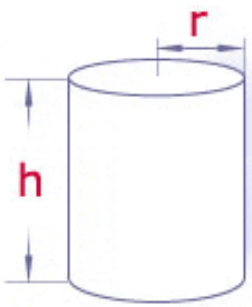
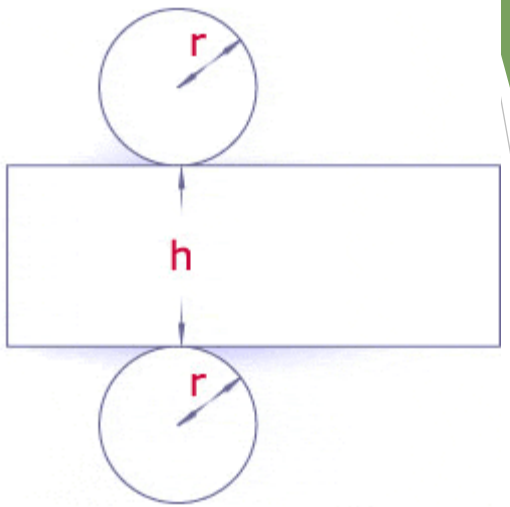
Volume of Non-Polyhedron solids

Cylinders





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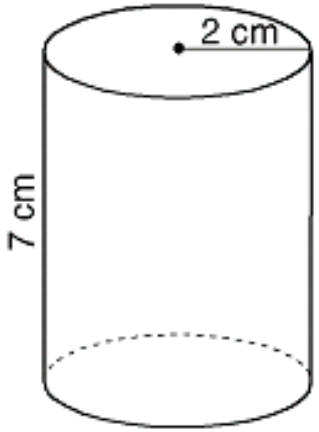
$$V = \pi r^2 h$$

Example #1: Calculate the volume of a cylinder

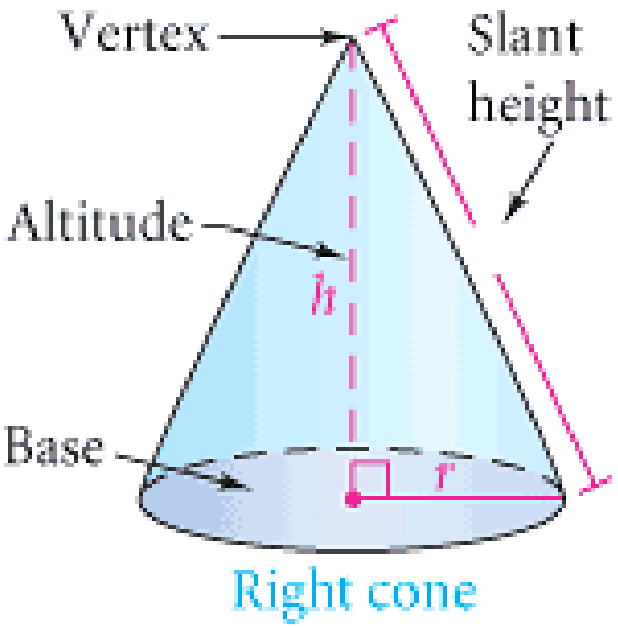
What is the volume of the cylinder shown below? [*Round your answer to the nearest cubic centimeter.*]

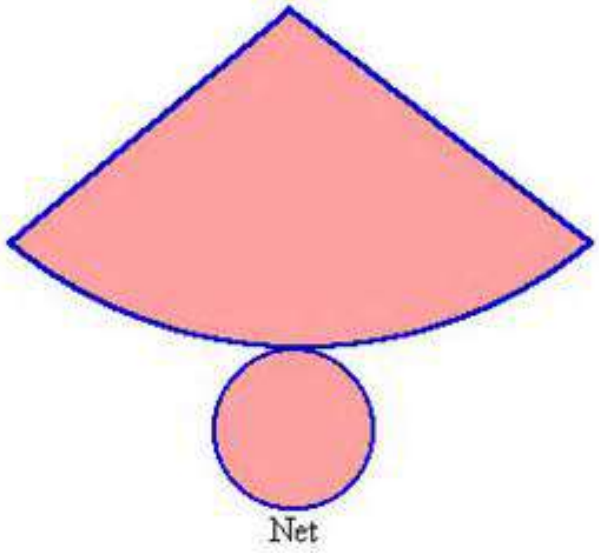
$$V = \pi\ r^2 h$$



$$V = \pi\ (2)^2 (7) = 28\pi \approx 88 cm^3$$

Cones





Example #1: Calculate the volume of a cone

Find the capacity of this cone to the nearest tenth of a cubic cm?

