Fraction Division

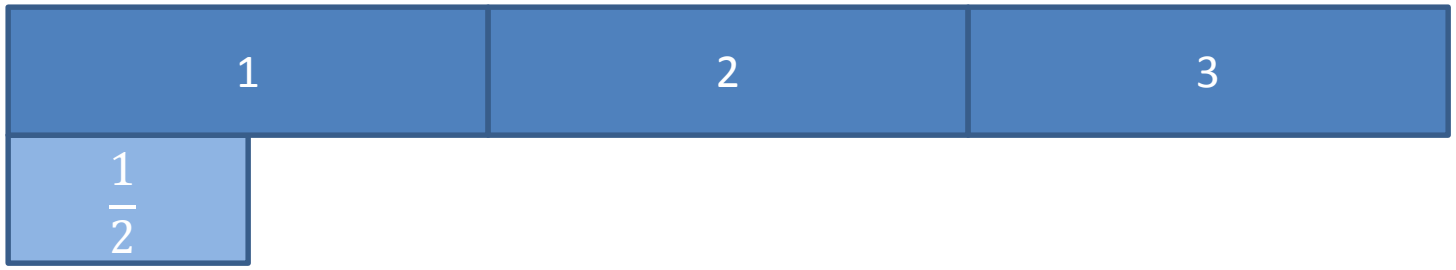
Using Fraction Strips

Brian Stockus

Whole Number Divided by a Unit Fraction (1 of 2)

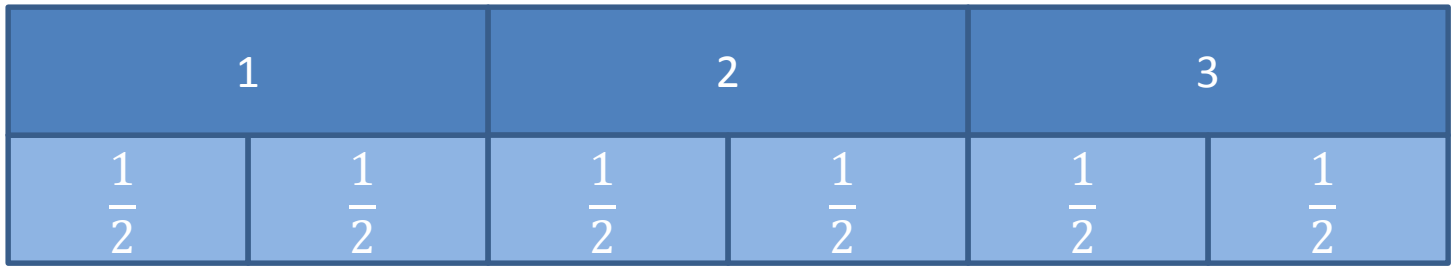
Example: $3 \div \frac{1}{2}$

When dividing by a fraction, I find it helpful to think to myself a question like, "How many one-halves are there in 3 wholes?



Whole Number Divided by a Unit Fraction (2 of 2)

Example: $3 \div \frac{1}{2}$



There are 6 one-halves in 3 wholes so, $3 \div \frac{1}{2} = 6$.

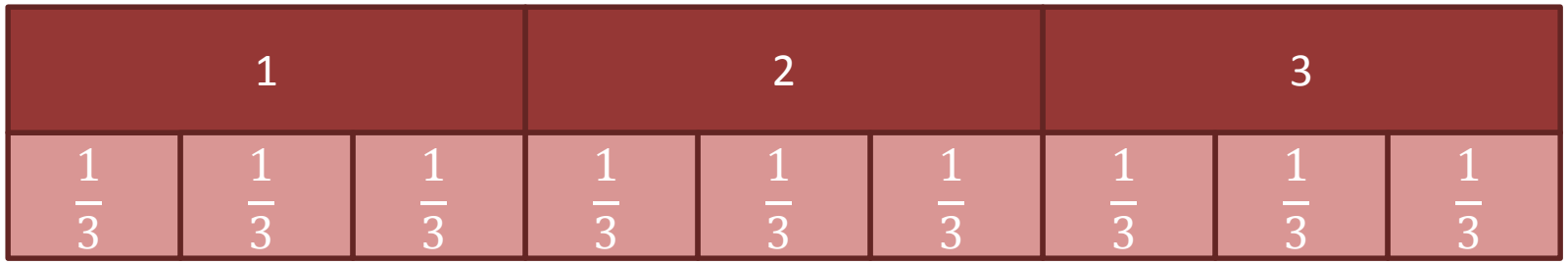
Whole Number Divided by a Unit Fraction (1 of 2)

Example: $3 \div \frac{1}{3}$

“How many one-thirds are there in 3 wholes?

Whole Number Divided by a Unit Fraction (2 of 2)

Example: $3 \div \frac{1}{3}$

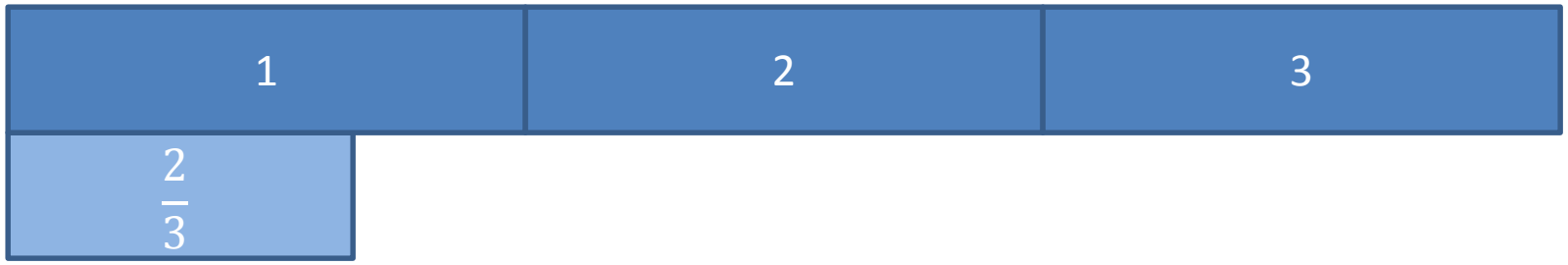


There are 9 one-thirds in 3 wholes so, $3 \div \frac{1}{3} = 9$.

