# Experimental Uncertainty

Uncertainty is a representation of how precise a measurement is. When the weather report says it will snow 3-6 inches today that is an example of uncertainty. A more useful way to represent this snowfall uncertainty is: $4.5\ in \pm 1.5\ in$. This makes it more clear what the average value is and how large the uncertainty is compared to that average. In lab you will often need to determine the uncertainty in the measurements you make and in the quantities you calculate from those measurements. This document will show you how.

## Random Uncertainty

Your friend asks you to measure their height. You use a tape measure (with centimeter markings) and get 180 cm. Your friend asks you to quantify how certain you are about that measurement. To answer this you decide to ask two more people to measure your friend's height. You record these *multiple trials* of the height measurement in a table.

| Trial | Height (cm) |
|---|---|
| 1 | 180 cm |
| 2 | 179 cm |
| 3 | 182 cm |
| Average | 180.3 cm |

You realize that by doing multiple trials and finding the average you've estimated how certain the measurement is. Trial 3 is furthest from the average, $182\ cm - 180.3\ cm = 1.7\ cm$ from the average to be precise. This is called the *random uncertainty* of the measurement. The complete statement of the measurement of your friend's height is $180.3\ cm \pm 1.7\ cm$. This means your friend's height is somewhere between $178.6\ cm$ and $182.0\ cm$.

There is no way to measure your friend's height exactly. There's no way to make any measurement exactly. Every measurement has uncertainty and the only way to estimate that uncertainty is to do multiple trials of the measurement. Notice the word 'estimate' in the previous sentence. Doing multiple trials doesn't determine the uncertainty in a measurement, it only estimates it. If you want to estimate the uncertainty better you need to do even more trials. You find one more person measure your friend's height.

| Trial | Height (cm) |
|---|---|
| 1 | 180 cm |
| 2 | 179 cm |
| 3 | 182 cm |
| 4 | 178 cm |
| Average±random uncertainty | $179.8\ cm \pm 2.2\ cm$ |

Two things to notice here: 1) the average decreased slightly which makes sense since trial 4 was less than the previous average, 2) the estimate of the uncertainty in the measurement <u>increased</u>. Here's how that makes sense: Remember that multiple trials is how you *estimate* the random uncertainty in a measurement. By doing even more trials you get an *even better* *estimate*. If you take a very small number of trials you will always underestimate the uncertainty in the measurement. Because of this it's best to do as many trials as time allows for.

## Instrument Uncertainty

You ask the same 4 people to use a ruler (with millimeter markings) to measure the length of your phone.

| Trial | Length (cm) |
|---|---|
| 1 | 14.3 cm |
| 2 | 14.3 cm |
| 3 | 14.3 cm |
| 4 | 14.3 cm |
| Average±random uncertainty | $14.3\ cm \pm ???\ cm$ |

All 4 trials ended up being the same. It is rare that this happens. If we used the method for determining random uncertainty we would get $\pm 0.0\ cm$ which is not reasonable. No measurement is exact. Notice that the length of the phone was measured with a ruler that has millimeter markings. In this case it's the measurement instrument that is limiting the precision of the measurement. This is called *instrument uncertainty*, and it's equal to half the smallest increment. For the ruler this would be $0.1\ cm/2 = 0.05\ cm$. The measurement of the length of your phone with uncertainty would then be $14.3\ cm \pm 0.05\ cm$.

### Random uncertainty vs. instrument uncertainty and the importance of multiple trials

The only time you need to address instrument uncertainty is when all trials are identical. If even one trial is different from the others then random uncertainty is larger and you can ignore instrument uncertainty. Estimate random uncertainty using the method described in the random uncertainty section.

## The Weakest Link Rule

Now that you've determined your friend's height including uncertainty ($179.8\ cm \pm 2.2\ cm$) you decide to solve the following experimental problem:

Determine the smallest rectangular volume that your friend could stand up in.

