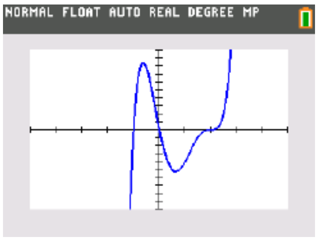
Given $f(x) = -x^3(x-2)^2(x+3)$

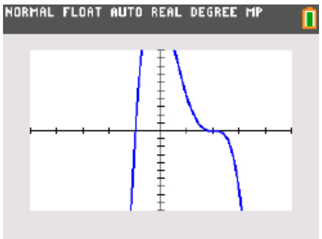
a. Identify the roots and list their multiplicities.

b. Which of the below graphs would model f(x)?

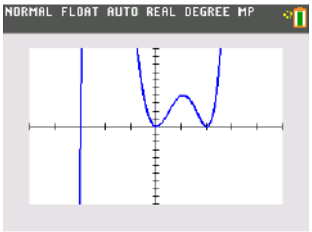
A.



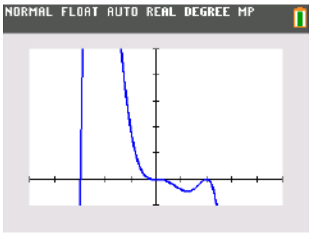
C.



B.



D.



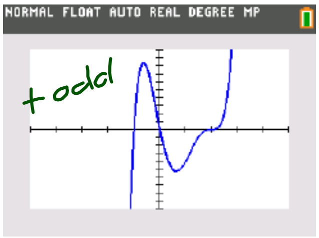
Given $f(x) = -x^3(x-2)^2(x+3)$

a. Identify the roots and list their multiplicities.

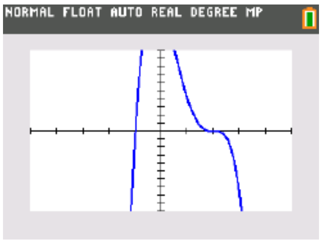
0 m.3, 2 m.2, -3 m.1 neg. deg 6 ↙ ↓

b. Which of the below graphs would model f(x)?

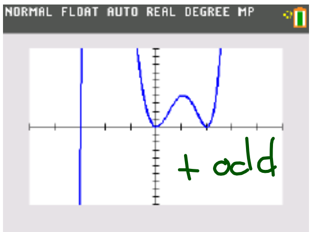
~~A.~~



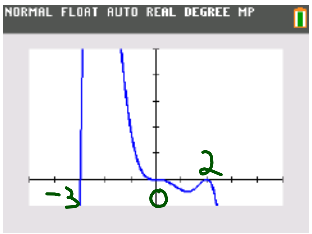
C.



~~B.~~



D.

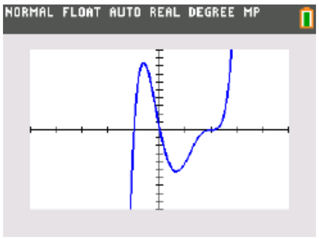


Given $f(x) = x^2(x-2)^2(x+3)$

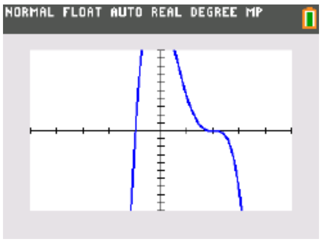
a. Identify the roots and list their multiplicities.

b. Which of the below graphs would model f(x)?

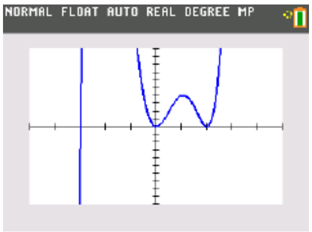
A.



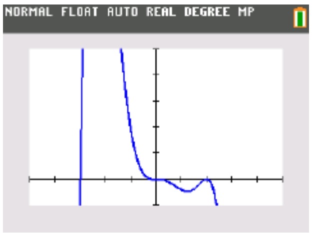
C.



B.



D.



