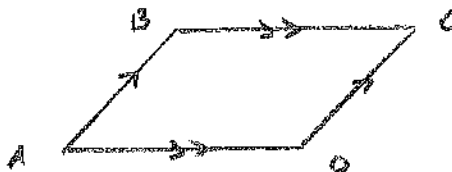


Parallelogram:

quadrilateral with both pairs of opposite sides parallel
($\square$)

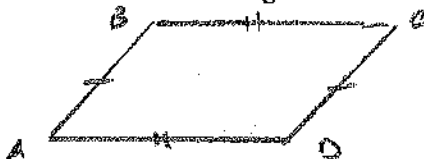


$$\overline{AB} \parallel \overline{CD}$$

$$\overline{BC} \parallel \overline{AD}$$

Opposite sides of a $\square$ are congruent

Sides of a Parallelogram

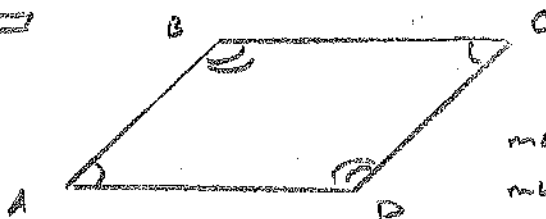


$$\overline{AB} \cong \overline{CD}$$

$$\overline{BC} \cong \overline{AD}$$

Opposite angles of a $\square$ are congruent

Angles of a Parallelogram



$$\angle A \cong \angle C$$

$$\angle B \cong \angle D$$

Consecutive angles of a $\square$ are supplementary

$$m\angle A + m\angle B = 180^\circ$$

$$m\angle B + m\angle C = 180^\circ$$

$$m\angle C + m\angle D = 180^\circ$$

$$m\angle D + m\angle A = 180^\circ$$

Diagonals of a $\square$ bisect each other

Diagonals of a Parallelogram



$$\overline{AE} \cong \overline{CE}$$

$$\overline{BE} \cong \overline{DE}$$

Algebra Find the values of x and y in $\square PQRS$.

14. $PT = 2x$, $TR = y + 4$, $QT = x + 2$, $TS = y$

15. $PT = x + 2$, $TR = y$, $QT = 2x$, $TS = y + 3$



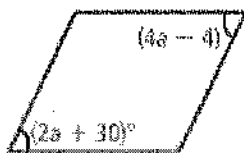
Solving a System by Substitution

$$\begin{aligned} \textcircled{14} \quad 2x &= y + 4 & |x+2| &= y \\ 2x &= (x+2) + 4 & \rightarrow 6+2 &= y \\ 2x &= x+2+4 & |8=y| \\ 2x &= x+6 & \\ -x &= -x & \\ \hline x &= 6 \end{aligned}$$

$$\begin{aligned} \textcircled{15} \quad |x+2| &= y & 2x &= y+3 \\ 5+2 &= y & 2x &= (x+2)+3 \\ 7 &= y & 2x &= x+2+3 \\ 2x &= x+5 & \\ -x &= -x & \\ \hline x &= 5 \end{aligned}$$

Algebra Find the value(s) of the variable(s) in each parallelogram.

26.



$$\begin{aligned} 4a - 4 &= 2a + 30 \\ -2a & -2a \\ \hline 2a - 4 &= 30 \\ +4 & +4 \\ \hline 2a &= 34 \\ \frac{2a}{2} &= \frac{34}{2} \\ a &= 17 \end{aligned}$$

$a = 17$

27.



$$\begin{aligned} 2x - 5 &= x + 7 \\ -x & -x \\ \hline x - 5 &= 7 \\ +5 & +5 \\ \hline x &= 12 \end{aligned}$$

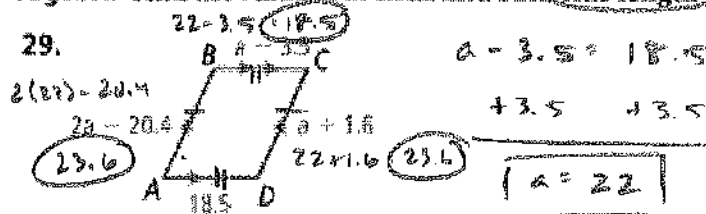
$x = 12$

$$\begin{aligned} 6y + 1 &= 4y + 9 \\ -4y & -4y \\ \hline 2y + 1 &= 9 \\ -1 & -1 \\ \hline 2y &= 8 \\ \frac{2y}{2} &= \frac{8}{2} \\ y &= 4 \end{aligned}$$

$y = 4$

Algebra Find the value of a . Then find each side length or angle measure.