Whole Number Divided by a Non-Unit Fraction (1 of 6)

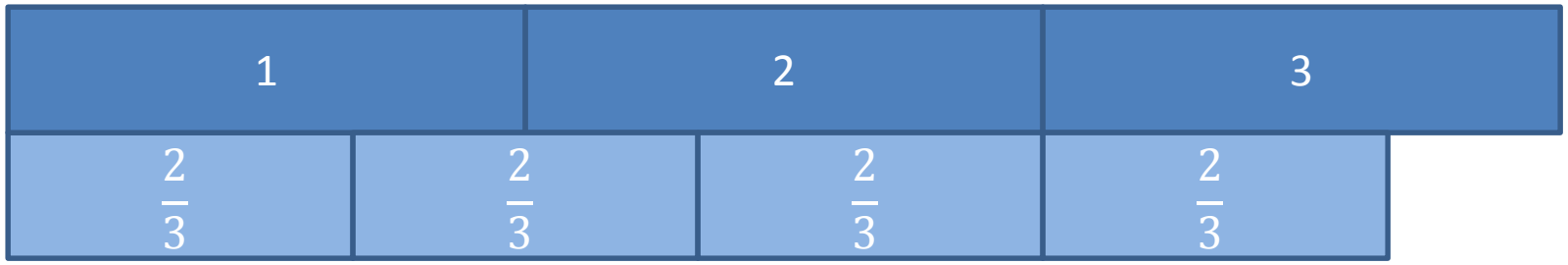
Example: $3 \div \frac{2}{3}$

My measurement division question stays the same, “How many two-thirds are there in 3 wholes?



Whole Number Divided by a Non-Unit Fraction (2 of 6)

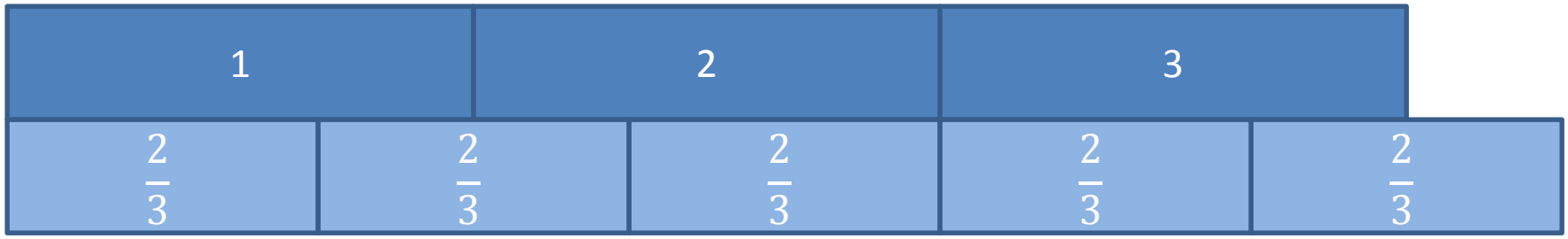
Example: $3 \div \frac{2}{3}$



Hmm, if I stop at 4 two-thirds there is some extra space.

Whole Number Divided by a Non-Unit Fraction (3 of 6)

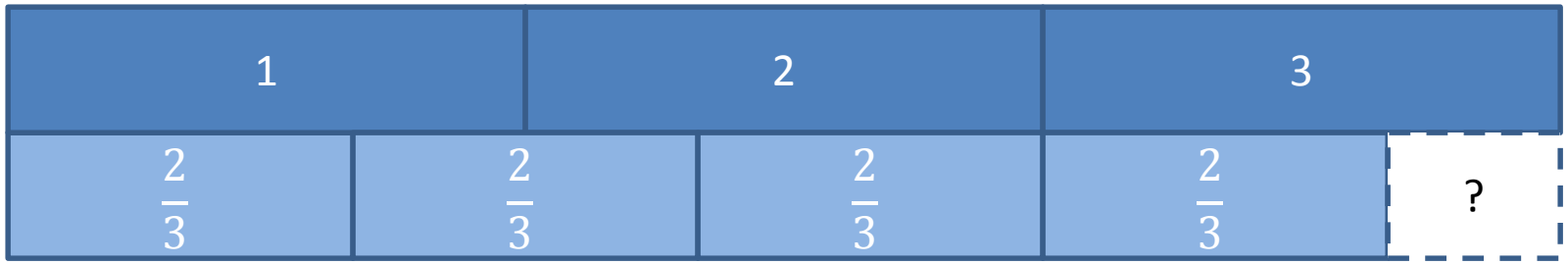
Example: $3 \div \frac{2}{3}$



But if I use 5 two-thirds, then I go past 3 wholes.

Whole Number Divided by a Non-Unit Fraction (4 of 6)

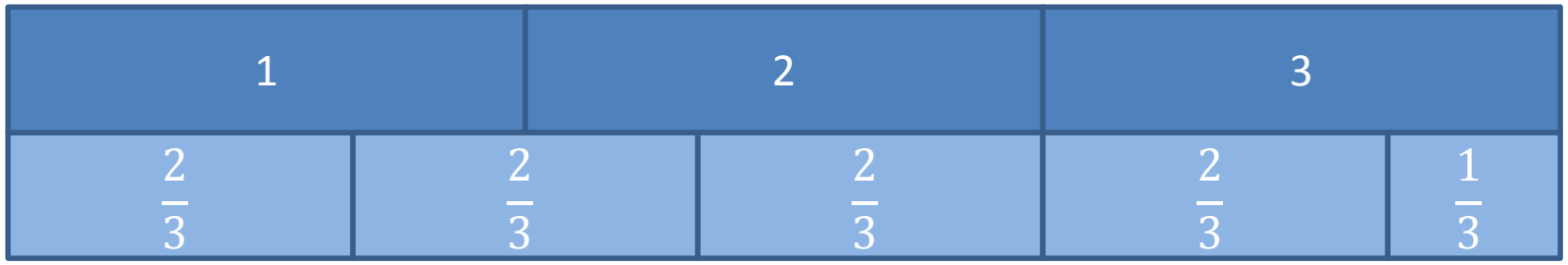
Example: $3 \div \frac{2}{3}$



That means there are more than 4 two-thirds in 3 wholes, but less than 5 two-thirds. That means $4 < (3 \div \frac{2}{3}) < 5$.

