Advanced Algebra II
Spring Semester Review: Part #1

Name Key

Find the quotient and the remainder for each of the given expressions:

1) $\dfrac{2x^5+x^4-5x^3-3x^2-6x+8}{x+2}$

$$\begin{array}{r|rrrrrr} -2 & 2 & 1 & -5 & -3 & -6 & 8 \\ & \downarrow & -4 & 6 & -2 & 10 & -8 \\ \hline & 2 & -3 & 1 & -5 & 4 & \boxed{0} \end{array}$$

$2x^4-3x^3+x^2-5x+4$

2) $\dfrac{x^5+3x^4-2x^3+x^2-3x+6}{x^2-3}$

$x^3+3x^2+x+10+\frac{36}{x^2-3}$

$x^2-3 \overline{)\,x^5+3x^4-2x^3+x^2-3x+6}$

$-(x^5 \quad -3x^3)$

$3x^4+x^3+x^2-3x+6$

$-(3x^4 \quad -9x^2)$

x^3+10x^2-3x+6

$-(x^3 \quad -3x)$

$10x^2+6$

$-(10x^2-30)$

$+36$

Use the remainder theorem and synthetic substitution to evaluate each of the following:

3) Given $f(x)=2x^5-3x^4+x^3-2x^2-8x+3$, Find $\boxed{f(2)=3}$

$$\begin{array}{r|rrrrrr} 2 & 2 & -3 & 1 & -2 & -8 & 3 \\ & & 4 & 2 & 6 & 8 & 0 \\ \hline & 2 & 1 & 3 & 4 & 0 & \boxed{3} \end{array}$$

4) $g(x)=4x^5-x^4+3x^3-2x+8$, Find $\boxed{g(-3)=-1120}$

$$\begin{array}{r|rrrrrr} -3 & 4 & -1 & 3 & 0 & -2 & 8 \\ & & -12 & 39 & -126 & 378 & -1128 \\ \hline & 4 & -13 & 42 & -126 & 376 & -1120 \end{array}$$

Derive a polynomial function, in standard form, for the given zeros below and call it $h(x)$.

5) Zeros at $x=2$, $x=-1$, and $x=3i$

$(x-2)(x+1)(x+3i)(x-3i)=0$

$(x^2-x-2)(x^2+9)=0$

$x^4-x^3-2x^2+9x^2-9x-18=0$

$\boxed{x^4-x^3+7x^2-9x-18=0}$

$\boxed{h(x)=x^4-x^3+7x^2-9x-18}$

6) Zeros $x=2$, $x=3$, and $x=2+i$ $x=2-i$

$(x-2)(x-3)(x-(2+i))(x-(2-i))=0$

$(x^2-5x+6)(x-2-i)(x-2+i)$

$(x^2-5x+6)((x-2)^2-i^2)=0$

$(x^2-5x+6)(x^2-4x+4+1)=0$

$(x^2-5x+6)(x^2-4x+5)=0$

$x^4-4x^3+5x^2$
$-5x^3+20x^2-25x$
$+6x^2-24x+30$ $=0$

$\boxed{x^4-9x^3+31x^2-49x+30=0}$

Solve each of the following equations by factoring. Find all real and imaginary solutions. If needed, use the calculator to identify possible factors of the polynomial and then synthetic division to verify the factors.

7) $x^5+2x^3=24x$

$x^5+2x^3-24x=0$

$x(x^4+2x^2-24)=0$

$x(x^2+6)(x^2-4)=0$

$x(x^2+6)(x+2)(x-2)=0$

$\boxed{x=0}$ $x^2+6=0$ $\boxed{x=-2}$ $\boxed{x=2}$

$x^2=-6$

$\boxed{x=\pm i\sqrt{6}}$

8) $x^4-64x=0$ (Simple factor and then use a difference of ~~squares~~ cubes)

$x(x^3-64)=0$

$x(x-4)(x^2+4x+16)=0$

$\boxed{x=0}$ $x-4=0$ $\boxed{x=4}$

$x^2+4x+16=0$

$x^2+4x+4=-16+4$

$\sqrt{(x+2)^2}=\sqrt{-12}$

$x+2=\pm 2i\sqrt{3}$

$\boxed{x=-2\pm 2i\sqrt{3}}$

9) $x^5 - 5x^3 = 36x$

$$x^5 - 5x^3 - 36x = 0$$
$$x(x^4 - 5x^2 - 36) = 0$$
$$x(x^2 - 9)(x^2 + 4) = 0$$
$$x(x+3)(x-3)(x^2+4) = 0$$