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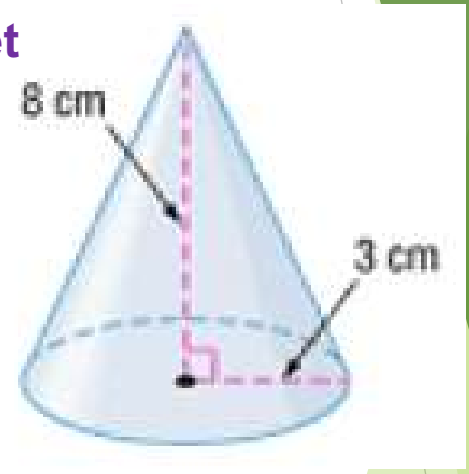


$$V = \frac{1}{3}\pi\ r^2 h$$

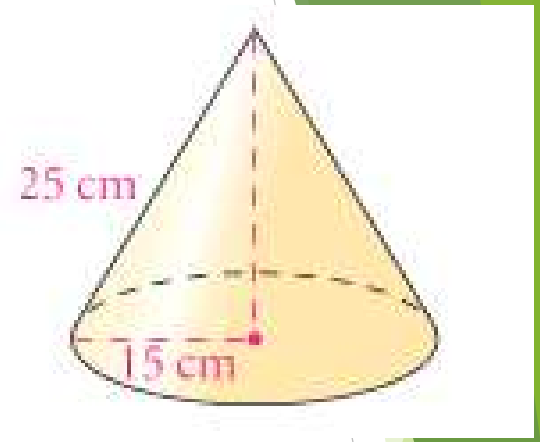
$$V = \frac{1}{3}\pi\ (3)^2(8)$$

$$V = 24\pi \approx 75.4cm^3$$

Example #2: Finding the volume of a cone given the slant height in terms of pi.

$$V = \frac{1}{3}\pi\, r^2 h$$

We have the radius is 15 cm,
but we DO NOT have the vertical height!



$$V = \frac{1}{3}\pi\, (15)^2 h$$

Oh, Pythagorean theorem!

$$25^2 = 15^2 + h^2$$

$$400 = h^2$$

$$20 = h$$

$$V = \frac{1}{3}\pi\, (15)^2 (20)$$

$$V = 1500\pi$$

Example #2: try this one!

If you rolled up an 8 1/2 by 11 in sheet of paper into the largest possible cylinder 8 1/2 in high, what would be the volume of the cylinder?

$$V = \pi\ r^2 h$$

$$V = \pi\ r^2 (8.5)$$

$$11 = \pi d$$

Need to find the diameter of the cylinder first!
Hmm… Oh we have the circumference of the circle!

