Fun with Definite Integration
AP Calculus

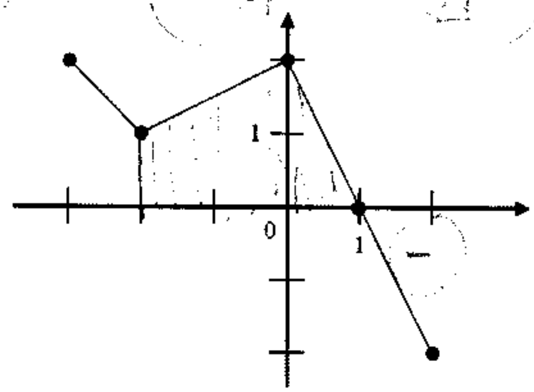
Name: Answers

1) Evalute: $\int_{\frac{\pi}{8}}^{\frac{\pi}{2}} \sin(2x) + \cos(2x)\, dx$ $= -\frac{1}{2}\cos(2x) + \frac{1}{2}\sin(2x) \Big|_{\frac{\pi}{8}}^{\frac{\pi}{2}}$

$= -\frac{1}{2}\cos\pi + \frac{1}{2}\sin\pi \Big) - \Big(-\frac{1}{2}\cos\frac{\pi}{4} + \frac{1}{2}\sin\frac{\pi}{4}\Big)$

$= \frac{1}{2} + 0 \Big) - \Big(-\frac{\sqrt{2}}{4} + \frac{\sqrt{2}}{4}\Big) = \boxed{\frac{1}{2}}$

2)



Graph of f

Add up area under curve

The graph of the piecewise linear function f is shown in the figure above. If $g(x) = \int_{-2}^{x} f(t)\,dt$, which of the following values is greatest?

(A) $g(-3)$ (B) $g(-2)$ (C) $g(0)$ (D) $g(1)$ (E) $g(2)$

3) If $\int_{-5}^{2} f(x)\,dx = -17$ and $\int_{5}^{2} f(x)\,dx = -4$, what is the value of $\int_{-5}^{5} f(x)\,dx$?

$\int_2^5 f(x) = 4$

(A) -21 (B) -13 (C) 0 (D) 13 (E) 21

$\int_{-5}^{2} f(x) + \int_{2}^{5} f(x) = -17 + 4$

4) $\int_3^5 \frac{x}{x^2-4}\,dx = \frac{1}{2}\int_5^{21} u^{-1}\,du = \frac{1}{2}\ln|u| \Big|_5^{21}$

$u = x^2 - 4$
$du = 2x\,dx$
$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\big(\ln(21) - \ln(5)\big) = \boxed{\frac{1}{2}\ln\left|\frac{21}{5}\right|}$

limits of integ. $3 \to (3)^2 - 4 = 5$
$5 \to (5)^2 - 4 = 21$

5) Given $\int_1^3 (2x+5)dx = 5\int_0^k (x+1)^4 dx$ find the exact value of k that solves the equation

(Leave answer in simplest radical form.)

$$x^2+5x\Big|_1^3 = 5\cdot\frac{1}{5}(x+1)^5\Big|_0^{k}$$

$$((9+15)-(1+5)) = ((k+1)^5-(0+1)^5)$$

$$18 = (K+1)^5-1$$

$$19 = (K+1)^5$$

$$\sqrt[5]{19} = K+1$$

$$\boxed{K = \sqrt[5]{19}-1}$$

6) $\int_0^{\frac{\pi}{2}} \cos(x)\sqrt{\sin(x)}\,dx$

$u = \sin x$

$du = \cos x\,dx$

limits of integ:

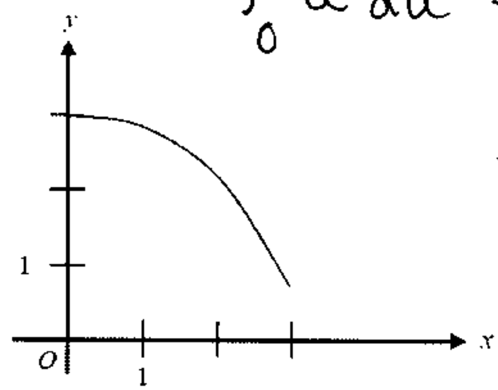
$0 \to \sin 0 \to 0$

$\frac{\pi}{2} \to \sin\frac{\pi}{2} \to 1$

$$\int_0^1 u^{\frac{1}{2}}du = \frac{2}{3}u^{\frac{3}{2}}\Big|_0^1$$

$$= \frac{2}{3}(1)^{3/2} - \frac{2}{3}(0)^{3/2}$$

$$= \boxed{\frac{2}{3}}$$



Graph of f

7)

10. The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_1^3 f(x)\,dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(C) Right Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length

(E) Trapezoidal sum approximation of $\int_1^3 f(x)\,dx$ with 4 subintervals of equal length