29.



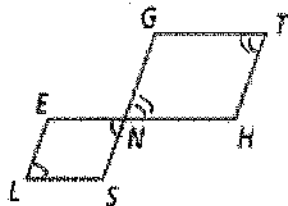
$$\begin{aligned} a - 3.5 &= 18.5 \\ +3.5 & +3.5 \\ \hline a &= 22 \end{aligned}$$

$a = 22$

Proof

33. Given: $\square LENS$ and $\square NGTH$

Prove: $\angle L \cong \angle T$



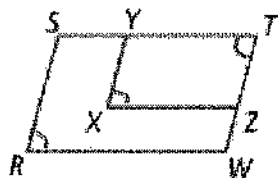
- ① $\square LENS$ and $\square NGTH$
- ② $\angle L \cong \angle ENS$
 $\angle T \cong \angle GNT$
- ③ $\angle ENS \cong \angle GNT$
- ④ $\angle L \cong \angle T$

- | S | R |
|--|---------------------------------------|
| ① $\square LENS$ and $\square NGTH$ | ① Given |
| ② $\angle L \cong \angle ENS$
$\angle T \cong \angle GNT$ | ② Opp. $\angle$'s of $\square \cong$ |
| ③ $\angle ENS \cong \angle GNT$ | ③ Vert. $\angle$'s Thm |
| ④ $\angle L \cong \angle T$ | ④ Transitive Prop |

Proof

36. Given: $\square RSTW$ and $\square XYTZ$

Prove: $\angle R \cong \angle X$

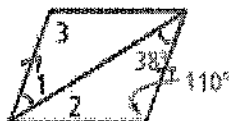


- ① $\square RSTW$ and $\square XYTZ$
- ② $\angle R \cong \angle T$
 $\angle T \cong \angle X$
- ③ $\angle R \cong \angle X$

- | S | R |
|--|---------------------------------------|
| ① $\square RSTW$ and $\square XYTZ$ | ① Given |
| ② $\angle R \cong \angle T$
$\angle T \cong \angle X$ | ② Opp. $\angle$'s of $\square \cong$ |
| ③ $\angle R \cong \angle X$ | ③ Transitive Prop |

Find the measures of the numbered angles for each parallelogram.

38.

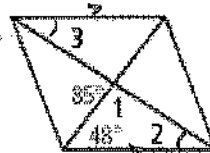


$$\begin{aligned} m\angle 1 &= 380 \\ (\text{Alt. Int. } \angle) \end{aligned}$$

$$\begin{aligned} (m\angle 3 &= 110^\circ) \\ (\text{Opp. } \angle \cong) \end{aligned}$$

$$\begin{aligned} 110 + 38 &= 148 \\ 180 - 148 &= 32 \\ (m\angle 2 &= 32^\circ) \end{aligned}$$

40.

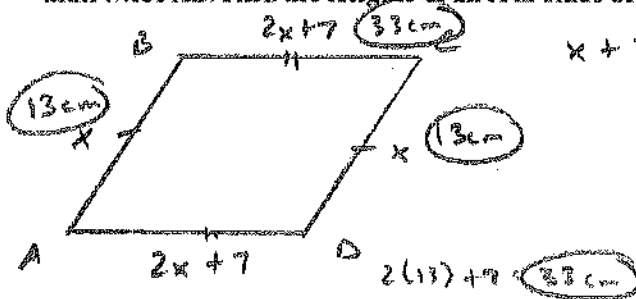


$$\begin{aligned} 180 - 85 &= 95 \\ (m\angle 1 &= 95^\circ) \end{aligned}$$

$$\begin{aligned} 95 + 48 &= 143 \\ 180 - 143 &= 37 \\ (m\angle 2 &= 37^\circ) \end{aligned}$$

$$\begin{aligned} (m\angle 3 &= 37^\circ) \\ (\text{Alt. Int. } \angle) \end{aligned}$$

41. Algebra The perimeter of $\square ABCD$ is 92 cm, AD is 7 cm more than twice AB . Find the lengths of all four sides of $\square ABCD$.