C= 11 inches

$$\frac{11}{\pi} \approx 3.5$$

$$d \approx 3.5$$

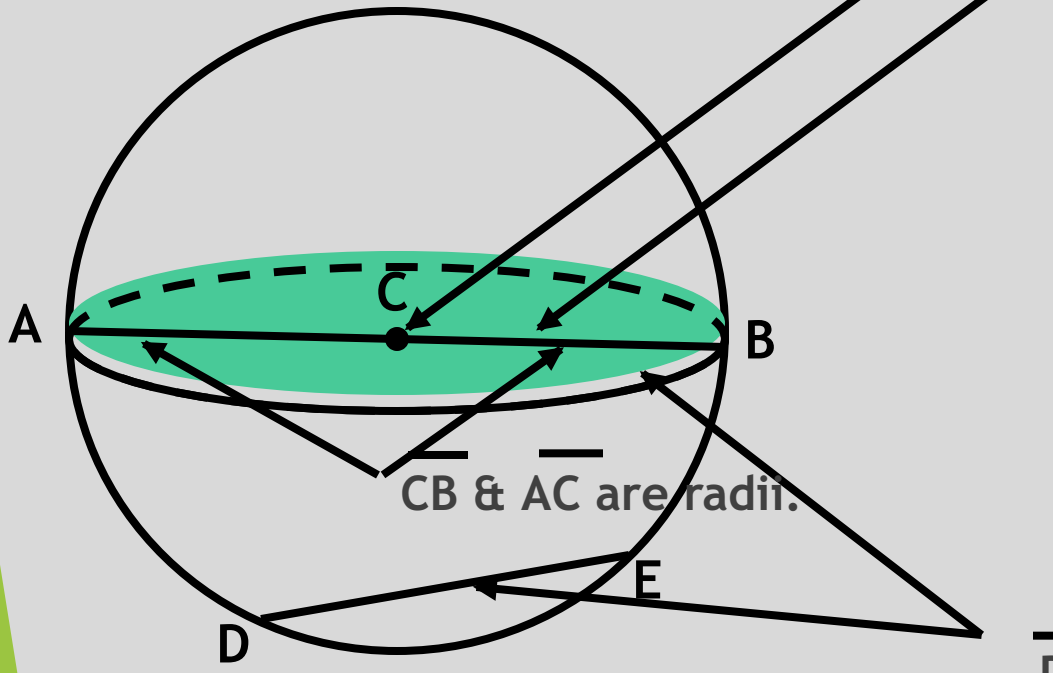
$$r \approx 1.75$$

$$V = \pi\ r^2 h$$

$$V = \pi\ (1.75)^2 (8.5)$$

$$V \approx 82\ \text{in}^3$$

Parts of a Sphere



C is the center of the sphere.

$\overline{AB}$ is a diameter.

$\overline{CB}$ & $\overline{AC}$ are radii.

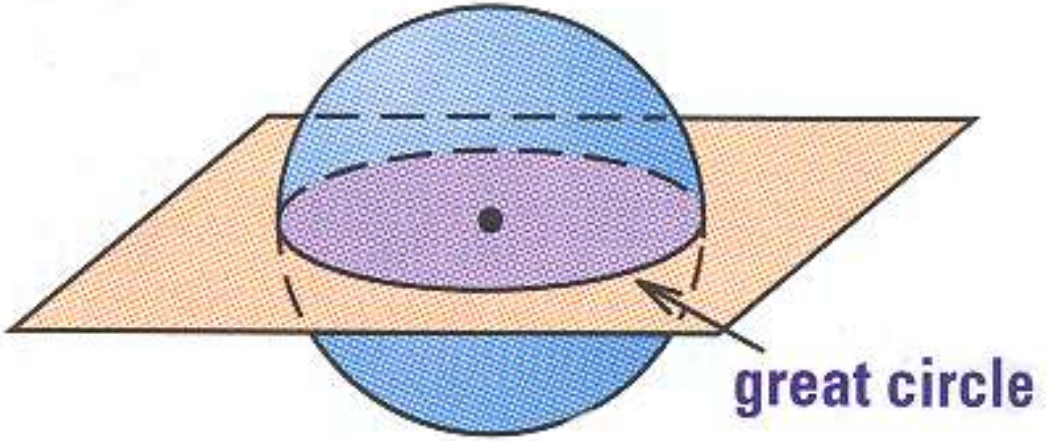
$\overline{DE}$ & $\overline{AB}$ are chords.

More Definitions

- Great Circle of a Sphere - the cross section of a sphere sliced by a plane through its center.
- Hemisphere - ½ of a sphere.

** Every great circle splits a sphere into 2 hemispheres.

More . . .

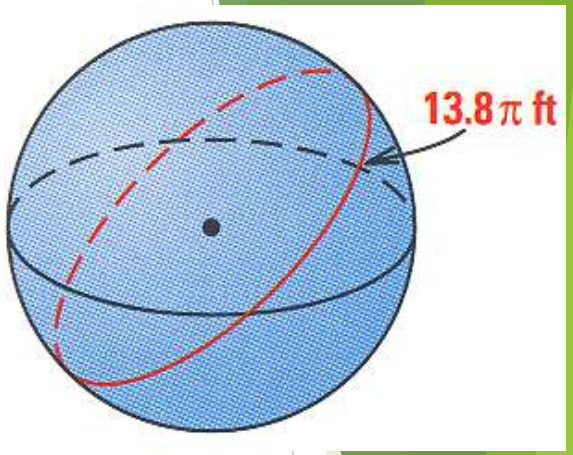


- If a plane intersects a sphere, the intersection is either a single point or a circle. If the plane contains the center of the sphere, then the intersection is a great circle of the sphere. Every great circle of a sphere separates a sphere into two congruent halves called hemispheres.

So the great circle is a Cross-section of the sphere!

