Final Days before AP Review 3
AP Calculus

Name: Answers

No calculator

6. The twice-differentiable function f is defined for all real numbers and satisfies the following conditions:

$$f(0) = 2,\ f'(0) = -4, \text{ and } f''(0) = 3.$$

(a) The function g is given by $g(x) = e^{ax} + f(x)$ for all real numbers, where a is a constant. Find $g'(0)$ and $g''(0)$ in terms of a. Show the work that leads to your answers.

(b) The function h is given by $h(x) = \cos(kx)f(x)$ for all real numbers, where k is a constant. Find $h'(x)$ and write an equation for the line tangent to the graph of h at $x = 0$.

a) $g'(x) = ae^{ax} + f'(x)$

$g'(0) = ae^{0} + f'(0) = a - 4$ $\qquad \boxed{g(0) = a - 4}$

$g''(x) = a^2e^{ax} + f''(x)$

$g''(0) = a^2e^{0} + f''(0) = a^2 + 3$ $\qquad \boxed{g''(0) = a^2 + 3}$

b) $h'(x) = -\sin(kx)\cdot k\cdot f(x) + \cos(kx)\cdot f'(x)$

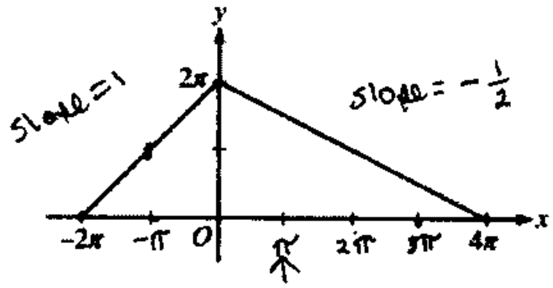
slope of tangent to $h(x)$ at $x = 0$: $-k\sin(0)\cdot f(0) + \cos(0)\cdot f'(0)$

slope $= f'(0) = -4$

point: $h(0) = \cos(0)f(0) = 1\cdot 2 = 2$ $\qquad (0, 2)$

$y - 2 = -4(x)$ or $\boxed{y = -4x + 2}$

No calculator



Graph of g

6. Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.

(a) Find $\int_{-2\pi}^{4\pi} f(x)\,dx$. Show the computations that lead to your answer.

(b) Find all x-values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.

(c) Let $h(x) = \int_0^{3x} g(t)\,dt$. Find $h'\left(-\frac{\pi}{3}\right)$.

a) Area $= \frac{1}{2}(2\pi)(2\pi) + \frac{1}{2}(4\pi)(2\pi) = 2\pi^2 + 4\pi^2 = 6\pi^2$

$$\int_{-2x}^{4x} f(x)dx = 6\pi^2 - \int_{-2\pi}^{4\pi} \cos\left(\tfrac{1}{2}x\right)$$

$$= 6\pi^2 - \left[2\sin\left(\tfrac{1}{2}x\right)\right]_{-2\pi}^{4\pi}$$

$$= 6\pi^2 - (2\sin(2\pi) - 2\sin(-\pi)$$

$$= 6\pi^2 - 0 \quad = \boxed{6\pi^2}$$

b) $f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right)$

$0 = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right)$

$g'(x) = -\frac{1}{2}\sin\left(\frac{x}{2}\right)$

at $x = \pi$ $\quad g'(x) = -\frac{1}{2}$ and $\frac{1}{2}\sin\left(\frac{3\pi}{2}\right) = \frac{1}{2}$

$\boxed{\text{C.P. at } x = \pi}$

c) $h'(x) = \Big[g(x)\cdot 3\Big]_0^{3x} = 3g(3x) - 3g(0)$

$h'\left(-\frac{\pi}{3}\right) = 3g\left(-\frac{3\pi}{3}\right) - 3g(0)$

$= 3g(-\pi) - 3g(0)$

$= 3\pi - 3\cdot 2\pi$

$= \boxed{-3\pi}$