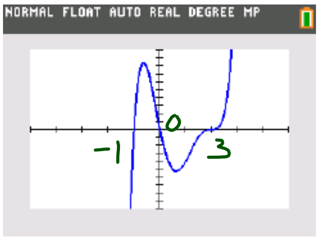
Given $f(x) = x^2(x-2)^2(x+3)$

a. Identify the roots and list their multiplicities.

0 m.2 , 2 m.2 , −3 m. 1    pos. deg 5

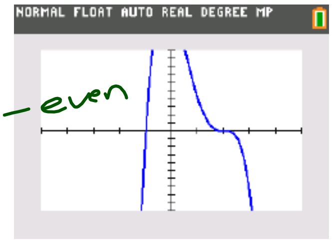
b. Which of the below graphs would model f(x)?

A.

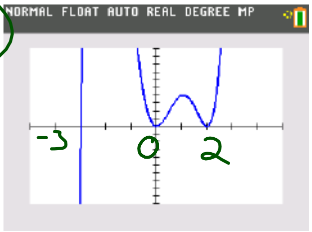


~~C.~~

– even

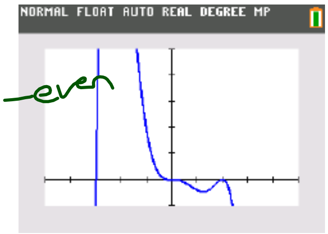


[✓ B.]



~~D.~~

– even

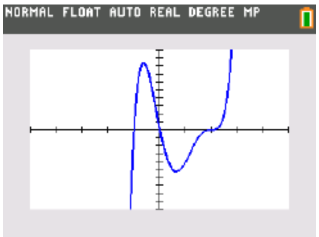


Given $f(x) = -2(x+1)(x-2)^3$

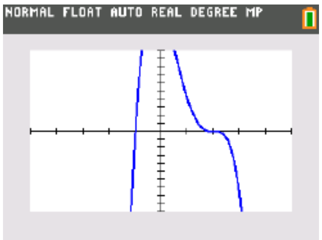
a. Identify the roots and list their multiplicities.

b. Which of the below graphs would model f(x)?

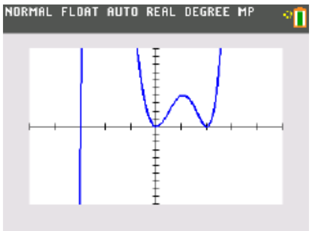
A.



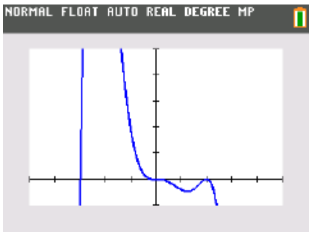
C.



B.



D.



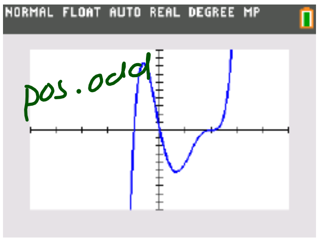
Given $f(x) = -2(x+1)(x-2)^3$

a. Identify the roots and list their multiplicities.

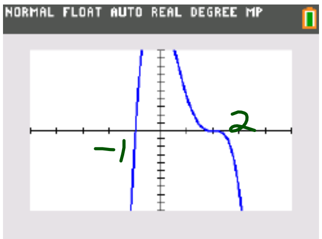
-1 m.1 , 2 m.3 neg. deg 4

b. Which of the below graphs would model f(x)?

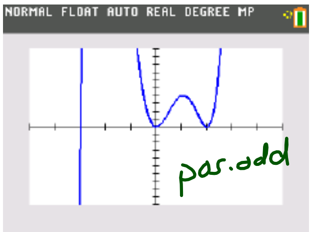
~~A~~.



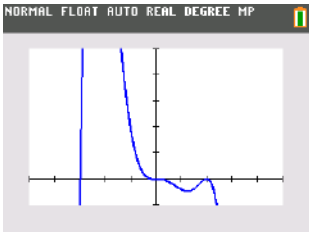
C.



~~B~~.



D.