Whole Number Divided by a Non-Unit Fraction (5 of 6)

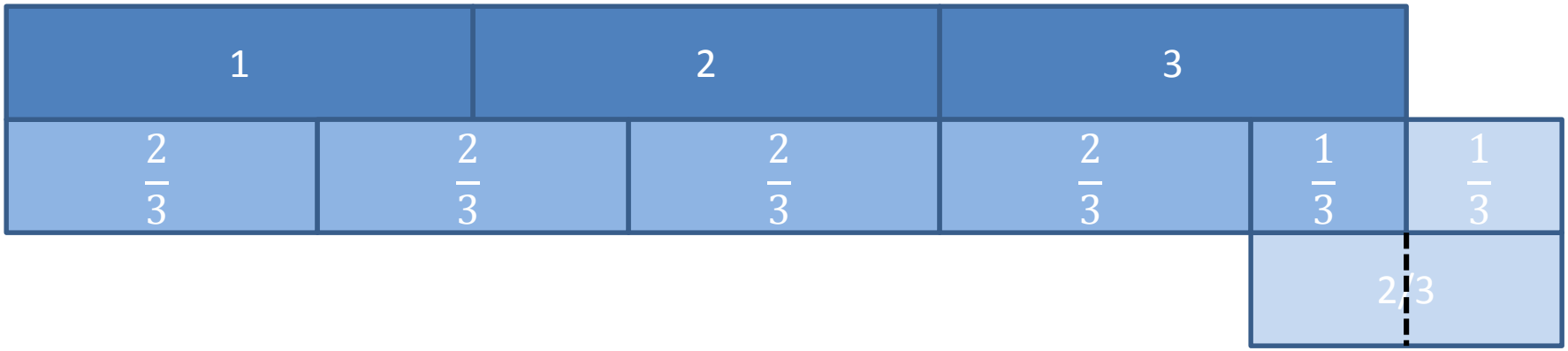
Example: $3 \div \frac{2}{3}$



If I think about it, in order to complete the third whole I need a one-third piece ($\frac{2}{3} + \frac{1}{3} = 1$). The question I have to answer is, what fraction of a two-third piece am I using?

Whole Number Divided by a Non-Unit Fraction (6 of 6)

Example: $3 \div \frac{2}{3}$



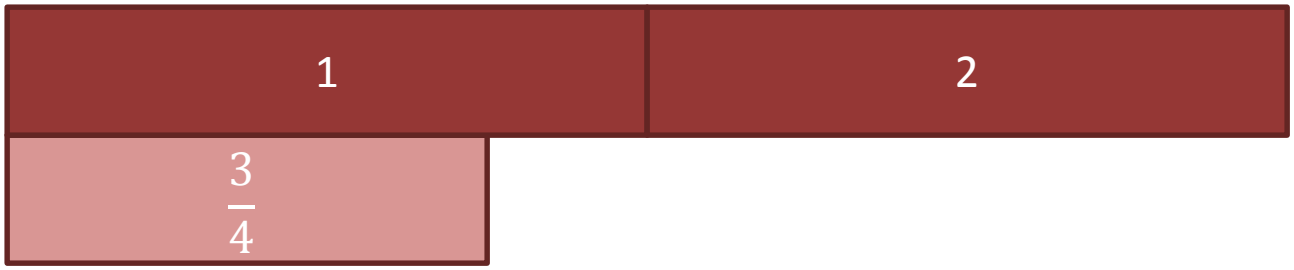
One-third is half of two-thirds, so I need half of a two-thirds piece. So altogether there are 4 two-thirds pieces and $\frac{1}{2}$ of a two-thirds piece in 3 wholes. That means $3 \div \frac{2}{3} = 4\frac{1}{2}$.

This is really the trickiest part of dividing by a fraction, understanding that you need a fraction of a piece which can result in a fraction you might not be expecting to see in the answer. In this case I divided by $\frac{2}{3}$, but ended up with $\frac{1}{2}$ in my answer because I needed half of a $\frac{2}{3}$-size piece.

Whole Number Divided by a Non-Unit Fraction (1 of 6)

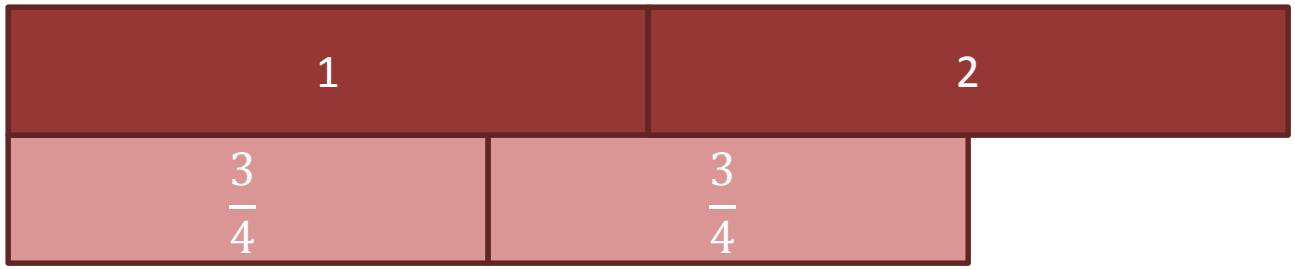
Example: $2 \div \frac{3}{4}$

My measurement division question stays the same, “How many three-fourths are there in 2 wholes?



