Fun with Binomial Expansion, Permutations, and Combinations Name: Answers

1. Expand the following binomials:

a) $(x+2)^4 = 1x^4 + 4x^3(2) + 6x^2(2)^2 + 4x(2)^3 + 1(2)^4$ ← use the 4th row to find coefficients

$= x^4 + 8x^3 + 24x^2 + 32x + 16$

b) $(3x-y^2)^5 = 1(3x)^5 + 5(3x)^4(-y^2) + 10(3x)^3(-y^2)^2 + 10(3x)^2(-y^2)^3 + 5(3x)(-y^2)^4 + (-y^2)^5$

$= 243x^5 - 405x^4y^2 + 270x^3y^4 - 90x^2y^6 + 15xy^8 - y^{10}$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36

2. Find the 3rd term of $(3x-7y)^9$.

$\_\_ + \_\_ + 36(3x)^7(-7y)^2 = 3,857,868x^7y^2$

3. Determine whether each of the following situations would require calculating a permutation or a combination. Then answer the question. How many ways could you:

   a. select a lead and an understudy for a school play from 30 people who auditioned

   permutation (order matters) $_{30}P_2$

   b. assign 19 students to their seats on the first day of school

   permutation (order matters) $_{19}P_{19}$

   c. select three students and two teachers to represent the school at an Honor Society conference in Washington, D.C. from a total of 20 students and 6 teachers.

   Combination $_{20}C_3 \cdot {}_6C_2$

~~4. How many 5 letter arrangements are there in the name RUSSOLELLO?~~

Skip this question!

5. A teacher is making a multiple-choice quiz. She wants to give each student the same questions but have each student's questions in a different order. If there are twenty-seven students in the class, what is the least number of questions the quiz must contain?

Let's try some! If there were 5 questions, we could arrange them in $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$ ways. (or $_5P_5$)

If there were 4 questions, we could arrange them in $4 \cdot 3 \cdot 2 \cdot 1 = 24$ ways (or $_4P_4$)

So I guess we need at least 5 questions.

6. The local Family Restaurant has a daily breakfast special in which the customer may choose one item from each of the following groups:

Breakfast Sandwiches	Accompaniments	Juice
egg and ham	breakfast potatoes	orange
egg and bacon	apple slices	cranberry
egg and cheese	fresh fruit cup	tomato
3 choices	pastry 4 choices	apple grape 5 choices

a. How many different breakfast specials are possible?

$3 \cdot 4 \cdot 5 =$ 60 meals

b. How many different breakfast specials without meat are possible?

$1 \cdot 4 \cdot 5 =$ 20 meals

7. How many three-digit numbers can be formed from the digits 0,1,2, …, 9?

$9 \cdot 10 \cdot 10 =$ 900 Numbers

(you can't start a 3 digit # with zero)

you can use 1-9 (1st digit) · 0-9 (2nd digit) · 0-9 (3rd digit)

How many of these numbers would be multiples of five?

Multiples of 5 end in 0 or 5, so you have two choices for the last digit

$\frac{9}{1\text{st digit}\ 1\text{-}9} \cdot \frac{10}{2\text{nd digit}\ 0\text{-}9} \cdot \frac{2}{3\text{rd digit}} =$ 180 numbers

8. There are fourteen juniors and twenty-three seniors in the Service Club. The club is to send four representatives to the State Conference.

a. How many different ways are there to select a group of four students to attend the conference?

$23 + 14 = 37$ students $= 37$

$_{37}C_4 =$ 66,045 ways

b. If the members of the club decide to send two juniors and two seniors, how many different groupings are possible?

$_{14}C_2 \cdot {}_{23}C_2 =$ 23,023 ways