Divide f(x) by d(x) using synthetic division to find the quotient and remainder.

$$f(x) = x^3 + 4x^2 + 7x - 9$$

$$d(x) = x + 3$$

Divide f(x) by d(x) using synthetic division to find the quotient and remainder.

$$f(x) = 2x^4 - 5x^3 + 7x^2 - 3x + 1$$

$$d(x) = x - 3$$

Divide f(x) by d(x) using synthetic division to find the quotient and remainder.

$$f(x) = x^3 + 4x^2 + 7x - 9$$

$$d(x) = x + 3$$

$$\begin{array}{r|rrrr} -3 & 1 & 4 & 7 & -9 \\ & \downarrow & -3 & -3 & -12 \\ \hline & 1 & 1 & 4 & \boxed{-21} \end{array}$$

$x^2 + x + 4$ r. $-21$

Divide f(x) by d(x) using synthetic division to find the quotient and remainder.

$$f(x) = 2x^4 - 5x^3 + 7x^2 - 3x + 1$$

$$d(x) = x - 3$$

$$\begin{array}{r|rrrrr} 3 & 2 & -5 & 7 & -3 & 1 \\ & \downarrow & 6 & 3 & 30 & 81 \\ \hline & 2 & 1 & 10 & 27 & \boxed{82} \end{array}$$

$2x^3 + x^2 + 10x + 27$ r. $82$

Divide f(x) by d(x) using synthetic division to find the quotient and remainder.

$$f(x) = 9x^3 + 7x^2 - 3x$$

$$d(x) = x - 10$$

Divide f(x) by d(x) using synthetic division to find the quotient and remainder.

$$f(x) = 5x^4 - 3x + 1$$

$$d(x) = x - 4$$

Divide f(x) by d(x) using synthetic division to find the quotient and remainder.

$$f(x) = 9x^3 + 7x^2 - 3x$$

$$d(x) = x - 10$$

$$\begin{array}{r|rrrr} 10 & 9 & 7 & -3 & 0 \\ & \downarrow & 90 & 970 & 9670 \\ \hline & 9 & 97 & 967 & \boxed{9670} \end{array}$$

$9x^2+97x+967$ r. 9670

Divide f(x) by d(x) using synthetic division to find the quotient and remainder.

$$f(x) = 5x^4 - 3x + 1$$

$$d(x) = x - 4$$

$$\begin{array}{r|rrrrr} 4 & 5 & 0 & 0 & -3 & 1 \\ & \downarrow & 20 & 80 & 320 & 1268 \\ \hline & 5 & 20 & 80 & 317 & \boxed{1269} \end{array}$$

$5x^3+20x^2+80x+317$ r. 1269

Determine a polynomial of minimum degree with real coefficients that has roots 1/4, 1 - 2i and a leading coefficient of 2.  Write it in standard form.

Determine a polynomial of minimum degree with real coefficients that has roots 1/4, 1 - 2i and a leading coefficient of 2.  Write it in standard form.

$x = \frac{1}{4}$ $x = 1 - 2i$ $x = 1 + 2i$

$(4x - 1)(x - 1 + 2i)(x - 1 - 2i)$

$x^2 - x - \cancel{2xi} - x + 1 + \cancel{2i} + \cancel{2xi} \cancel{-2}i \pm 4\cancel{i^2}$

$(4x - 1)(x^2 - 2x + 5)$

$4x^3 - 8x^2 + 20x - x^2 + 2x - 5$

$\frac{4x^3}{2} - \frac{9x^2}{2} + \frac{22x}{2} - \frac{5}{2}$

$2x^3 - \frac{9}{2}x^2 + 11x - \frac{5}{2}$

Determine a polynomial of minimum degree with real coefficients that has roots -1/2, 2 + 3i and a leading coefficient of 4.  Write it in standard form.

Determine a polynomial of minimum degree with real coefficients that has roots -1/2, 2 + 3i and a leading coefficient of 4.  Write it in standard form.

