Final Days before AP Review
AP Calculus

Name: Answers

Yes, calculator.

Let f be the function defined by

$$f(x) = \begin{cases} \sqrt{x+1} & \text{for } 0 \le x \le 3 \\ 5 - x & \text{for } 3 < x \le 5. \end{cases}$$

(a) Is f continuous at $x = 3$? Explain why or why not.

(b) Find the average value of $f(x)$ on the closed interval $0 \le x \le 5$.

(c) Suppose the function g is defined by

$$g(x) = \begin{cases} k\sqrt{x+1} & \text{for } 0 \le x \le 3 \\ mx + 2 & \text{for } 3 < x \le 5, \end{cases}$$

where k and m are constants. If g is differentiable at $x = 3$, what are the values of k and m?

answers $\boxed{m = \frac{2}{5}, K = \frac{8}{5}}$

a) $f(3) = \begin{cases} \sqrt{4} = 2 \\ 5-3 = 2 \end{cases}$ $\lim_{x \to 3^-} f(x) = 2$

$\lim_{x \to 3^+} f(x) = 2$

since $f(3) = 2 = \lim_{x \to 3} f(x)$ then f is cont. at 3

b) $\text{Ave} = \frac{1}{5-0} \int_0^5 f(x)dx = \frac{1}{5}\left(\int_0^3 f(x)dx + \int_3^5 f(x)\right)$

$\frac{1}{5}\left(\frac{14}{3} + 2\right)$

$= \boxed{\frac{4}{3}}$

c) $\begin{cases} k\sqrt{x+1} = mx + 2 \\ \frac{1}{2}k(x+1)^{-1/2} = m \end{cases}$

at $x = 3$: $\begin{cases} K\sqrt{4} = 3m + 2 \rightarrow 2K = 3m + 2 \rightarrow 2K = \frac{3}{4}K + 2 \\ \frac{1}{2}K\left(\frac{1}{\sqrt{4}}\right) = m \rightarrow \left(\frac{1}{4}K\right) = m \end{cases}$

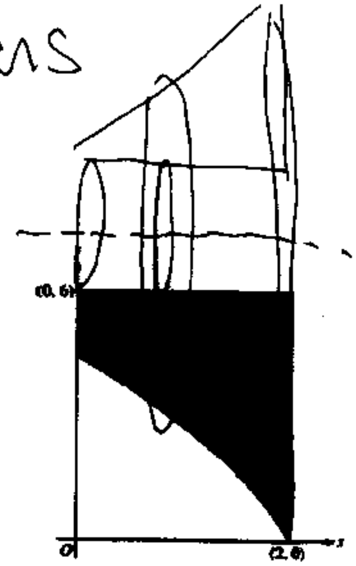
$1.25K = 2$

$\boxed{K = 1.6}$ $\boxed{m = .4}$

Answers

Volume and Area AP Problems – Mixed Practice **Name:**

AP Calculus



1) In the figure above, R is the shaded region in the first quadrant bounded by the graph of $y = 4\ln(3 - x)$, the horizontal line $y = 6$, and the vertical line $x = 2$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 8$.

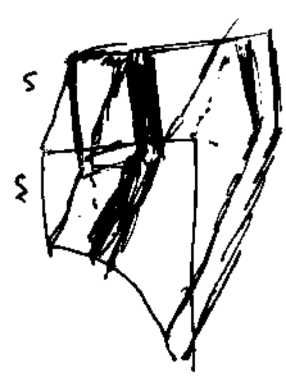
(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

a) $$A = \int_0^2 \left(6 - 4\ln(3-x)\right)dx = 6.817$$

b) $$V = \pi \int_0^2 \left(\left(8 - 4\ln(3-x)\right)^2 - (8-6)^2\right)dx = 168.180$$

c) $$V = \int_0^2 \left(6 - 4\ln(3-x)\right)^2 dx$$

$$= 26.267$$



Area of each square $= (\text{side})^2$