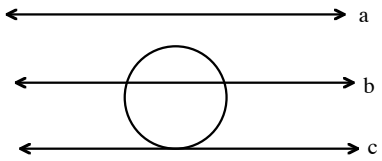
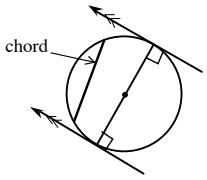
TANGENTS, SECANTS, AND CHORDS #19

The figure at right shows a circle with three lines lying on a flat surface. Line a does not intersect the circle at all. Line b intersects the circle in two points and is called a **SECANT**. Line c intersects the circle in only one point and is called a **TANGENT** to the circle.



TANGENT/RADIUS THEOREMS:

1. Any tangent of a circle is perpendicular to a radius of the circle at their point of intersection.
2. Any pair of tangents drawn at the endpoints of a diameter are parallel to each other.



A **CHORD** of a circle is a line segment with its endpoints on the circle.

DIAMETER/CHORD THEOREMS:

1. If a diameter bisects a chord, then it is perpendicular to the chord.
2. If a diameter is perpendicular to a chord, then it bisects the chord.

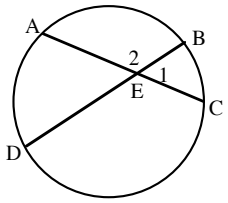
ANGLE-CHORD-SECANT THEOREMS:

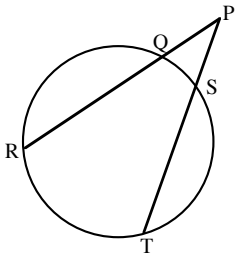
$m\angle 1 = \frac{1}{2}(m\overset{\frown}{AD} + m\overset{\frown}{BC})$

$AE \cdot EC = DE \cdot EB$

$m\angle P = \frac{1}{2}(m\overset{\frown}{RT} - m\overset{\frown}{QS})$

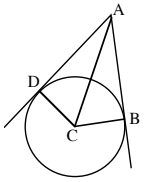
$PQ \cdot PR = PS \cdot PT$





Example 1

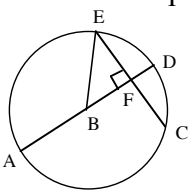
If the radius of the circle is 5 units and AC = 13 units, find AD and AB.



$\overline{AD} \perp \overline{CD}$ and $\overline{AB} \perp \overline{CD}$ by Tangent/Radius Theorem, so $(AD)^2 + (CD)^2 = (AC)^2$ or $(AD)^2 + (5)^2 = (13)^2$. So AD = 12 and $\overline{AB} \cong \overline{AD}$ so AB = 12.

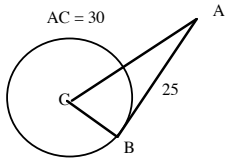
Example 2

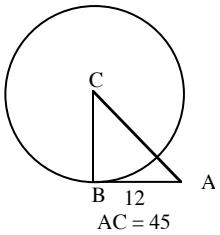
In $\odot$B, EC = 8 and AB = 5. Find BF. Show all subproblems.

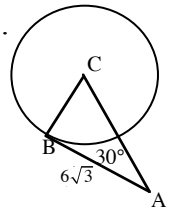


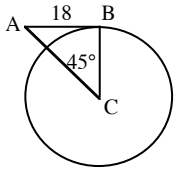
The diameter is perpendicular to the chord, therefore it bisects the chord, so EF = 4. $\overline{AB}$ is a radius and AB = 5. $\overline{EB}$ is a radius, so EB = 5. Use the Pythagorean Theorem to find BF: $BF^2 + 4^2 = 5^2$, BF = 3.

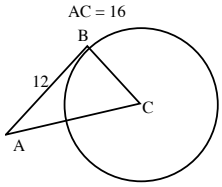
In each circle, C is the center and $\overline{AB}$ is tangent to the circle at point B. Find the area of each circle.

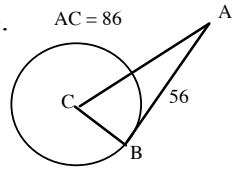
1. 

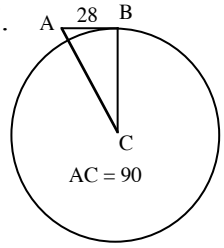
2. 