Whole Number Divided by a Non-Unit Fraction (2 of 6)

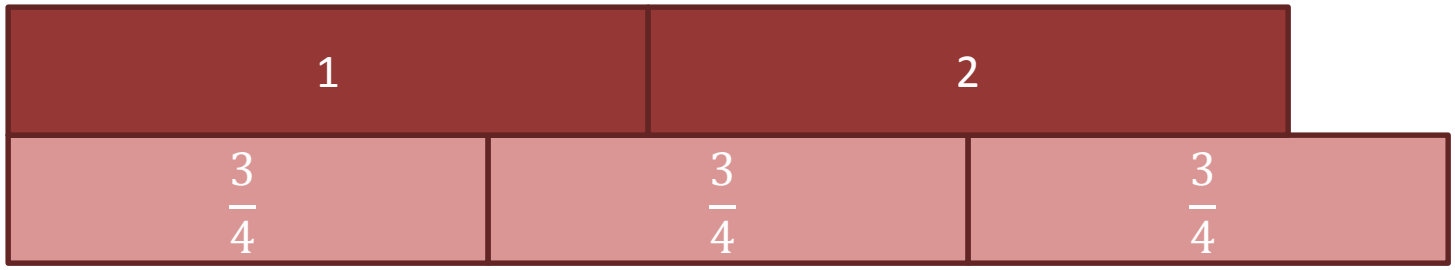
Example: $2 \div \frac{3}{4}$



Hmm, if I stop at 2 three-fourths there is some extra space.

Whole Number Divided by a Non-Unit Fraction (3 of 6)

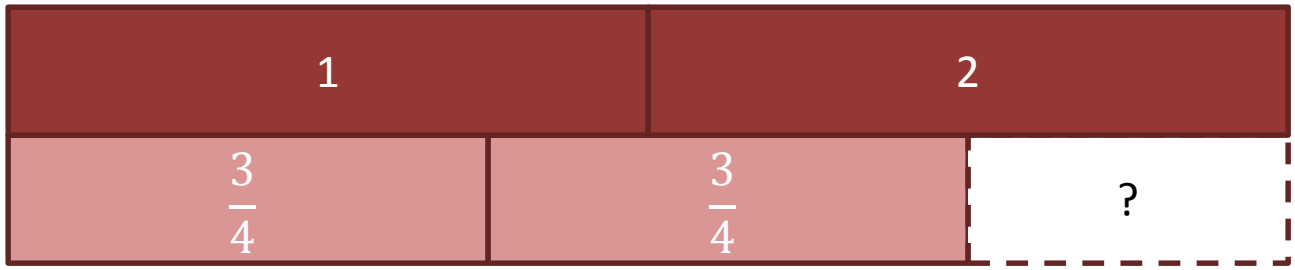
Example: $2 \div \frac{3}{4}$



But if I use 3 three-fourths, then I go past 2 wholes.

Whole Number Divided by a Non-Unit Fraction (4 of 6)

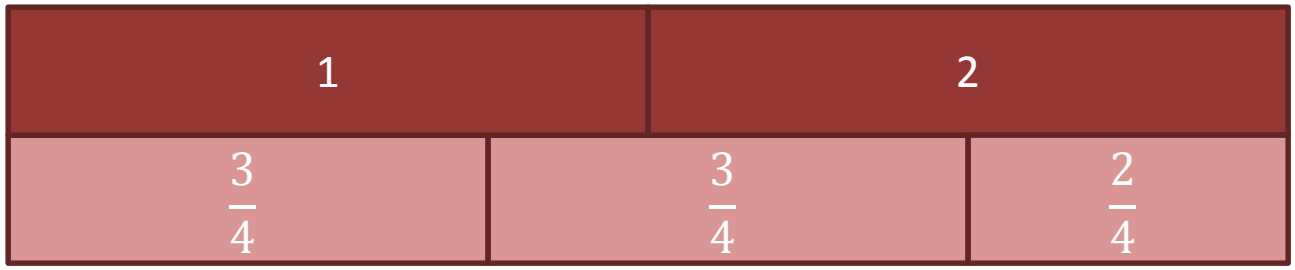
Example: $2 \div \frac{3}{4}$



That means there are more than 2 three-fourths in 2 wholes, but less than 3 three-fourths. That means $2 < (2 \div \frac{3}{4}) < 3$.

Whole Number Divided by a Non-Unit Fraction (5 of 6)

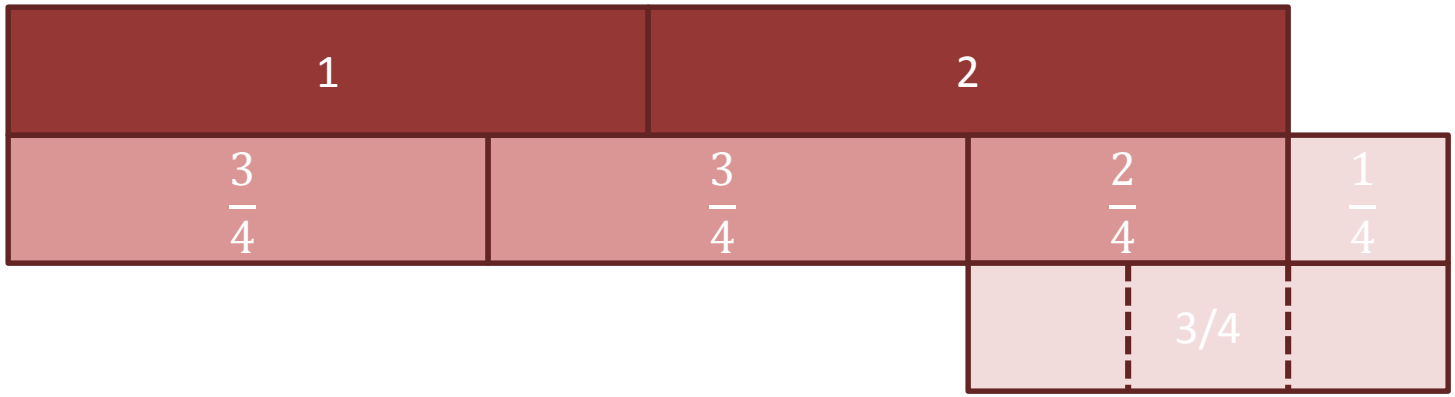
Example: $2 \div \frac{3}{4}$



If I think about it, so far I have used 2 three-fourths, which is equal to $1\frac{2}{4}$ ($\frac{3}{4} + \frac{3}{4} = 1\frac{2}{4}$) . In order to complete the second whole I need a two-fourths piece ($1\frac{2}{4} + \frac{2}{4} = 2$). The question I have to answer is, what fraction of a three-fourths piece am I using?

