Class: Math 8-2 and Math 8-3

Let's examine two different situations: One in linear, the other is exponential.

Two equipment rental companies have different penalty policies for returning a piece of equipment late:

Company 1: On day 1, the penalty is \$5. On day 2, the penalty is \$10. On day 3, the penalty is \$15. On day 4, the penalty is \$20 and so on, increasing by \$5 each day the equipment is late.

Company 2: On day 1, the penalty is \$0.01. On day 2, the penalty is \$0.02. On day 3, the penalty is \$0.04. On day 4, the penalty is \$0.08 and so on, doubling in amount each additional day late.

Jim rented a digger from Company 2 because he thought it had the better late return policy. The job he was doing with the digger took longer than he expected, but it did not concern him because the late penalty seemed so reasonable. When he returned the digger 15 days late, he was shocked by the penalty fee. What did he pay, and what would he have paid if he had used Company 1 instead?

Company 1		Company 2	
Day	Penalty	Day	Penalty
1		1	
2		2	
3		3	
4		4	
5		5	
6		6	
7		7	
8		8	
9		9	
10		10	
11		11	
12		12	
13		13	
14		14	
15		15	

1. Which company has a greater 15 day late charge?

F(n) = _____ where n begins at 1. 2. Write an explicit formula for option 1.

3. Write an explicit formula for option 2. F(n) = _____ where n begins at 1.

4. Which option is arithmetic?

5. Which option is geometric?

Now, let's use our graphing calculator to help us solve.

6) A typical thickness of toilet paper is 0.001 inches. Seems pretty thin, right? Let's see what happens when we start folding toilet paper.

a. How thick is the stack of toilet paper after 1 fold? After 2 folds? After 5 folds?

b. Write an explicit formula for the sequence that models the thickness of the folded toilet paper after n folds.

c. After many folds will the stack of folded toilet paper pass the 1 foot mark?

HINT: WE CAN USE THIS FORMULA WHEN SOLVING PROBLEMS THAT DEAL WITH EXPONENTIAL GROWTH:

You try:

7) A rare coin appreciates at a rate of 5.2% a year. If the initial value of the coin is \$500, after how many years will its value cross the \$3,000 mark? Show the formula that will model the value of the coin after t years.

8) A bucket is put under a leaking ceiling. The amount of water in the bucket doubles every minute. After 8 minutes, the bucket is full. After how many minutes is the container half full?

9) A three-bedroom house in Burbville was purchased for \$190,000. If housing prices are expected to increase 1.8% annually in that town, write an explicit formula that models the price of the house in *t* years. Find the price of the house in 5 years.

10) A local college has increased the number of graduates by a factor of 1.045 over the previous year for every year since 1999. In 1999, 924 students graduated. What explicit formula models this situation? Approximately how many students will graduate in 2014? Name:

Unit 5 Homework 5: Exponential Growth

Class: _____ Math 8-2 and 8-3

1. The population of a small city is increasing at a rate of 8.2% a year. If the population was 309,784 people in 2012, what will it be by 2020?

2. Toby receives a 3% raise each year. If his salary this year is \$62,000, how much will he be making in 7 years?

3. Two band mates have only 7 days to spread the word about their next performance. Jack thinks they can each pass out 100 fliers a day for 7 days and they will have done a good job in getting the news out. Meg has a different strategy. She tells 10 of her friends about the performance on the first day and asks each of her 10 friends to each tell a friend on the second day and then everyone who has heard about the concert to tell a friend on the third day and so on, for 7 days. Make an assumption that students make sure they are telling someone who has not already been told.

a) Over the first 7 days, Meg's strategy will reach fewer people than Jack's. Show that this is true.

b) If they had been given more than 7 days, would there be a day on which Meg's strategy would begin to inform more people than Jack's strategy? If not, explain why not. If so, which day would this occur on?

4) The number of cells in a small cancer cluster are increasing exponentially at a rate of 6% a day. After 30 days of research, there were 517 cells. How many cells were there at the start of the research?

5) You deposit \$1500 in an account that pays 5% interest yearly. How much money do you have after 6 years?

6) If I have \$500 in my account after 4 years investing at 2.5% per year, how much money did I start with?

7) A mouse population is 25,000 and is increasing in size at a rate of 20% per year. What is the mouse population after 3 years?

8) David owns a chain of fast food restaurants that operated 200 stores in 2005. If the rate of increase is 8% annually, how many stores does the restaurant operate in 2017 ?