Ex 2: Using the great circle

The circumference of a great circle of a sphere is 13.8π ft. What is the area of this cross-section of the sphere to the nearest tenth of a foot?



$$C = 2\pi r$$

$$13.8\pi = 2\pi r$$

$$13.8 = 2r$$

$$6.9 = r$$

$$A = \pi r^2$$

$$A = \pi(6.9)^2$$

$$A = \pi(47.61)$$

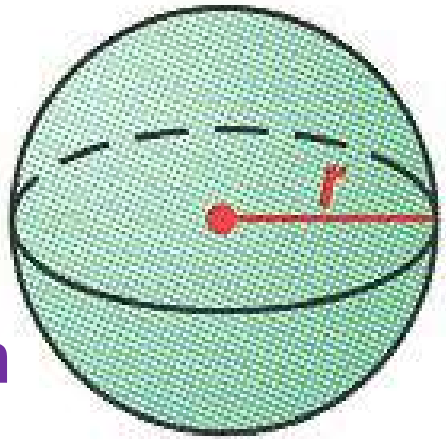
$$A = 47.61\pi \text{ ft}^2$$

Or about

149.6ft^2

Volume of a Sphere

- Finding the volume of a sphere with radius r



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$$V = \frac{4}{3}\pi r^3$$

Volume of a Sphere

Ex: Find the volume of a sphere with a radius of 3 ft in terms of pi and nearest tenth of a foot.

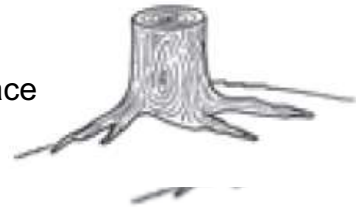
$$V = \frac{4}{3}\pi(3)^3 = \frac{4}{3}\pi(27)$$

V = 36π ft³ or 113.1 ft³

Cross sections

A section of a tree trunk is the shape of a cylinder as shown.

When a tree trunk is cut in order to see the tree rings, the plane surface of the trunk is called a ___Cross-section_____.



Cylinder

1. Suppose a plane intersects a cylinder parallel to its bases. What is the shape of the cross section? (figure 1)

   Circle with same area as its base

2. Suppose a plane intersects a cylinder perpendicular to its bases so that the plane passed through the centers of the bases. What is the shape of this cross section? (figure 2)

   Rectangle

3. Suppose a plane intersects a cylinder so that it is not parallel to its bases. What is the shape of this cross section? (figure 3)

   Ellipse(oval)

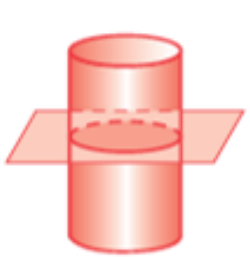


Figure 1

Figure 2

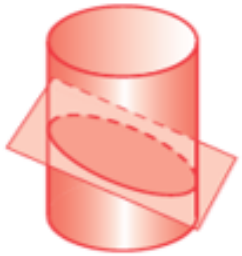
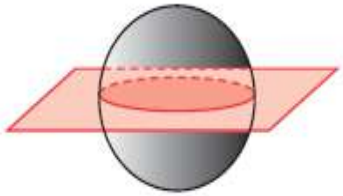


Figure 3

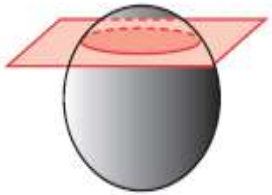
Sphere

Suppose a plane intersects a sphere.

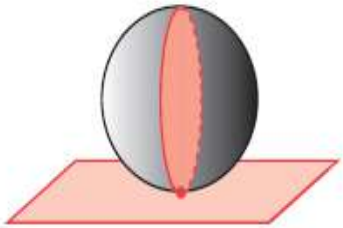
Describe the three different cross sections when a plane intersects a sphere.



Great circle –circle with largest area!



Circle

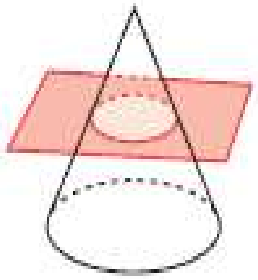


A single point

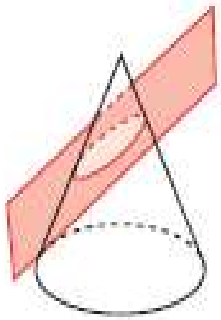
Cone

Suppose a plane intersects a cone.

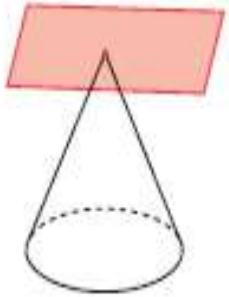
Describe the different cross sections when a plane intersects a cone.