$x = -\frac{1}{2}$ $\quad x = 2 + 3i \quad x = 2 - 3i$

$(2x+1)(x-2-3i)(x-2+3i)$

$x^2 - 2x + \cancel{3xi} - 2x + 4 \cancel{-6i} \cancel{-3xi} + \cancel{6i} \overset{+}{-} 9\cancel{i^2}$

$(2x+1)(x^2-4x+13)$

$2x^3 - 8x^2 + 26x + x^2 - 4x + 13$

$2(2x^3 - 7x^2 + 22x + 13)$

$4x^3 - 14x^2 + 44x + 26$

Determine if x + 5 is a factor of $f(x) = 3x^3 + 2x^2 + x - 5$

Show your work and explain your answer.

Determine if x - 1 is a factor of $f(x) = x^3 - x^2 + x - 1$

Show your work and explain your answer.

Determine if x + 5 is a factor of $f(x) = 3x^3 + 2x^2 + x - 5$

Show your work and explain your answer.

$f(-5) = 3(-5)^3 + 2(-5)^2 + (-5) - 5 \quad = -335$

no... remainder is not 0.

Determine if x - 1 is a factor of $f(x) = x^3 - x^2 + x - 1$

Show your work and explain your answer.

$f(1) = 1^3 - 1^2 + 1 - 1 = 0$

yes... remainder is 0 !!

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 6x^3 - 5x - 1$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 6x^4 - x^3 - 6x^2 - x - 12$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 6x^3 - 5x - 1$$

P: 1

Q: 1, 2, 3, 6

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 6x^4 - x^3 - 6x^2 - x - 12$$

P: 1, 2, 3, 4, 6, 12

Q: 1, 2, 3, 6

$\pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{6}$

$\pm 2, \pm \frac{2}{3} \quad \pm 4, \pm \frac{4}{3}$

$\pm 3, \pm \frac{3}{2} \quad \pm 6, \pm 12$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 2x^3 - x^2 - 9x + 9$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 4x^3 + 7x^2 - x + 6$$

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 2x^3 - x^2 - 9x + 9$$

P: 1, 3, 9 $\pm 1, \pm \frac{1}{2}, \pm 3, \pm \frac{3}{2}, \pm 9, \pm \frac{9}{2}$

Q: 1, 2

Use the Rational Zeros Theorem to write list of potential rational zeros.

$$f(x) = 4x^3 + 7x^2 - x + 6$$

P: 1, 2, 3, 6 $\pm 1, \pm \frac{1}{2}, \pm \frac{1}{4}, \pm 2, \pm 3, \pm \frac{3}{2}, \pm \frac{3}{4}, \pm 6$

Q: 1, 2, 4

Given $f(-1)=0$, find all remaining zeros of $f(x)=x^3-2x^2-13x-10$

Given $f(1)=0$, find all remaining zeros of $f(x)=3x^3+4x^2-5x-2$

Given $f(-1)=0$, find all remaining zeros of $f(x)=x^3-2x^2-13x-10$

$$\begin{array}{r|rrrr} -1 & 1 & -2 & -13 & -10 \\ & \downarrow & -1 & 3 & 10 \\ \hline & 1 & -3 & -10 & \boxed{0} \end{array}$$

$x^2-3x-10$

$x^2-3x-10$

$(x-5)(x+2)$

$\boxed{x=5, -2, -1}$

Given $f(1)=0$, find all remaining zeros of $f(x)=3x^3+4x^2-5x-2$

$$\begin{array}{r|rrrr} 1 & 3 & 4 & -5 & -2 \\ & \downarrow & 3 & 7 & 2 \\ \hline & 3 & 7 & 2 & \boxed{0} \end{array}$$

$3x^2+7x+2$

$3x^2+7x+2$

$(3x+1)(x+2)$

$\boxed{x=-\frac{1}{3}, -2, 1}$

Given $f(2)=0$ , find all remaining zeros of $f(x)=x^3-3x-2$

Given $f(3)=0$ , find all remaining zeros of $f(x)=2x^3-3x^2-5x-12$

Given $f(2)=0$ , find all remaining zeros of $f(x)=x^3-3x-2$

$$\begin{array}{r|rrrr} 2 & 1 & 0 & -3 & -2 \\ & \downarrow & 2 & 4 & 2 \\ \hline & 1 & 2 & 1 & \boxed{0} \end{array}$$

