EOC Constructed Response Practice #2

1. Given a sequence determined by $a_n = 2n + 9$, where a_1 is the first term:
   a. What is the 300th term of the sequence? Show your work.
   b. What is an expression in sigma notation for the sum of the first 300 terms of this sequence?
   c. What is the sum of the first 300 terms of this sequence? Show your work and explain the approach you used to find your answer.
   d. What is the sum of the 301st through the 700th term of this sequence? Show your work and explain the approach you used to find your answer.

2. Consider the function $g(x) = -(x-2)^2 + 4$.
   a. Describe the transformations that occur to the graph of $f(x) = x^2$ to get $g(x)$.
   b. Graph $g(x)$ using the vertex and at least 3 points on one side of the vertex. Show your work algebraically.
   c. What are the domain and range of $g(x)$?

3. Consider the real valued functions $f(x) = x^2 + 5$ and $g(x) = 3x - 1$.
   a. What are the domain and range of each function? Explain how you found your answers.
   b. Find $f(g(x))$. Show your algebraic work and explain how you found your answer.
   c. What are the domain and range of $f(g(x))$? Show your algebraic work and explain how you found your answer.

4. Consider the complex numbers $q = 5 + i$ and $r = 2 - 3i$.
   a. What is $2q + 3r$ expressed in $a + bi$ form, where a and b are real values? Show your algebraic work.
   b. What is $(q)(r)$ expressed in $a + bi$ form? Show your algebraic work.
   c. What is q/r expressed in $a + bi$ form? Show your algebraic work and explain how you found your answer.

5. Use polynomial long division to find $\dfrac{-2x^2 + 3x^3 + x^4 + x - 8}{x^2 + x - 1}$. Show your work algebraically. Explain how you found your answer as if you were writing to a student who was not in class when the concept was taught.

6. Consider the graph of the function $f(x) = x^4 - 2x^3 - 8x^2 + 21x$, which has one x-intercept at (–3, 0). Find all other zeros of the function algebraically. Show your work and explain the approach you used to find your answer.

1a. $a_{300} = 2(300) + 9 = 600 + 9 = \boxed{609}$

b. $\sum_{n=1}^{300} (2n+9)$

c. I will use the formula for an arithmetic series,

$S_n = \frac{n}{2}(a_1 + a_n)$. I need to calculate a_1 first.

$a_1 = 2(1) + 9 = 11$

$S_{300} = \frac{300}{2}(11 + 609) = 150(620) = \boxed{93{,}000}$

I used $n = 300$, $a_1 = 11$, and $a_{300} = 609$ in the formula.

d. $a_{700} = 2(700) + 9 = \boxed{1409}$

$S_{700} = \frac{700}{2}(11 + 1409) = 350(1420) = 497{,}000$

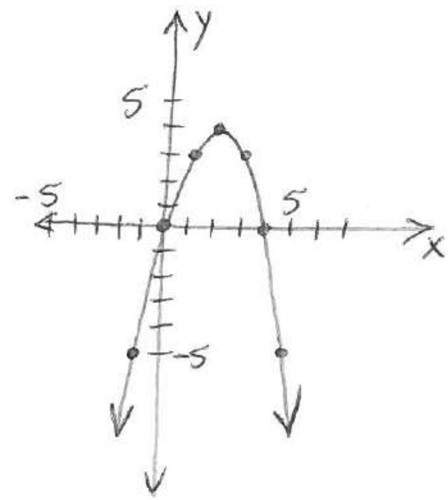
$S_{700} - S_{300} = 497{,}000 - 93{,}000 = \boxed{404{,}000}$

I found a_{700} by using the original formula for the sequence. Then, I found S_{700} using $n = 700$, $a_1 = 11$, and $a_{700} = 1409$. To find the sum of the 301st through the 700th term, I subtracted the sum of first 300 terms from the sum of the first 700 terms.

2.a. I reflected the graph over the x-axis and translated it 2 units to the right and 4 units up.

b. The vertex is at (2,4).

x	y
-1	-5
0	0
1	3
2	4
3	3
4	0
5	-5



$g(-1) = -(-1-2)^2+4 = -(-3)^2+4 = -9+4 = -5$

$g(0) = -(0-2)^2+4 = -(-2)^2+4 = -4+4 = 0$

$g(1) = -(1-2)^2+4 = -(-1)^2+4 = -1+4 = 3$

$g(3) = -(3-2)^2+4 = -(1)^2+4 = -1+4 = 3$

$g(4) = -(4-2)^2+4 = -(2)^2+4 = -4+4 = 0$

$g(5) = -(5-2)^2+4 = -(3)^2+4 = -9+4 = -5$

c. The domain is all real numbers, because any number can be substituted in for x in g(x). The range is $y \leq 4$, because g(x) is a parabola with the vertex at (2,4). The range will be all numbers less than or equal to 4, because the parabola opens downward.

