \cdot 4 = 8$

Subtract to find the area of the shaded region.

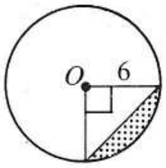
Area of sector AOB − area of $\triangle AOB = 4\pi - 8$

Find the area of the shaded region. Point *O* marks the center of each circle.

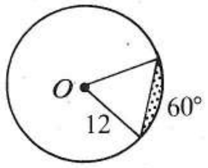
13.



14.



15.



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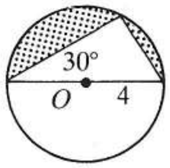

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11–7 Ratios of Areas

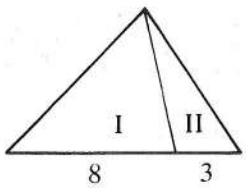
Objectives: Find the ratio of the areas of two triangles. Understand and apply the relationships between scale factors, perimeters, and areas of similar figures.

Comparing Areas of Triangles

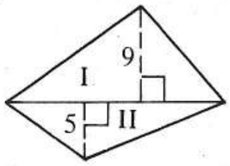
1. If two triangles have equal heights, then the ratio of their areas equals the ratio of their bases.
2. If two triangles have equal bases, then the ratio of their areas equals the ratio of their heights.
3. If two triangles are similar, then the ratio of their areas equals the square of their scale factor.

Example 1 Find the ratio of the area of $\triangle$I to the area of $\triangle$II.

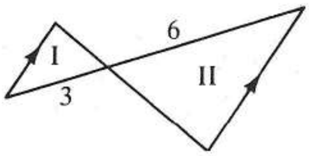
a.



b.



c.



Solution

a. The two triangles have the same height, so: ratio of areas = ratio of bases

$$\frac{\text{area of }\triangle\text{I}}{\text{area of }\triangle\text{II}} = \frac{8}{3}$$

b. The two triangles have the same base, so: ratio of areas = ratio of heights

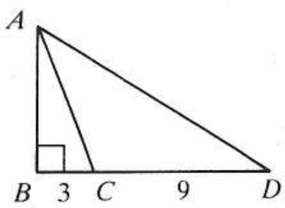
$$\frac{\text{area of }\triangle\text{I}}{\text{area of }\triangle\text{II}} = \frac{9}{5}$$

c. The two triangles are similar, so: ratio of areas = (scale factor)^2

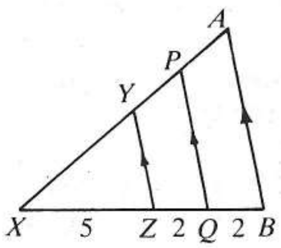
$$\frac{\text{area of }\triangle\text{I}}{\text{area of }\triangle\text{II}} = \left(\frac{3}{6}\right)^2 = \frac{1}{4}$$

Find the ratio of the areas of:

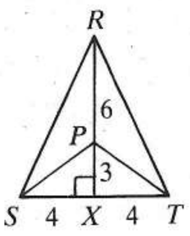
1. a. $\triangle ABD$ to $\triangle ADC$
b. $\triangle ABC$ to $\triangle ABD$



2. a. $\triangle XYZ$ to $\triangle XPQ$
b. $\triangle XAB$ to $\triangle XYZ$



3. a. $\triangle RST$ to $\triangle PST$
b. $\triangle XRT$ to $\triangle XPT$



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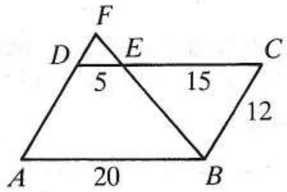
11–7 Ratios of Areas *(continued)*

***ABCD* is a parallelogram. Find each ratio.**



4. $\frac{\text{area of } \triangle DEF}{\text{area of } \triangle ABF}$

5. $\frac{\text{area of } \triangle DEF}{\text{area of } \triangle CEB}$

If the scale factor of two similar figures is $a:b$, then
(1) the ratio of the perimeters is $a:b$.
(2) the ratio of the areas is $a^2:b^2$.

Example 2

a. The scale factor of two similar figures is 3:5. Find the ratio of the perimeters and the ratio of the areas.

b. The ratio of the areas of two similar figures is 1:4. Find the ratio of their perimeters.

Solution

a. $a:b = 3:5$
ratio of perimeters $= a:b = 3:5$
ratio of areas $= a^2:b^2 = 9:25$

b. ratio of areas $= a^2:b^2 = 1:4$,
so $a:b = 1:2$.
ratio of perimeters $= a:b = 1:2$

The table refers to similar figures. Complete the table.

	6.	7.	8.	9.	10.	11.	12.
Scale factor	3:4	5:2	$x:7y$	?	?	?	?
Ratio of perimeters	?	?	?	8:7	2:13	?	?
Ratio of areas	?	?	?	?	?	36:1	4:9

13. The lengths of two similar rectangles are 3 m and 7 m, respectively. What is the ratio of their areas?
14. The longest sides of two similar trapezoids have lengths $4x$ and x^2, respectively. What is the ratio of their areas?
15. Two circles have radii 8 cm and 12 cm, respectively. Find the ratio of the circumferences and the ratio of the areas. (*Hint:* All circles are similar. The scale factor is the ratio of their radii.)
16. The areas of two circles are 25π and 100π.
    a. What is the ratio of their diameters?
    b. What is the ratio of their circumferences?
17. A hexagon with sides 5 cm, 6 cm, 7 cm, 8 cm, 9 cm, and 10 cm has area 123 cm^2. If a similar hexagon has longest side of 40 cm, what is its area?
18. Two similar triangles have corresponding sides with lengths 3 and 9. If the area of the first triangle is 12, what is the area of the second triangle?
19. Two similar triangles have areas 144 cm^2 and 169 cm^2. If a side of the larger triangle is 26 cm, find the length of the corresponding side of the smaller triangle.