Circle-smaller area than base



Ellipse(oval)



A single point

Summer is coming…

An ice cream cone is 11 cm deep and 5 cm across the opening of the cone. Two hemisphere-shaped scoops of ice cream, which also have diameters of 5 cm, are placed on top of the cone. If the ice cream were to melt into the cone, will it overflow?

Find the capacity of the cone:

$$\text{Volume(cone)} = \frac{1}{3}\pi\left(\frac{5}{2}\right)^2 \cdot (11)$$

$$\text{Volume(cone)} \approx 72$$

The volume the cone can hold is roughly 72 cm^3.

What solid resembles the ice cream scoop? A sphere

Find the volume of a sphere with the same radius:

$$\text{Volume(ice cream)} = \frac{4}{3}\pi\left(\frac{5}{2}\right)^3$$

$$\text{Volume(ice cream)} \approx 65.4$$

The volume of ice cream is roughly 65.4 cm^3.

The volume of the ice cream is significantly less, so no overflow here!!!!!

The end!

Why did the inches obey the yardstick?

He was their ruler!

Why were the similar triangles weighing themselves?

They were finding their scale!

Practice #1:
A tank in the form of a right circular cylinder is used for storing water. It has a diameter of 12 feet and a height of 14 feet. About how many gallons of water will it hold? (1 cubic foot contains 7.5 gallons).

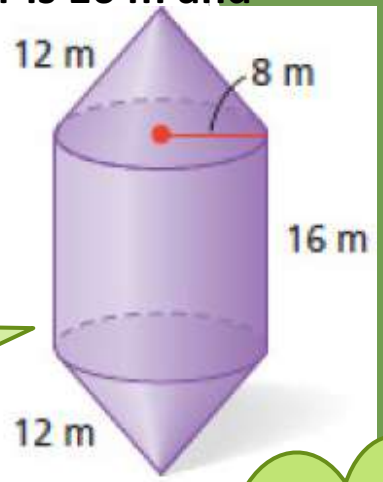
d=12 ft r=6 ft. h= 14 ft

$$V = \pi r^2 h = \pi(6)^2(14) = 1583.3626$$

1583.3626 cu. ft (7.5 gal /cu.ft)

11,875 gallons

Practice #2: Find the volume of the shape below where the radius of the cylinder is 8m, height of the cylinder is 16 m and the slant height of the cone is 12m. Round to the nearest whole cubic meter.



What are the 3D solids in the diagram?

2 cones and a cylinder

Leave in terms of pi for now!

Volume of cylinder= $\pi r^2 h$

$V=\pi 8^2 16 = 1024\pi$

Volume of the cone= $\frac{1}{3}\pi r^2 h$

Volume of the cone= $1/3\ \pi 8^2 h$

$8^2 + h^2 = 12^2$

$h \approx 8.944 \ldots$

Volume of the cone= $1/3\ \pi 8^2 (8.944..)$

$=190.8111\ldots\ \pi$

Oh no, they gave us the slant height! We need the vertical height!
So grateful for the Pythagorean theorem!

But wait there are 2 cones!

So multiply by 2

$(190.8111\ldots\ \pi)\ (2) = 381.622\ldots\ \pi$

Now let's add them together!

$$1024\pi + 381.622\ldots\ \pi$$

Total volume $\approx$4,416cu. meters

Practice #3: Fifty metal spheres with radii of 4m are melted and this melted solution is poured into rectangular prism with a base of 22 m by 60 m.

Find the height that the prism to nearest whole meter filled with the solution. Assume the metal solution will be filled to the top of the prism but you do not want it to overflow.

$V_{sphere} = \frac{4}{3}\pi r^3$

$V_{sphere} = \frac{4}{3}\pi(4^3)$

$V_{sphere} = 85\frac{1}{3}\pi$

$50 \text{ spheres} = 4266\frac{2}{3}\pi$

Vprism = Bh

V=(22)(60)h

V=1320h

4266 2/3 π =1320h

$\frac{4266\frac{2}{3}\pi}{1320} = \frac{1320h}{1320}$

10,1546... = h

So, we need a height of 11meters to ensure no overflow.

Study Lessons U8D2 and U8D3 for your quiz next block!

Want some practice for the quiz?!? Open this link ☺

Want more insider info on this quiz?
Play the water tank problem video and triangular prism and pyramid

Q: What do you get when you cross a pebble with a sphere?