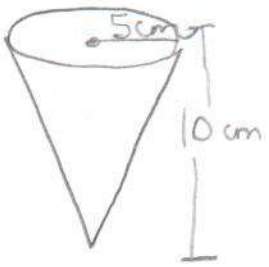
AP Calculus Extra Practice 2.6 Name Key

1. A container has the shape of an open circular cone. The height of the container is 10 cm and the diameter of the opening is 10 cm. Water in the container is evaporating so that its depth is changing at the constant rate of $\frac{-3}{10}$ cm/hr.



$\frac{dh}{dt} = \frac{-3}{10}\ \frac{cm}{hr}$

$V = \frac{125\pi}{12}\ cm^3$

a) Find the volume V of water in the container when h = 5cm. Indicate units of measure.

$V = \frac{\pi}{3}r^2h$

$V = \frac{\pi}{3}\left(\frac{5}{2}\right)^2(5)$

$V = \frac{\pi \cdot 25 \cdot 5\ cm^3}{3 \cdot 4}$

$V = \frac{125\pi}{12}\ cm^3$

$\frac{r}{h} = \frac{5}{10}$

$10r = 5h$

$r = \frac{h}{2}$

$\frac{dV}{dt} = \frac{-15\pi}{8}\ \frac{cm^3}{hr}$

b) Find the rate of change of the volume of water in the container, with respect to time, when h = 5cm. Indicate units of measure.

Find $\frac{dV}{dt}$ when $h = 5cm$

$V = \frac{\pi}{3}r^2h$

$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2\left(\frac{h}{1}\right)$

$V = \left(\frac{\pi}{3}\right)\left(\frac{h^2}{4}\right)\left(\frac{h}{1}\right)$

$V = \frac{\pi}{12}h^3$

$\frac{dV}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$

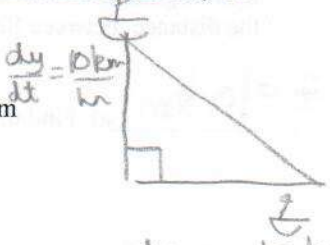
$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$\frac{dV}{dt} = \left(\frac{\pi}{4}\right)\left(\frac{5}{1}\right)^2\left(\frac{-3}{10}\right)$

$\frac{dV}{dt} = \frac{\pi}{4} \cdot \frac{25}{1} \cdot \frac{-3}{10}$ (2)

$\frac{dV}{dt} = \frac{-15\pi}{8}\ \frac{cm^3}{hr}$

2. Ship A is traveling due west toward Lighthouse Rock at a speed of 15 kilometers per hour. Ship B is traveling due north away from Lighthouse Rock at a speed of 10 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t.



$\frac{dy}{dt} = \frac{10\ km}{hr}$

$z = 5km$

a) Find the distance, in kilometers, between Ship A and Ship B when x = 4 km and y = 3 km

$\frac{dz}{dt} = -6\ \frac{km}{hr}$

$\frac{dx}{dt} = -15\ \frac{km}{hr}$

b) Find the rate of change, in km/hr, of the distance between the two ships when x = 4 km and y = 3 km.

$x^2 + y^2 = z^2$

$\frac{d}{dt}\left[x^2 + y^2 = z^2\right]$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$(4)(-15) + (3)(10) = (5)\frac{dz}{dt}$

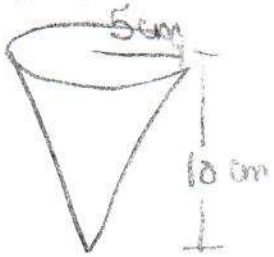
$-30 = 5\frac{dz}{dt}$

$-6 = \frac{dz}{dt}$

AP Calculus More Extra Practice 2.6 Name_____

1 A container has the shape of an open circular cone. The height of the container is 10 cm and the diameter of the opening is 10 cm.

Water in the container is evaporating so that its depth is changing at the constant rate of $\frac{-8}{15}$ cm/hr.



$\frac{dh}{dt} = -\frac{8}{15}\ \frac{cm}{hr}$

$V = 18\pi\ cm^3$

a) Find the volume V of water in the container when $h = 6$cm. Indicate units of measure.

$V = \frac{\pi}{3}(3)^2(6)$

$V = \frac{\pi}{3}(54) = 18\pi$

$\frac{r}{h} = \frac{5}{10}$

$r = \frac{h}{2}$

$\frac{dV}{dt} = \frac{-24\pi}{5}\ \frac{cm^3}{hr}$

b) Find the rate of change of the volume of water in the container, with respect to time, when $h = 6$cm. Indicate units of measure.

$V = \frac{\pi}{3}r^2h$

$V = \frac{\pi}{3}\left(\frac{h}{2}\right)^2\left(\frac{h}{1}\right)$

$V = \frac{\pi}{3}\left(\frac{h^2}{4}\right)\left(\frac{h}{1}\right)$

$V = \frac{\pi}{12}h^3$

$\frac{d}{dt}\left[V = \frac{\pi}{12}h^3\right]$

$\frac{dV}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$

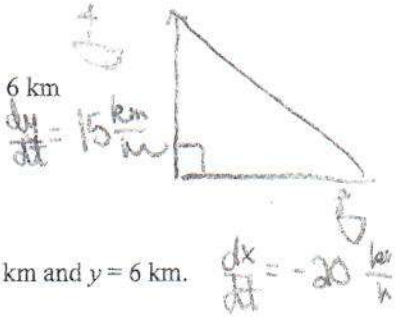
$\frac{dV}{dt} = \frac{\pi}{4}h^2\frac{dh}{dt}$

$\frac{dV}{dt} = \frac{\pi}{4}(6)^2\left(-\frac{8}{15}\right)$

$\frac{dV}{dt} = \frac{\pi}{1}\cdot\frac{36}{1}\cdot\frac{-8}{15}$

$\frac{dV}{dt} = -\frac{24\pi}{5}\ \frac{cm^3}{hr}$

12. Ship A is traveling due west toward Lighthouse Rock at a speed of 20 kilometers per hour. Ship B is traveling due north away from Lighthouse Rock at a speed of 15 km/hr. Let x be the distance between Ship A and Lighthouse Rock at time t, and let y be the distance between Ship B and Lighthouse Rock at time t.



$z = 10$ km

a) Find the distance, in kilometers, between Ship A and Ship B when $x = 8$ km and $y = 6$ km

$\frac{dz}{dt} = -7\ \frac{km}{hr}$

b) Find the rate of change, in km/hr, of the distance between the two ships when $x = 8$ km and $y = 6$ km.

$x^2 + y^2 = z^2$

$\frac{d}{dt}\left[x^2 + y^2 = z^2\right]$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$(8)(-20) + (6)(15) = (10)\frac{dz}{dt}$

$-160 + 90 = 10\frac{dz}{dt}$

$-70 = 10\frac{dz}{dt}$

$\left[\frac{dz}{dt} = -7\ \frac{km}{hr}\right]$