Whole Number Divided by a Non-Unit Fraction (6 of 6)

Example: $2 \div \frac{3}{4}$



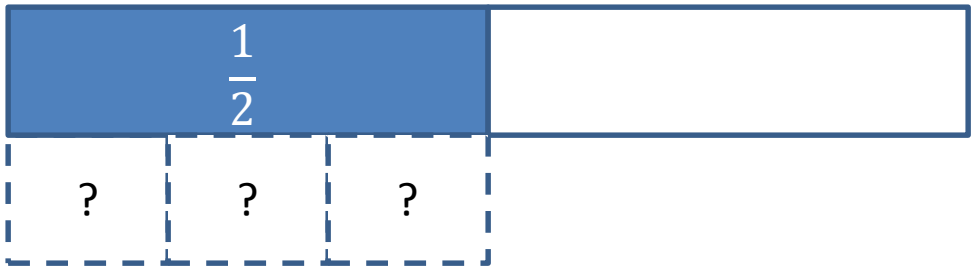
Two-fourths is $\frac{2}{3}$ of three-fourths, so I need $\frac{2}{3}$ of a three-fourths piece. So altogether there are 2 three-fourths pieces and $\frac{2}{3}$ of a three-fourths piece in 2 wholes. That means $2 \div \frac{3}{4} = 2\frac{2}{3}$.

Again, you needed a fraction of a piece which results in a fraction you might not be expecting to see in the answer. In this case I divided by $\frac{3}{4}$, but ended up with $\frac{2}{3}$ in my answer because I needed $\frac{2}{3}$ of a $\frac{3}{4}$-size piece.

Unit Fraction Divided by a Whole Number (1 of 3)

Example: $\frac{1}{2} \div 3$

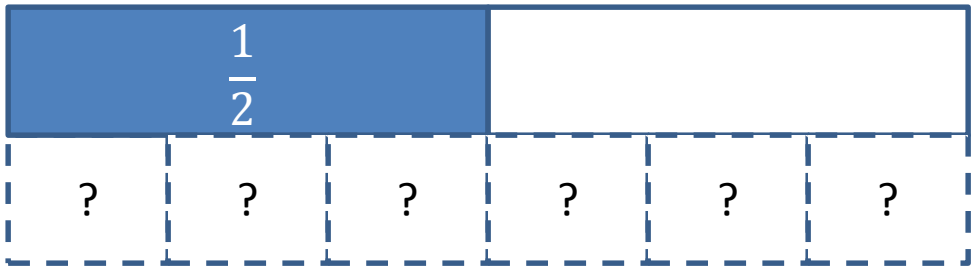
When dividing by a whole number, I tend to think a bit differently, “How large will each part be if I divide this fraction into 3 equal parts?”



When I divide $\frac{1}{2}$ into three equal parts, I need to consider the whole to help me understand what is happening.

Unit Fraction Divided by a Whole Number (2 of 3)

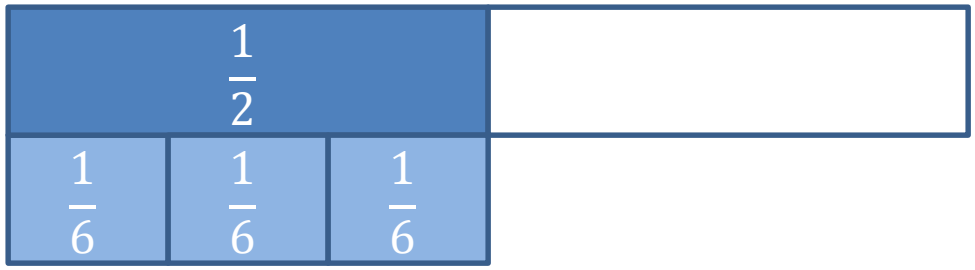
Example: $\frac{1}{2} \div 3$



It would take 6 of these equal parts to make a whole, so what I've done is divide the whole into sixths.

Unit Fraction Divided by a Whole Number (3 of 3)

Example: $\frac{1}{2} \div 3$

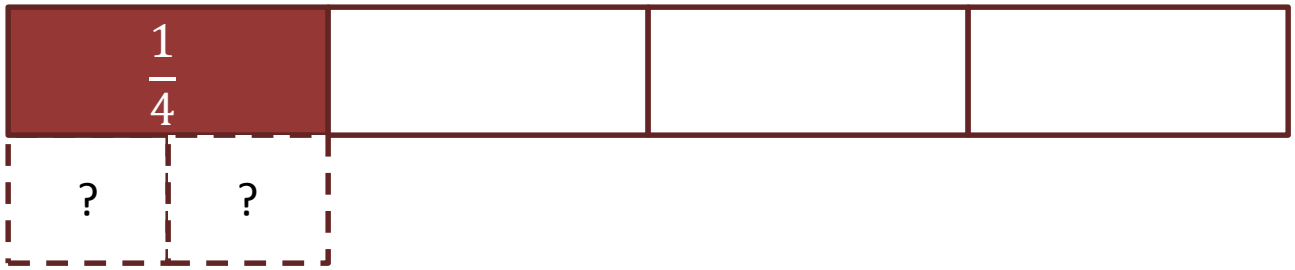


That means the size of each piece is $\frac{1}{6}$, so $\frac{1}{2} \div 3 = \frac{1}{6}$.

Unit Fraction Divided by a Whole Number (1 of 3)

Example: $\frac{1}{4} \div 2$

"How large will each part be if I divide this fraction into 2 equal parts?"