Malik bought a new car for \$15,000. As he drove it off the lot, his best friend, Will, told him that the car's value just dropped by 15% and that it would continue to depreciate 15% of its current value each year. If the car's value is now \$12,750 (according to Will), what will its value be after 5 years?

Complete the table below to determine the car's value after each of the next five years.

Number of years, <i>t</i> , passed since driving the car off the lot	Car value after <i>t</i> years	15% depreciation of current car value	Car value minus the 15% depreciation
0	\$12,750.00	\$1,912.50	\$10,837.50
1	10,837.50		
2			
3			
4			
5			

a) Write an explicit formula for the sequence that models the value of Malik's car t years after driving it off the lot.

b) Use the formula from part (a) to determine the value of Malik's car five years after its purchase. Round your answer to the nearest cent. Compare the value with the value in the table. Are they the same?

c) Use the formula from part (a) to determine the value of Malik's car 7 years after its purchase. Round your answer to the nearest cent.

Hint: We can use the same formula as we did in our last lesson when solving problems that involve exponential decay. You have to make sure you SUBTRACT the percent of decay or depreciation from 1.

You try:

1. Identify the initial value in each formula below, and state whether the formula models exponential growth or exponential decay. Justify your responses.

Initial Amount:

Growth or Decay?

- a. $f(t) = 2\left(\frac{2}{5}\right)^t$
- b. $f(t) = 2\left(\frac{5}{3}\right)^t$
- C. $f(t) = \frac{2}{3}(3)^t$
- d. $f(t) = \frac{2}{3} \left(\frac{1}{3}\right)^t$
- e. $f(t) = \frac{3}{2} \left(\frac{2}{3}\right)^t$
- 2. If a person takes a given dosage (*d*) of a particular medication, then the formula $f(t) = d (0.8)^t$ represents the concentration of the medication in the bloodstream *t* hours later. If Charlotte takes 200 mg of the medication at 6:00 a.m., how much remains in her bloodstream at 10:00 a.m.? How long does it take for the concentration to drop below 1 mg?

3. When you breathe normally, about 12% of the air in your lungs is replaced with each breath. Write an explicit formula for the sequence that models the amount of the original air left in your lungs, given that the initial volume of air is 500 mL. Use your model to determine how much of the original 500 mL remains after 50 breaths.

4. Ryan bought a new computer for \$2,100. The value of the computer decreases by 50% each year. When will the value drop below \$300?

5. Kelli's mom takes a 400 mg dose of aspirin. Each hour, the amount of aspirin in a person's system decreases by about 29%. How much aspirin is left in her system after 6 hours?

6. After 3 years, a car is worth \$17,909. If it is depreciating by 8% a year, what was its initial value?

7. A stock fell at the same rate for 2 years. It started at a value of \$14, and is now is worth \$6.86. At what rate was it falling?

Name: _____ Unit 5 Lesson 6 Homework: Exponential Decay

Class: _____ Math 8-2 and 8-3

1. From 2000 to 2013, the value of the U.S. dollar has been shrinking. The value can be modeled by the following formula:

 $v(t) = 1.36 (0.9758)^t$, where t is the number of years since 2000.

a. How much was a dollar worth in the year 2005?

b. Estimate the year in which the value of the dollar fell below \$1.00.

- 2. A construction company purchased some equipment costing \$300,000. The value of the equipment depreciates (decreases) at a rate of 14% per year.
 - a. Write a formula that models the value of the equipment.
 - b. What is the value of the equipment after 9 years?
 - c. Estimate when the equipment will have a value of \$50,000.

3. The number of newly reported cases of HIV (in thousands) in the United States from 2000 to 2010 can be modeled by the following formula:

 $f(t) = 41(0.9842)^t$, where t is the number of years after 2000.

- a. Identify the growth factor.
- b. Calculate the estimated number of new HIV cases reported in 2004.
- c. During what year did the number of newly reported HIV cases drop below 36,000?

- 4. Doug drank a soda with 130 mg of caffeine. Each hour, the caffeine in the body diminishes by about 12%.
 - a. Write formula to model the amount of caffeine remaining in Doug's system.
 - b. How much caffeine remains in Doug's system after 2 hours?
 - c. How long will it take for the level of caffeine in Doug's system to drop below 50 mg?
- 5. 64 teams participate in a softball tournament in which half the teams are eliminated after each round of play.
 - a. Write a formula to model the number of teams remaining after any given round of play.
 - b. How many teams remain in play after 3 rounds?
 - c. How many rounds of play will it take to determine which team wins the tournament?