The method you come up with is to ask the same 4 people to help measure your friend's left-to-right width and front-to-back depth. Together with the previously measured height, you'll determine the resulting volume $V = W \times D \times H$.

| Trial | Width (cm) |
|---|---|
| 1 | 52 cm |
| 2 | 50 cm |
| 3 | 53 cm |
| 4 | 52 cm |
| Average±random uncertainty | $51.8\ cm \pm 1.8\ cm$ |

| Trial | Depth (cm) |
|---|---|
| 1 | 28 cm |
| 2 | 31 cm |
| 3 | 30 cm |
| 4 | 33 cm |
| Average$\pm$random uncertainty | $30.5\ cm \pm 2.5\ cm$ |

The smallest rectangular volume that your friend could stand up in is then (using the average values of width, depth, and height):

$$V = W \times D \times H = (51.8\ cm)(30.5\ cm)(179.8\ cm) = 2.84 \times 10^5 cm^3 = 0.284\ m^3$$

We're not done. This calculated volume is based on measurements that have uncertainty. That means this volume has uncertainty too. We need to determine it so we can represent how precise our calculation is. This is done using the *weakest link rule* and here's how:

1. **Determine percent uncertainties**: For each quantity used in calculating the volume convert its uncertainty to a percent. This is done by dividing the uncertainty (the $\pm$ value) by the average. By converting to percentages the uncertainties can be meaningfully compared to one another.

| | |
|---|---|
| Width | $\frac{1.8\ cm}{51.8\ cm} = 3.47\%$ |
| Depth | $\frac{2.5\ cm}{30.5\ cm} = 8.20\%$ |
| Height | $\frac{2.2\ cm}{179.8\ cm} = 1.22\%$ |

2. **Identify the weakest link**: The uncertainty in the width, depth, and height all contribute to the uncertainty in the volume, but one of them contributes the most. This is the *weakest link*, and in this case it's the depth. It has the highest percent uncertainty of all the measurements.

3. **Determine the uncertainty in the calculated quantity**: This is done by applying the weakest link's (the depth's) percent uncertainty to the calculated quantity (the volume).

$$\Delta V = (0.284\ m^3)(0.082) = 0.023\ m^3$$

Now we're done! The smallest rectangular volume your friend could stand up in is:

$$V = 0.284\ m^3 \pm 0.023\ m^3$$

# Additional Important Details About Experimental Uncertainty

## Independent trials

You've already learned the purpose and importance of multiple trials. For multiple trials to accurately estimate random uncertainty those trials must be *independent*. Independent means the trials must not be correlated in any way. To guarantee this each trial should be performed by a different person when possible and the result should be kept private until all trials are completed.

## Reducing uncertainty

Measurements with lower uncertainty are always preferable. It's tempting to think that multiple trials reduces uncertainty, but this is not what multiple trials do. <u>Remember, multiple trials are how random uncertainty is estimated, NOT how uncertainty is reduced</u>. Taking additional trials leads to a better estimate of uncertainty, not a smaller uncertainty. To reduce uncertainty you have two options in general:

- **Reducing random uncertainty**: Make changes to the design of your experiment so that it's more consistent and repeatable.
- **Reducing instrument uncertainty**: In the rare case where all of your trials are the same value it means the largest source of uncertainty is the measurement device itself. If available, choose one with smaller instrument uncertainty.

## When there's no clear weakest link

In the above example where we determined the smallest rectangular volume your friend could stand up in the percent uncertainties in the measurements were: width – 3.47%, depth – 8.20%, height – 1.22%. Depth was the clear weakest link. But, what if the percent uncertainty in the width had instead been 7.47%. There no longer is a clear weakest link since two of the measurements have similar percent uncertainties. When this happens <u>you need to add all the similar percent uncertainties together</u> instead of just using one of them, then apply that to the calculated values. Even better, you could always add all of the percent uncertainties together instead of choosing a weakest link in the first place.