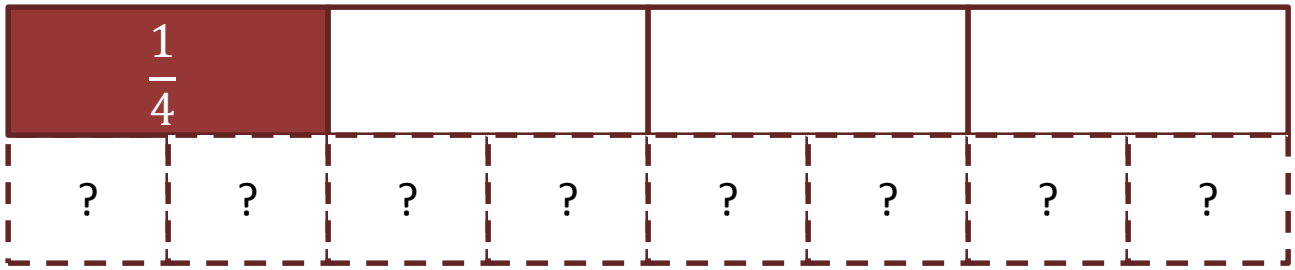


When I divide $\frac{1}{4}$ into two equal parts, I need to consider the whole to help me understand what is happening.

Unit Fraction Divided by a Whole Number (2 of 3)

Example: $\frac{1}{4} \div 2$

"How large will each part be if I divide this fraction into 2 equal parts?"

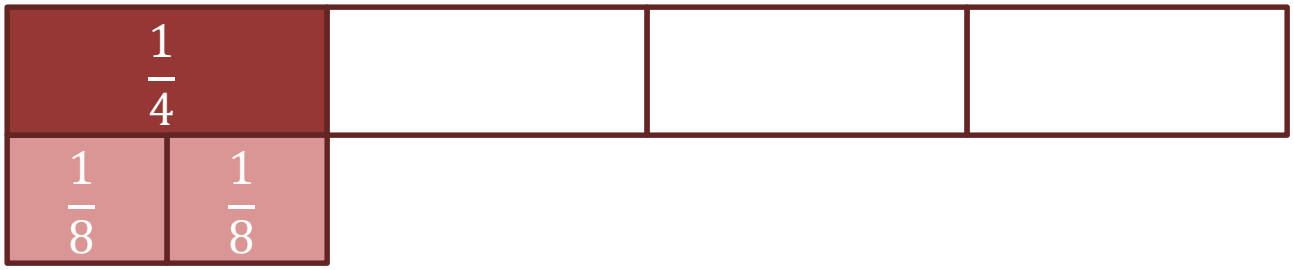


It would take 8 of these equal parts to make a whole, so what I've done is divide the whole into eighths.

Unit Fraction Divided by a Whole Number (3 of 3)

Example: $\frac{1}{4} \div 2$

"How large will each part be if I divide this fraction into 2 equal parts?"



That means the size of each piece is $\frac{1}{8}$, so $\frac{1}{4} \div 2 = \frac{1}{8}$.

Unit Fraction Divided by a Whole Number (1 of 2)

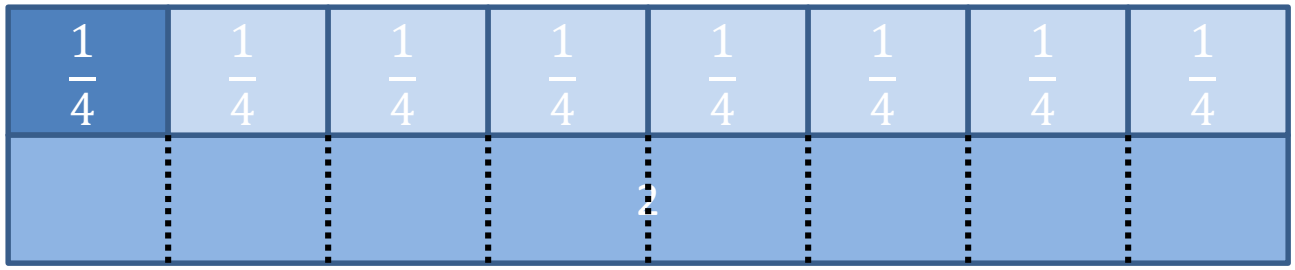
Example: $\frac{1}{4} \div 2$

Another way to think of it is "How many 2-whole size pieces can I get out of a piece that is $\frac{1}{4}$ of a whole?"

The question is somewhat silly because clearly I can't even get a 2-whole size piece one time without going over. I'm going to get some fraction of a 2-whole size piece. That means $(\frac{1}{4} \div 2) < 1$.

Unit Fraction Divided by a Whole Number (2 of 2)

Example: $\frac{1}{4} \div 2$

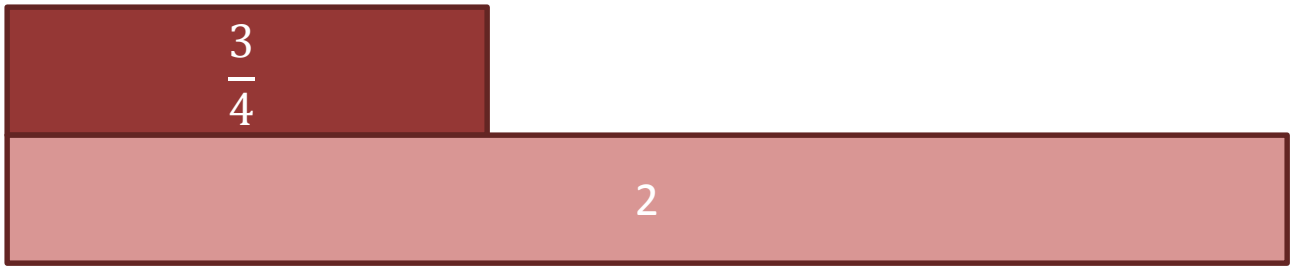


One-fourth is $\frac{1}{8}$ of 2 wholes, so I have $\frac{1}{8}$ of a 2-whole size piece. That means $\frac{1}{4} \div 2 = \frac{1}{8}$.