$x^2+2x+1$

$x^2+2x+1$

$(x+1)(x+1)$

$\boxed{x=-1 \text{ m.2.}, 2}$

Given $f(3)=0$ , find all remaining zeros of $f(x)=2x^3-3x^2-5x-12$

$$\begin{array}{r|rrrr} 3 & 2 & -3 & -5 & -12 \\ & \downarrow & 6 & 9 & 12 \\ \hline & 2 & 3 & 4 & \boxed{0} \end{array}$$

$2x^2+3x+4$

$2x^2+3x+4$

$$x=\frac{-3\pm\sqrt{3^2-4\cdot1\cdot2\cdot4}}{2\cdot2}$$

$$x=\frac{-3\pm\sqrt{-23}}{4}=\frac{-3\pm i\sqrt{23}}{4}$$

Given $4i$ is a zero of $f(x) = x^4 + 13x^2 - 48$
find all remaining zeros.

Given $4i$ is a zero of $f(x) = x^4 + 13x^2 - 48$
find all remaining zeros.

$x = 4i \qquad x = -4i$

$(x - 4i)\,(x + 4i)$

$x^2 + 16\cancel{i^2}$

$$
\begin{array}{r|l}
 & \quad x^2 - 3 \\
\hline
x^2 + 0x + 16 & x^4 + 0x^3 + 13x^2 + 0x - 48 \\
 & -\,x^4 \overset{-}{+} 0x^3 \overset{-}{+} 16x^2 \\
\hline
 & \qquad\qquad -3x^2 + 0x - 48 \\
 & \qquad\qquad \overset{+}{-}3x^2 \overset{-}{+} 0x \overset{+}{-} 48 \\
\hline
 & \qquad\qquad\qquad\qquad 0
\end{array}
$$

$x^2 - 3 = 0$

$x^2 = 3$

$$\boxed{x = \pm\sqrt{3} \quad , \pm 4i}$$

Given 1 - 2i is a zero of $f(x) = 4x^4 + 17x^2 + 14x + 65$
find all remaining zeros.

Given 1 - 2i is a zero of $f(x) = 4x^4 + 17x^2 + 14x + 65$
find all remaining zeros.

$x = 1 - 2i \qquad x = 1 + 2i$

$(x - 1 + 2i)(x - 1 - 2i)$

$x^2 - x - \cancel{2xi} - x + 1 + \cancel{2i} + \cancel{2xi} - \cancel{2i} - \overset{+}{4}\cancel{i^2}$

$x^2 - 2x + 5$

$$\begin{array}{r} 4x^2 + 8x + 13 \\ x^2 - 2x + 5 \,\overline{\big)\, 4x^4 + 0x^3 + 17x^2 + 14x + 65} \\ -4x^4 \pm 8x^3 \mp 20x^2 \\ \hline 8x^2 - 3x^2 + 14x \\ -8x^2 \pm 16x^2 \mp 40x \\ \hline 13x^2 - 26x + 65 \\ -13x^2 \pm 26x \mp 65 \\ \hline 0 \end{array}$$

$4x^2 + 8x + 13$

$$x = \frac{-8 \pm \sqrt{8^2 - 4 \cdot 4 \cdot 13}}{2 \cdot 4}$$

$$x = \frac{-8 \pm \sqrt{-144}}{8} = \frac{-8 \pm 12i}{8} = \boxed{\frac{-2 \pm 3i}{2},\ 1 \pm 2i}$$

Determine the end behavior of the function

$$f(x) = -12x^5 + x^3 + 10x^2 + 8$$

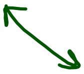
Determine the end behavior of the function

$$f(x) = 13x^4 + 7x^3 - 10x^2 + x - 3$$

Determine the end behavior of the function

$$f(x) = -12x^5 + x^3 + 10x^2 + 8$$

negative odd



$$\lim_{x \to -\infty} f(x) = \infty$$

$$\lim_{x \to \infty} f(x) = -\infty$$

Determine the end behavior of the function

$$f(x) = 13x^4 + 7x^3 - 10x^2 + x - 3$$

postive even

$$\lim_{x \to -\infty} f(x) = \infty$$

$$\lim_{x \to \infty} f(x) = \infty$$