$\boxed{x=0}$ $\quad x+3=0 \quad x-3=0 \quad x^2+4=0$

$\boxed{x=-3}$ $\quad \boxed{x=3}$ $\quad \sqrt{x^2} = \sqrt{-4}$

$\boxed{x = \pm 2i}$

10) $2x^4 - 128x = 0$

$$2x(x^3 - 64) = 0$$
$$2x(x-4)(x^2+4x+16) = 0$$

$2x = 0 \quad x-4=0 \quad x^2+4x+16=0$

$\boxed{x=0}$ $\quad \boxed{x=4}$ $\quad x^2+4x+4 = -16+4$

$$\sqrt{(x+2)^2} = \sqrt{12}$$
$$x+2 = \pm 2i\sqrt{3}$$
$$\boxed{x = -2 \pm 2i\sqrt{3}}$$

Use binomial expansion and Pascal's Triangle to expand the following expression:

11) $(2x+1)^4$

$$(1)(2x)^4(1)^0 + 4(2x)^3(1)^1 + 6(2x)^2(1)^2 + 4(2x)(1)^3 + 1(1)^4$$

$$16x^4 + 32x^3 + 24x^2 + 8x + 1$$

1
1 1 1
1 2 1 2
1 3 3 1 3
1 4 6 4 1 4

Which term determines the end behavior of a curve and why?

the Leading term ; the highest degree term

Determine the end behavior of each of the following functions:

12) $f(x) = 2x^4 - 15x^3 - 2x + 4x - 5$

$As\ x \to -\theta,\ y \to$ $+\infty$

$As\ x \to +\theta,\ y \to$ $+\infty$

13) $p(x) = -2x^5 + 3x^3 + 7x^2 - 4x - 90$

$As\ x \to -\theta,\ y \to$ $+\infty$

$As\ x \to +\theta,\ y \to$ $-\infty$

14) $h(x) = 2x^4 - 3x^2 + 6x + 10$

$As\ x \to -\theta,\ y \to$ $+\infty$

$As\ x \to +\theta,\ y \to$ $+\infty$

Graph the following function. Label all of the turning points and the zero's to the nearest tenth of a unit.

15) $f(x) = x^4 - 6x^3 - x^2 + 30x$

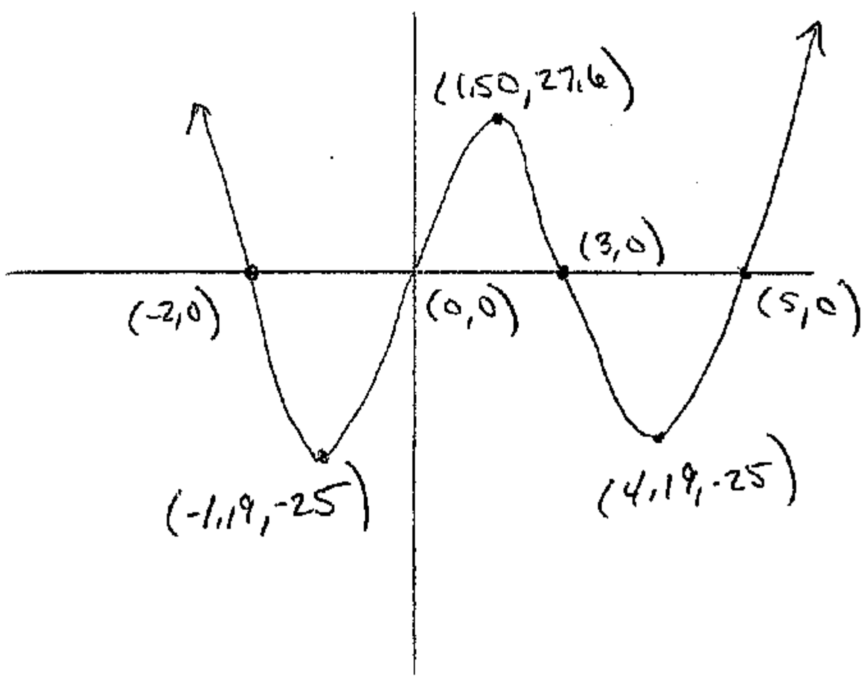
$f(x) = x(x^3 - 6x^2 - x + 30)$

Describe the end behavior of the curve.