Non-Unit Fraction Divided by a Whole Number (1 of 2)

Example: $\frac{3}{4} \div 2$

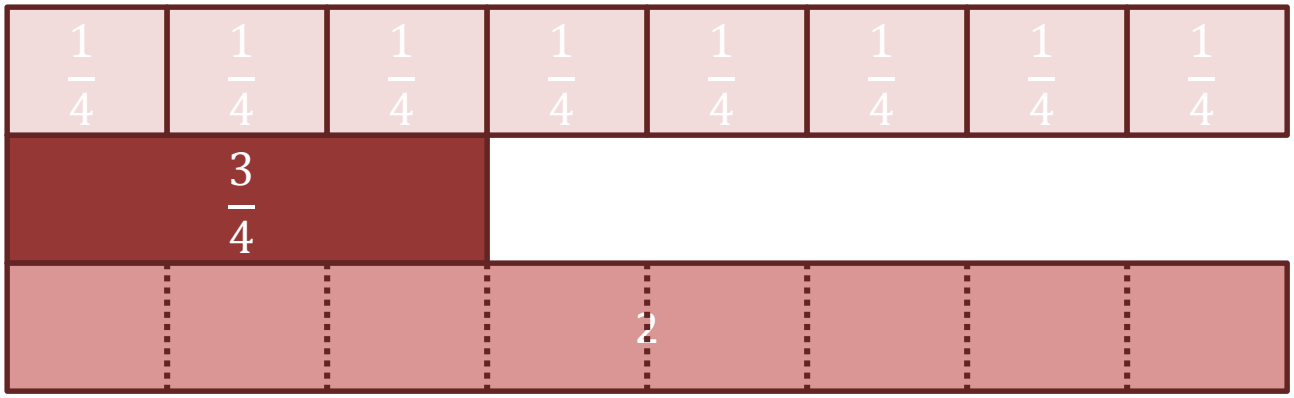
"How many 2-whole size pieces can I get out of a piece that is $\frac{3}{4}$ of a whole?"



If I try to measure out 2 wholes, it immediately goes too far. I can't even do it one time. That means $(\frac{3}{4} \div 2) < 1$.

Non-Unit Fraction Divided by a Whole Number (2 of 2)

Example: $\frac{3}{4} \div 2$

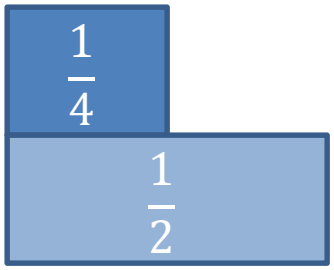


I know one-fourth is $\frac{1}{8}$ of 2 wholes, so I have $\frac{3}{8}$ of a 2-whole size piece. That means $\frac{3}{4} \div 2 = \frac{3}{8}$.

Fraction Divided by a Fraction (1 of 2)

Example: $\frac{1}{4} \div \frac{1}{2}$

"How many $\frac{1}{2}$-size pieces can I get out of a piece that is $\frac{1}{4}$ of a whole?"



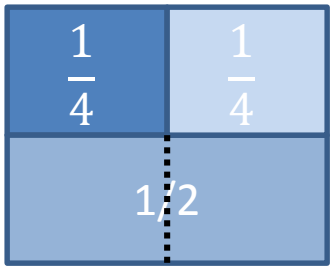
If I try to measure out $\frac{1}{2}$, it immediately goes too far. I can't even do it one time.

That means $(\frac{1}{4} \div \frac{1}{2}) < 1$.

Fraction Divided by a Fraction (1 of 2)

Example: $\frac{1}{4} \div \frac{1}{2}$

"How many $\frac{1}{2}$-size pieces can I get out of a piece that is $\frac{1}{4}$ of a whole?"

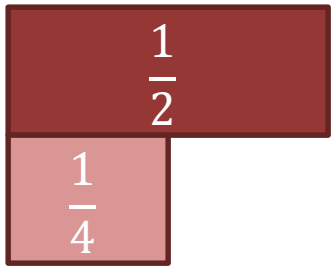


One-fourth is $\frac{1}{2}$ of one half, so I have $\frac{1}{2}$ of a $\frac{1}{2}$-size piece. That means $\frac{1}{4} \div \frac{1}{2} = \frac{1}{2}$.

Fraction Divided by a Fraction (1 of 2)

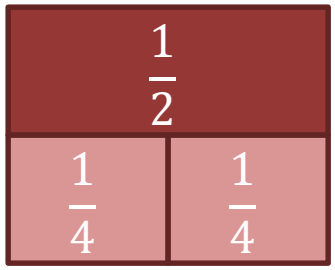
Example: $\frac{1}{2} \div \frac{1}{4}$

"How many $\frac{1}{4}$-size pieces can I get out of a piece that is $\frac{1}{2}$ of a whole?"



Fraction Divided by a Fraction (2 of 2)

Example: $\frac{1}{2} \div \frac{1}{4}$

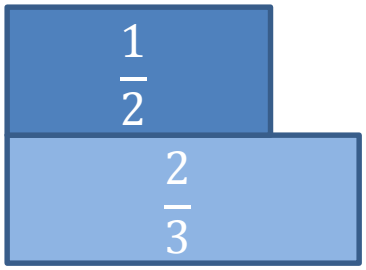


There are 2 one-fourths in $\frac{1}{2}$ so, $\frac{1}{2} \div \frac{1}{4} = 2$.

Fraction Divided by a Fraction (1 of 3)

Example: $\frac{1}{2} \div \frac{2}{3}$

"How many $\frac{2}{3}$-size pieces can I get out of a piece that is $\frac{1}{2}$ of a whole?"



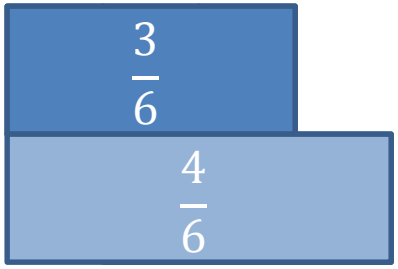
If I try to measure out $\frac{2}{3}$, it immediately goes too far. I can't even do it one time.