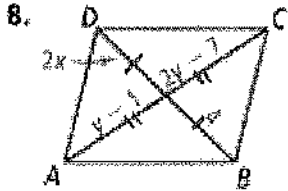
$$x + 2x + 7 + x + 2x + 7 = 92$$

$$6x + 14 = 92$$

$$\begin{array}{r} -14 \\ \hline 6x = 78 \end{array}$$

$$\frac{6x}{6} = \frac{78}{6} \rightarrow x = 13$$

Algebra For what values of x and y must $ABCD$ be a parallelogram?



$$\frac{2x}{2} = \frac{y}{2}$$

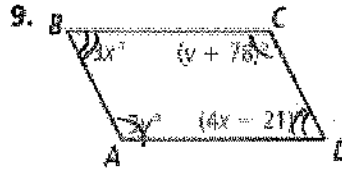
$$x = 2$$

$$y-1 = 2y-7$$

$$\begin{array}{r} -y \\ \hline -1 = y-7 \end{array}$$

$$+7 \quad +7$$

$$6 = y$$



$$3x = 4x - 21$$

$$\begin{array}{r} -4x \\ \hline -1x = -21 \end{array}$$

$$\begin{array}{r} -1 \\ \hline x = 21 \end{array}$$

$$x = 21$$

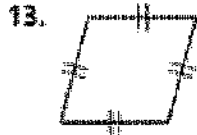
$$3y = y + 78$$

$$\begin{array}{r} -y \\ \hline 2y = 78 \end{array}$$

$$\frac{2y}{2} = \frac{78}{2}$$

$$y = 39$$

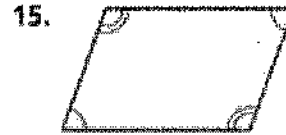
Can you prove that the quadrilateral is a parallelogram based on the given information? Explain.



Yes b/c both pairs of opposite sides are congruent

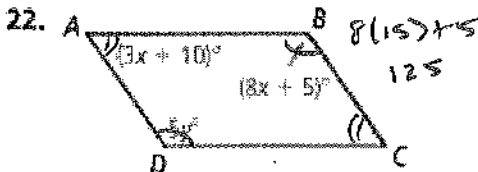


No b/c only one diagonal is bisected.



Yes b/c both pairs of opposite angles are congruent.

Algebra For what values of the variables must $ABCD$ be a parallelogram?



$$3x + 10 + 8x + 5 = 180$$

$$11x + 15 = 180$$

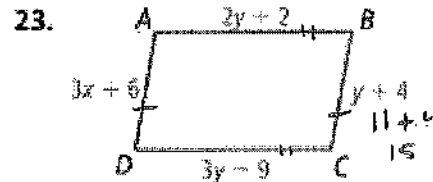
$$\begin{array}{r} -15 \\ \hline 11x = 165 \end{array}$$

$$\frac{11x}{11} = \frac{165}{11}$$

$$x = 15$$

$$\frac{5x}{5} = \frac{125}{5}$$

$$x = 25$$



$$3x + 6 = 15$$

$$\begin{array}{r} -6 \\ \hline 3x = 9 \end{array}$$

$$\frac{3x}{3} = \frac{9}{3}$$

$$x = 3$$

$$2y + 2 = 3y - 9$$

$$\begin{array}{r} -2y \\ \hline 2 = y - 9 \end{array}$$

$$+9 \quad +9$$

$$11 = y$$

$$y = 11$$