3\. a. For $f(x)$:

Domain: All real numbers

Range: $y \geq 5$

For $g(x)$:

Domain: All real numbers

Range: All real numbers

The domain for each function is all real numbers, because any number can be substituted for x in $f(x)$ or $g(x)$. For $f(x)$ the range is $y \geq 5$, because the graph is a parabola which opens upward with a vertex at $(0,5)$. For $g(x)$ the range is all real numbers, because the graph extends from $-\infty$ to ∞.

b. $f(g(x)) = f(3x-1) = (3x-1)^2+5$

$$= (3x-1)(3x-1)+5$$

$$= 9x^2-3x-3x+1+5$$

$$\boxed{f(g(x)) = 9x^2-6x+6}$$

I substituted $3x-1$ for $g(x)$ in $f(g(x))$ which meant that I substituted $3x-1$ for x in $f(x)$. I squared $(3x-1)$ using FOIL and simplified.

C. The x-coordinate of the vertex is $x = \frac{-b}{2a} = \frac{-(-6)}{2(9)} = \frac{6}{18} = \frac{1}{3}$

$f(g(\frac{1}{3})) = 9(\frac{1}{3})^2 - 6(\frac{1}{3})+6 = 9(\frac{1}{9})-2+6 = 1-2+6 = 5$

The vertex is $(\frac{1}{3},5)$.

The domain is all real numbers, because any number can be substituted for x in $f(g(x))$. The range is $y \geq 5$, because $f(g(x)) = 9x^2-6x+6$ is a parabola which opens upward with a vertex at $(\frac{1}{3},5)$.

4.a. $2q+3r = 2(5+i)+3(2-3i)$

$= 10+2i+6-9i = \boxed{16-7i}$

b. $(q)(r) = (5+i)(2-3i) = 10-15i+2i-3i^2$

$= 10-13i-3(-1)$

$= 10-13i+3 = \boxed{13-13i}$

c. $\frac{q}{r} = \frac{5+i}{2-3i}\left(\frac{2+3i}{2+3i}\right) = \frac{10+15i+2i+3i^2}{4+6i-6i-9i^2}$

$= \frac{10+17i+3(-1)}{4-9(-1)}$

$= \frac{10+17i-3}{4+9}$

$= \frac{7+17i}{13} = \boxed{\frac{7}{13}+\frac{17}{13}i}$

I multiplied the numerator and denominator by the complex conjugate of the denominator. Next, I simplified by combining like terms and substituting -1 for i^2. I put the answer in $a+bi$ form.

5.

$$
\begin{array}{r l}
 & x^2+2x-3 \\
x^2+x-1 & \overline{)\,x^4+3x^3-2x^2+x-8} \\
(-) & \underline{x^4+x^3-x^2} \\
 & 2x^3-x^2+x-8 \\
(-) & \underline{2x^3+2x^2-2x} \\
 & -3x^2+3x-8 \\
(-) & \underline{-3x^2-3x+3} \\
 & 6x-11
\end{array}
$$

$$\boxed{\text{The answer is } x^2+2x-3+\frac{6x-11}{x^2+x-1}.}$$

First, I put the numerator in standard form. Then I divided using long division. First, I divided the leading term of the numerator by the leading term of the denominator to get x^2. I put x^2 above the x^2 term of the numerator. Then, I multiplied x^2 by x^2+x-1 and wrote the product under the numerator. I subtracted this expression from the numerator. I repeated this process until I was left with a remainder which had a degree less than the denominator.

6. $\underline{-3}|\ 1 \quad -2 \quad -8 \quad 21 \quad 0$

$\qquad\quad -3 \quad 15 \quad -21 \quad 0$

$\qquad 1 \quad -5 \quad 7 \quad 0 \quad 0$

$x^3 - 5x^2 + 7x = 0$

$x(x^2 - 5x + 7) = 0$

$\boxed{x=0} \quad x^2 - 5x + 7 = 0$

Use the Quadratic Formula:

$a = 1 \quad b = -5 \quad c = 7$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(7)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{-3}}{2}$$

$$x = \boxed{\frac{5 \pm i\sqrt{3}}{2}}$$

The other zeros were 0, $\frac{5+i\sqrt{3}}{2}$, and $\frac{5-i\sqrt{3}}{2}$.

I know $x = -3$ is a zero of the function, because $(-3,0)$ is an x-intercept. I used synthetic division to divide $x^4 - 2x^3 - 8x^2 + 21x$ by $x+3$. I got $x^3 - 5x^2 + 7x$. I factored out an x to get $x(x^2 - 5x + 7)$. I set each factor equal to zero and used the Quadratic Formula to solve $x^2 - 5x + 7 = 0$.