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11–8 Geometric Probability

Objective: Use lengths and areas to solve problems involving geometric probability.

Probability is the chance or likelihood that an event will occur. A probability near one means an event is very likely to occur; a probability near zero means an event is very unlikely. Geometric probability uses lengths and areas to solve probability problems. This lesson will use two basic principles.

The first principle involves the lengths of segments.

Suppose a point P of $\overline{XY}$ is picked at random. Then:

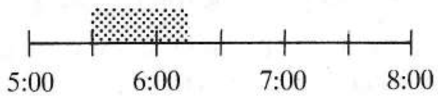
$$\text{probability that } P \text{ is on } \overline{XQ} = \frac{\text{length of } \overline{XQ}}{\text{length of } \overline{XY}}$$

Example 1

A friend promises to call you at home sometime between 5 P.M. and 8 P.M. At 5:30, you must leave your house unexpectedly for 45 minutes. What is the probability you miss the first call?

Solution

Think of a time line. The shaded segment represents the interval you are not at home. Between 5 P.M. and 8 P.M. you are not at home for three-quarters of an hour.



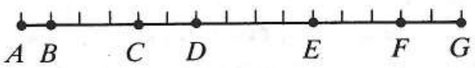
$$\text{probability you miss the call} = \frac{\text{length of shaded segment}}{\text{length of the whole segment}}$$

$$= \frac{\frac{3}{4}}{3} = \frac{3}{4} \cdot \frac{1}{3} = \frac{1}{4}$$

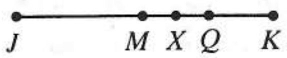
Solve.

1. A point X is picked at random on $\overline{AF}$. What is the probability that X is on:

   a. $\overline{AC}$? **b.** $\overline{CD}$? **c.** $\overline{CE}$?
   d. $\overline{DF}$? **e.** $\overline{AG}$? **f.** $\overline{FG}$?



2. M is the midpoint of $\overline{JK}$, Q is the midpoint of $\overline{MK}$, and X is the midpoint of $\overline{MQ}$. If a point on $\overline{JK}$ is picked at random, what is the probability that the point is on $\overline{MX}$?



3. Every 20 minutes a bus pulls up outside a busy department store and waits for five minutes while passengers get on and off. Then the bus leaves. If a person walks out of the department store at a random time, what is the probability that a bus is there?

4. A piece of rope 20 ft long is cut into two pieces at a random point. What is the probability that both pieces of rope will be at least 3 ft long?

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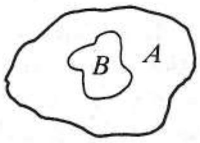
11–8 Geometric Probability *(continued)*

The second principle of geometric probability involves areas of regions.

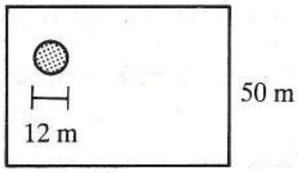
Suppose a point P of region A is picked at random. Then:

probability that P is in region B = $\dfrac{\text{area of region } B}{\text{area of region } A}$



Example 2

A parachutist jumps from an airplane and lands in the rectangular field shown. What is the probability that the parachutist avoids the tree represented by a circle in the diagram? (Assume that the person is unable to control the landing point.)



Solution

probability of avoiding the tree $= \dfrac{\text{area of rectangle} - \text{area of shaded region}}{\text{area of rectangle}}$

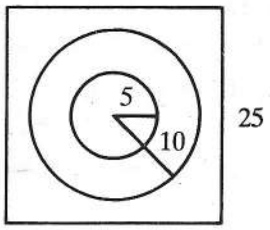
area of the rectangle $= 70 \cdot 50 = 3500$

area of shaded region $= \pi r^2 \approx 3.14 \cdot 6^2 = 113$

$\dfrac{\text{area of rectangle} - \text{area of shaded region}}{\text{area of rectangle}} = \dfrac{3500 - 113}{3500} \approx 0.97$

If a value of π is required in the following exercises, use $\pi \approx 3.14$.

5. A dart lands at a random point on the square dartboard shown.
   a. What is the probability that the dart lands within the larger circle?
   b. What is the probability that the dart lands within the smaller circle?



Exercise 5

6. A circular archery target has diameter 60 cm. Its bull's eye has diameter 10 cm.
   a. If an amateur shoots an arrow and the arrow hits a random point on the target, what is the probability that the arrow hits the bull's eye?
   b. After many shots, 100 arrows have hit the target. Estimate the number hitting the bull's eye.
7. A dart is thrown at a board 200 cm long and 157 cm wide. Attached to the board are 10 balloons, each with radius 12 cm. Assuming that each balloon lies entirely on the board, what is the probability that a dart that hits the board also hits a balloon?
8. A ship has sunk in the ocean in a square region 5 miles on a side. A salvage vessel anchors at a random spot in this square. Divers search half a mile in all directions from the point on the ocean floor directly below the vessel.
   a. What is the approximate probability that they locate the sunken ship at the first place they anchor?
   b. What is the approximate probability that they locate the sunken ship in five tries? (Assume that the tries do not overlap.)