$As\ x \to -\theta,\ y \to$ $+\infty$

$As\ x \to +\theta,\ y \to$ $+\infty$

Maximum # of turning points possible? 3



What does Descartes' Rule say about the number of positive real roots and negative real roots for the polynomial function below?

16) $h(x)=2x^6-5x^5-x^4+3x^3-2x^2-5x-1$ A) Number of positive real solutions? 3 oR 1

$+ \quad + \quad - \quad - \quad - \quad + \quad -$ B) Number of negative real solutions? 3 oR 1

The table below right gives the average life expectancy, in years, in the United States from 1900 until the year 2000. Let time "t" $=0$ in 1900 and complete each of the following;

17) Find a linear model for the data. Round each term to 3 places past the decimal.

$y=.269t+51.33$

18) What is the R^2 value of the linear model? Round to 4 places past the decimal.

$R^2=.9607$

19) Find a cubic model and round to 3 places past the decimal.

$y=(-2.106\times10^{-5})x^3+.001x^2+.331x+49.47$

20) What is the R^2 value of the cubic model? Round to 4 places.

$R^2=.9999$

21) Which model fits the data better and briefly explain why?

the Cubic Appears to fit the data better but heads down eventually, Linear will people Always get older

22) Which model is the better predictor over time and why?

Neither Really. One goes up forever, the other goes down past 0.

23) Use your model to predict the Life Expectancy of a person in the year 2050.

Linear

in 2050 the linear says 91.68 yrs old.

Year	Average Life Span
1900	49.5
1920	56.4
1940	63.6
1960	69.9
1980	73.9
2000	75.4

Use your calculator and synthetic division to factor the following function layer by layer and then identify all of the zeros.

24) $f(x) = 2x^4 - 5x^3 - 15x^2 + 10x + 8$ Possible Rational Zeros: $\frac{p}{q} = \frac{\pm 1, 2, 4, 8}{\pm 2}$

$f(x) = (x+2)(2x^3 - 9x^2 + 3x + 4)$

$f(x) = (x+2)(x-1)(2x^2 - 7x - 4)$

$f(x) = (x+2)(x-1)(2x+1)(x-4)$

$x = -2 \quad x = 1 \quad x = -\frac{1}{2} \quad x = 4$

Zeros at $(-2,0)$ $(1,0)$ $(-\frac{1}{2},0)$ $(4,0)$

-2	2	-5	-15	10	8
		-4	18	-6	-8
1	2	-9	3	4	0
		2	-7	-4	
	2	-7	-4	0	

$\pm 1, \pm 2, \pm 4, \pm 8, \pm \frac{1}{2}$

Determine a cubic function that is obtained from the parent function $f(x) = x^3$ after the following sequence of transformations:

- Vertical stretch by a factor of 4.
- A reflection over the "y" axis.
- A horizontal shift of 3 units to the left.
- A vertical shift of 2 units down.

25) Final Function: $f(x) = -4(x+3)^3 - 2$

- Vertical compression by a factor of $\frac{2}{3}$
- A reflection over the "x" axis.
- A horizontal shift of 2 units to the right.
- A vertical shift of 4 units up.

26) Final Function: $f(x) = \frac{2}{3}(x-2)^3 + 4$

Solve each of the following equations. Check for extraneous solutions.

27) $\sqrt{2x+5} - 1 = x$

$(\sqrt{2x+5})^2 = (x+1)^2$

$\cancel{2x} + 5 = x^2 + \cancel{2x} + 1$

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

$\cancel{x = -2}$ $\boxed{x = 2}$

28) $(\sqrt{3x+10})^2 = (x+4)^2$

$3x + 10 = x^2 + 8x + 16$

$0 = x^2 + 5x + 6$

$x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$\boxed{x = -2 \quad x = -3}$