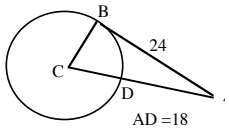
3. 

4. 

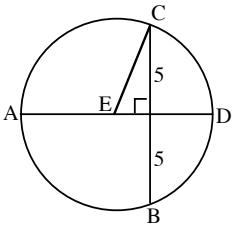
5. 

6. 

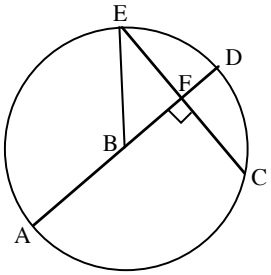
7. 

8. 

9. In the figure at right, point E is the center and m∠CED = 55°. What is the area of the circle?



In the following problems, B is the center of the circle. Find the length of $\overline{BF}$ given the lengths below.



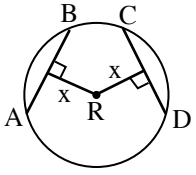
10. EC = 14, AB = 16

11. EC = 35, AB = 21

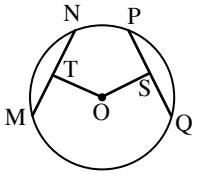
12. FD = 5, EF = 10

13. EF = 9, FD = 6

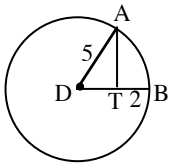
14. In ⊙R, if $AB = 2x - 7$ and $CD = 5x - 22$, find x.



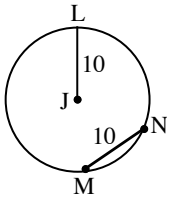
15. In ⊙O, $\overline{MN} \cong \overline{PQ}$, $MN = 7x + 13$, and $PQ = 10x - 8$. Find PS.



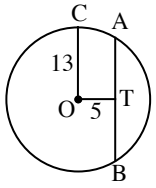
16. In ⊙D, if AD = 5 and TB = 2, find AT.



17. In $\odot$J, radius JL and chord MN have lengths of 10 cm. Find the distance from J to $\overline{MN}$.

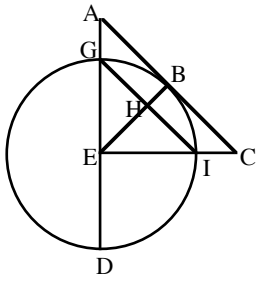


18. In $\odot$O, OC = 13 and OT = 5. Find AB.

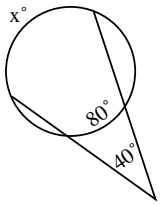


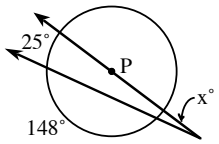
19. If $\overline{AC}$ is tangent to circle E and $\overline{EH} \perp \overline{GI}$, is $\Delta GEH \sim \Delta AEB$? Prove your answer.

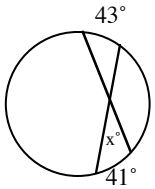
20. If $\overline{EH}$ bisects $\overline{GI}$ and $\overline{AC}$ is tangent to circle E at point B, are $\overline{AC}$ and $\overline{GI}$ parallel? Prove your answer.

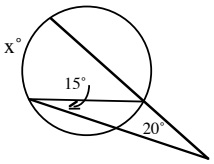


Find the value of x.

21. 

22. 

23. 

24. 

In $\odot$F, $m\widehat{AB} = 84°$, $m\widehat{BC} = 38°$, $m\widehat{CD} = 64°$, $m\widehat{DE} = 60°$. Find the measure of each angle and arc.