That means $(\frac{1}{2} \div \frac{2}{3}) < 1$.

Fraction Divided by a Fraction (2 of 3)

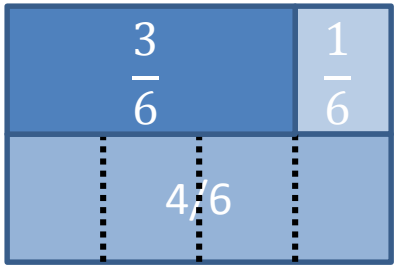
Example: $\frac{1}{2} \div \frac{2}{3}$

I don't know off the top of my head what fraction of $\frac{2}{3}$ one half is so I'm going to use common denominators to help me. I'm going to change the denominators to 6, so I end up with $\frac{3}{6} \div \frac{4}{6}$:



Fraction Divided by a Fraction (3 of 3)

Example: $\frac{1}{2} \div \frac{2}{3}$



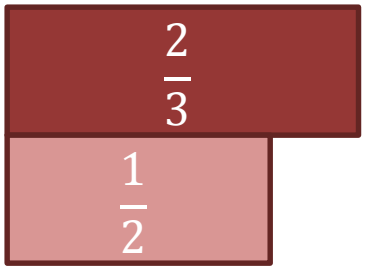
Three-sixths is $\frac{3}{4}$ of four-sixths, so I need $\frac{3}{4}$ of a four-sixths piece. That means $\frac{3}{6} \div \frac{4}{6} = \frac{3}{4}$.

In other words, $\frac{1}{2} \div \frac{2}{3} = \frac{3}{4}$.

Fraction Divided by a Fraction (1 of 6)

Example: $\frac{2}{3} \div \frac{1}{2}$

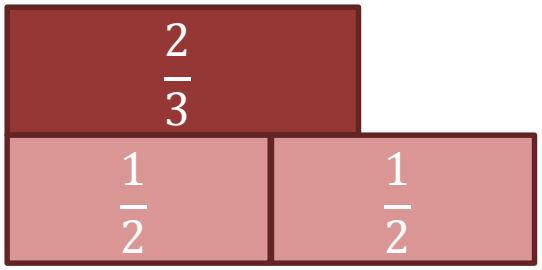
"How many $\frac{1}{2}$-size pieces can I get out of a piece that is $\frac{2}{3}$ of a whole?"



Hmm, if I stop at 1 one-half there is some extra space.

Fraction Divided by a Fraction (2 of 6)

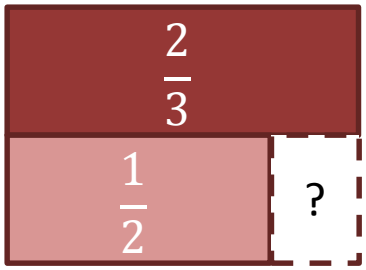
Example: $\frac{2}{3} \div \frac{1}{2}$



But if I use 2 one-halves, then I go past $\frac{2}{3}$.

Fraction Divided by a Fraction (3 of 6)

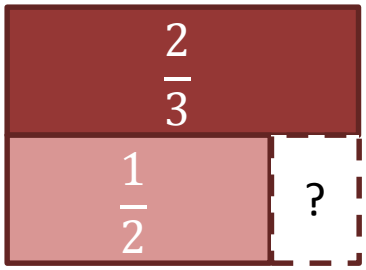
Example: $\frac{2}{3} \div \frac{1}{2}$



That means there is more than 1 one-half in $\frac{2}{3}$, but less than 2 one-halves. That means $1 < (\frac{2}{3} \div \frac{1}{2}) < 2$.

Fraction Divided by a Fraction (4 of 6)

Example: $\frac{2}{3} \div \frac{1}{2}$

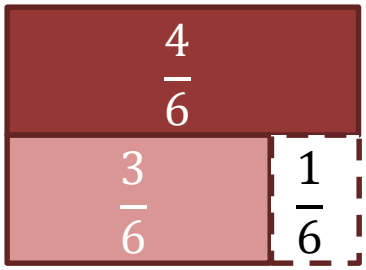


I know $\frac{1}{2}$ + ? = $\frac{2}{3}$. For the sake of consistency right now, I'm going to use common denominators again to help me find what fraction is missing.

Fraction Divided by a Fraction (5 of 6)

Example: $\frac{2}{3} \div \frac{1}{2}$

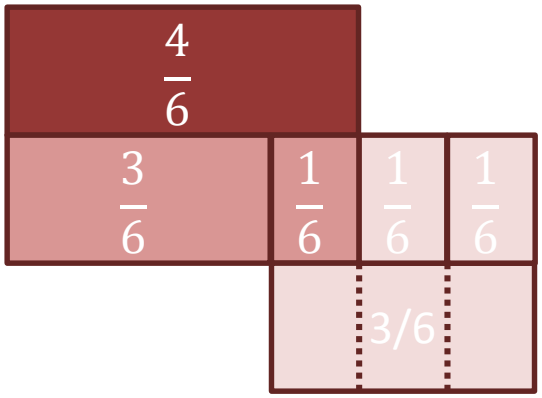
I'll use 6 again, so this time I'll end up with $\frac{4}{6} \div \frac{3}{6}$.



Now it's much easier to see that the missing piece is $\frac{1}{6}$ because $\frac{3}{6} + \frac{1}{6} = \frac{4}{6}$.

Fraction Divided by a Fraction (6 of 6)

Example: $\frac{2}{3} \div \frac{1}{2}$



One-sixth is $\frac{1}{3}$ of three-sixths, so I need $\frac{1}{3}$ of a three-sixths piece. So altogether there is 1 three-sixths piece and $\frac{1}{3}$ of a three-sixths piece in $\frac{4}{6}$. That means $\frac{4}{6} \div \frac{3}{6} = 1\frac{1}{3}$.

In other words, $\frac{2}{3} \div \frac{1}{2} = 1\frac{1}{3}$.