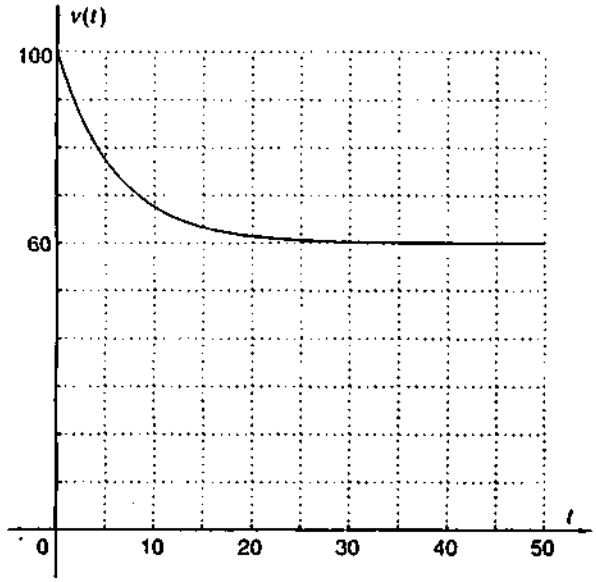
Name _____ Group _____ Date _____

Exploration 1-3: Introduction to Definite Integrals

Objective: Find out what a definite integral is by working a real-world problem that involves the speed of a car.

As you drive on the highway you accelerate to 100 feet per second to pass a truck. After you have passed, you slow down to a more moderate 60 ft/sec. The diagram shows the graph of your velocity, $v(t)$, as a function of the number of seconds, t, since you started slowing.



1. What does your velocity seem to be between t = 30 and t = 50 seconds? How far do you travel in the time interval [30, 50]?

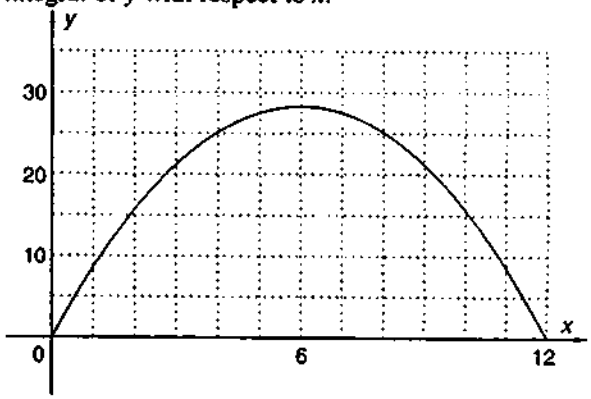
2. Explain why the answer to Problem 1 can be represented as the area of a *rectangular* region of the graph. Shade this region.

3. The distance you travel between $t = 0$ and $t = 20$ can also be represented as the area of a region bounded by the (curved) graph. Count the number of squares in this region. Estimate the area of parts of squares to the nearest 0.1 square space. For instance, how would you count this partial square?

4. How many feet does each small square on the graph represent? How far, therefore, did you go in the time interval [0, 20]?

5. Problems 3 and 4 involve finding the product of the x-value and the y-value for a function where y may *vary* with x. Such a product is called the **definite integral** of y with respect to x. Based on the units of t and $v(t)$, explain why the definite integral of $v(t)$ with respect to t in Problem 4 has feet for its units.

6. The graph shows the cross-sectional area, y square inches, of a football as a function of the distance, x inches, from one of its ends. Estimate the definite integral of y with respect to x.



7. What are the units of the definite integral in Problem 6? What, therefore, do you suppose the definite integral represents?

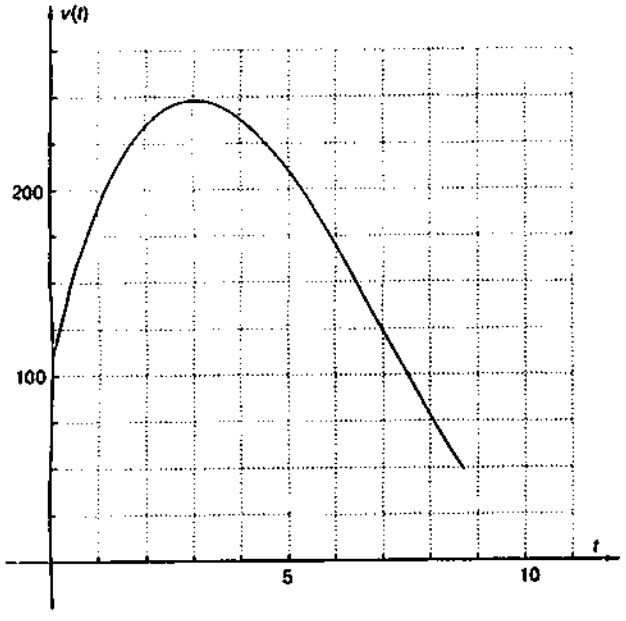
8. What did you learn as a result of doing this Exploration that you did not know before? (Over)

Name _____ Group _____ Date _____

Exploration 1-4: Definite Integrals by Trapezoidal Rule

Objective: Estimate the definite integral of a function numerically rather than graphically by counting squares.

Rocket Problem: Ella Vader (Darth's daughter) is driving in her rocket ship. At time $t = 0$ minutes she fires her rocket engine. The ship speeds up for a while, then slows down as Alderaan's gravity takes its effect. The graph of her velocity, $v(t)$ miles per minute, is shown below.



1. What mathematical concept would be used to estimate the distance Ella goes between $t = 0$ and $t = 8$?

2. Estimate the distance in Problem 1 geometrically.

3. Ella figures that her velocity is given by

   $$v(t) = t^3 - 21t^2 + 100t + 110.$$

   Plot this graph on your grapher. Does the graph confirm or refute what Ella figures? Tell how you arrive at your conclusion.

4. Divide the region under the graph from $t = 0$ to $t = 8$, which represents the distance, into four vertical strips of equal width. Draw four trapezoids whose areas approximate the areas of these strips, and whose parallel sides extend from the x-axis to the graph. By finding the areas of these trapezoids, estimate the distance Ella goes. Does the answer agree with Problem 2?

5. The technique in Problem 4 is the **trapezoidal rule.** Put a program into your grapher to use this rule. The function equation may be stored as y_1. The input should be the starting time, the ending time, and the number of trapezoids. The output should be the value of the definite integral. Test your program by using it to answer Problem 4.

6. Use the program from Problem 5 to estimate the definite integral using 20 trapezoids.

7. The *exact* value of the definite integral is the *limit* of the estimates by trapezoids as the width of each trapezoid approaches zero. By using the program from Problem 5, make a conjecture about the exact value of the definite integral.

8. What is the fastest Ella went? At what time was that?

9. Approximately what was Ella's rate of change of velocity when $t = 5$? Was she speeding up or slowing down at that time?

10. At what time does Ella stop? Based on the graph, does she stop abruptly or gradually?

11. What did you learn as a result of doing this Exploration that you did not know before? (Over)