25. $m\widehat{EA}$

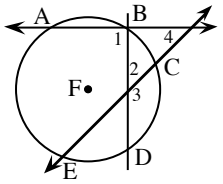
26. $m\widehat{AEB}$

27. $m\angle 1$

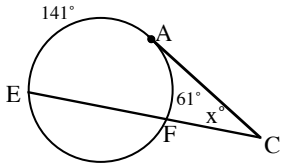
28. $m\angle 2$

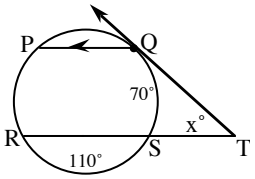
29. $m\angle 3$

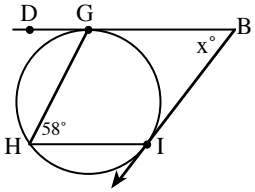
30. $m\angle 4$

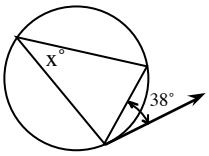


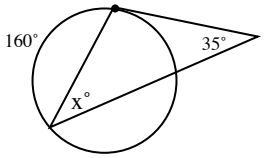
For each circle, tangent segments are shown. Use the measurements given find the value of x.

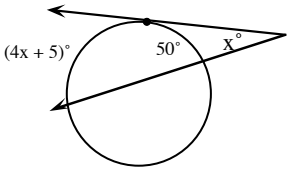
31. 

32. 

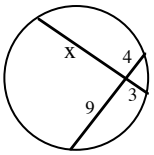
33. 

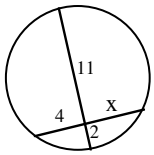
34. 

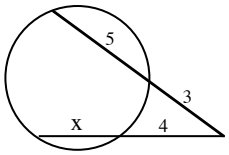
35. 

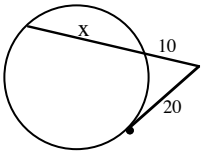
36. 

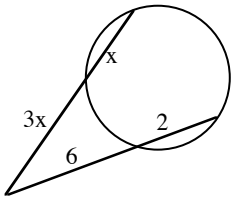
Find each value of x. Tangent segments are shown in problems 40, 43, 46, and 48.

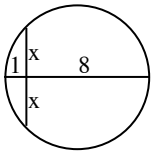
37. 

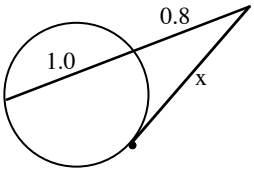
38. 

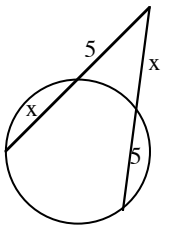
39. 

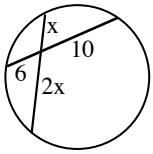
40. 

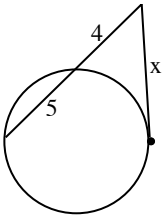
41. 

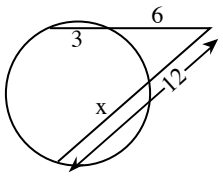
42. 

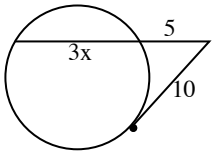
43. 

44. 

45. 

46. 

47. 

48. 

Answers

1. 275π sq. un.
2. 1881π sq. un.
3. 36π sq. un.
4. 324π sq. un.
5. 112π sq. un.
6. 4260π sq. un.
7. 7316π sq. un.
8. 49π sq. un.
9. ≈ 117.047 sq. un.
10. ≈ 14.4
11. ≈ 11.6
12. ≈ 7.5
13. 3.75
14. 5
15. 31
16. 4
17. $5\sqrt{3}$ cm.
18. $5\sqrt{3}$
19. Yes, $\angle GEH \cong \angle AEB$ (reflexive). $\overline{EB}$ is perpendicular to $\overline{AC}$ since it is tangent so $\angle GHE \cong \angle ABE$ because all right angles are congruent. So the triangles are similar by AA~.
20. Yes. Since $\overline{EH}$ bisects $\overline{GI}$ it is also perpendicular to it (SSS). Since $\overline{AC}$ is a tangent, $\angle ABE$ is a right angle. So the lines are parallel since the corresponding angles are right angles and all right angles are equal.
21. 160
22. 9
23. 42
24. 70
25. 114
26. 276
27. 87
28. 49
29. 131
30. 38
31. 40
32. 55
33. 64
34. 38
35. 45
36. 22.5
37. 12
38. $5\frac{1}{2}$
39. 2
40. 30
41. 2
42. $2\sqrt{2}$
43. 1.2
44. 5
45. $\sqrt{30}$
46. 6
47. 7.5
48. 5