AP Calculus Exam Prep Assignment #9 KEY

Multiple Choice

1)

$$u = 2x - 1 \Rightarrow x = \frac{u+1}{2} \Rightarrow y = \left(\frac{u+1}{2}\right)^2 + 2$$

$$\frac{dy}{du} = 2\left(\frac{u+1}{2}\right)\left(\frac{1}{2}\right) = \frac{u+1}{2} = \frac{(2x-1)+1}{2} = x$$

D)

2) $f'(x) = \dfrac{1(x+1) - 1(x-1)}{(x+1)^2} = \dfrac{2}{(x+1)^2} \Rightarrow f'(1) = \dfrac{2}{(1+1)^2} = \dfrac{1}{2}$ **D)**

3) $\dfrac{dy}{dx} = 2\cos 3x(-\sin 3x)(3) = -6\cos 3x \sin 3x$ **A)**

4) $h'(x) = 2f(x)[f'(x)] - 2g(x)[g'(x)] = 2f(x)[-g(x)] - 2g(x)[f(x)] = -4f(x)g(x)$ **C)**

5)

$$\frac{d^2}{dx^2}(f(x^3)) = \frac{d}{dx}\left(3x^2 f'(x^3)\right) = 6xf'(x^3) + 3x^2\left(\frac{d}{dx}f'(x^3)\right) = 6xg(x^3) + 3x^2\left(\frac{d}{dx}g(x^3)\right)$$

$$= 6xg(x^3) + 3x^2\left(3x^2 f(x^6)\right) = 6xg(x^3) + 9x^4 f(x^6)$$

D)

6) $A = \pi r^2 \quad \dfrac{dA}{dt} = 2\pi r\dfrac{dr}{dt} \quad \dfrac{dA}{dt} = 2\dfrac{dr}{dt} \quad 2\dfrac{dr}{dt} = 2\pi r\dfrac{dr}{dt} \Rightarrow r = \dfrac{1}{\pi}$ **C)**

7) $A = 6x^2 \quad \dfrac{dA}{dt} = 12x\dfrac{dx}{dt} \quad \dfrac{dA}{dt} = 6\dfrac{dx}{dt} \quad 6\dfrac{dx}{dt} = 12x\dfrac{dx}{dt} \Rightarrow x = \dfrac{1}{2}$ **E)**

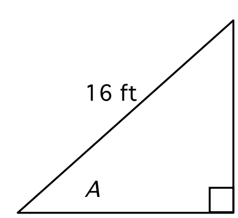
8) $\dfrac{dr}{dt} = 0.3 \quad \dfrac{dV}{dt} = 4\pi r^2\dfrac{dr}{dt} \quad \dfrac{dV}{dt} = (100\pi)(0.3) = 30\pi$ **E)**

9) [GC] The figure shows a 16-foot ladder leaning against a wall. The tip of the ladder is sliding down the wall at the rate of 5.6 feet per second. What is the rate of change, in radians per second, of the angle A at the instant when the tip of the ladder is 7.0 feet above the ground?

$$\sin A = \frac{y}{16} \Rightarrow y = 16\sin A \Rightarrow \frac{dy}{dt} = 16\cos A\left(\frac{dA}{dt}\right)$$

$$-5.6 = 16\left(\frac{\sqrt{207}}{16}\right)\frac{dA}{dt} \Rightarrow \frac{dA}{dt} = \frac{-5.6}{\sqrt{207}} \approx -0.389$$

C)



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Problems

10) [1995 AB 5] Water is draining from a right conical tank with height 12 feet and diameter 8 feet into a cylindrical tank that has a base with area 400π square feet. The depth h, in feet, of the water in the conical tank is changing at the rate of $(h - 12)$ feet per minute.
(The volume V of a cone with radius r and height h is $V = (1/3)\pi r^2 h$)

A) Write an expression for the volume of water in the conical tank as a function of h.

$$r = \frac{1}{3}h \quad V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{1}{27}\pi h^3$$

B) At what rate is the volume of water in the conical tank changing when $h = 3$?

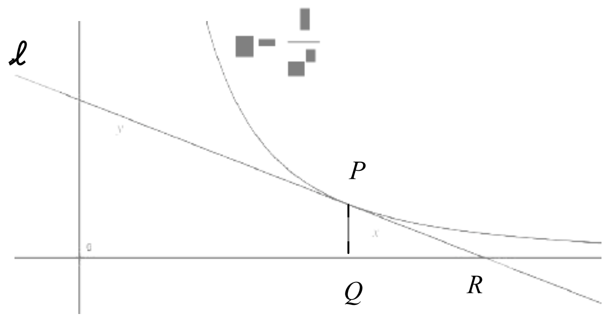
$$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt} = \frac{1}{9}\pi(3^2)(3-12) = -9\pi \text{ ft}^3/\text{min}$$

C) Let y be the depth, in feet, of the water in the cylindrical tank. At what rate is y changing when $h = 3$? Indicate units of measure.

Since water is draining from the conical tank at $-9\pi \text{ ft}^3/\text{min}$, the cylindrical tank is filling at a rate of $9\pi \text{ ft}^3/\text{min}$.

$$V = 400\pi y \quad \frac{dV}{dt} = 400\pi\frac{dy}{dt} \Rightarrow \frac{dy}{dt} = \frac{9\pi}{400\pi} = \frac{9}{400} \text{ ft/min}$$

11) In the figure at the right, line l is tangent to the graph of $y = \frac{1}{x^2}$ at the point P, with coordinates $\left(w, \frac{1}{w^2}\right)$, where $w > 0$. Point Q has coordinates $(w,0)$. Line l crosses the x-axis at point R, with coordinates $(k,0)$.



A) Find the value of k when $w = 3$.

$$\frac{dy}{dx} = -\frac{2}{x^3} = -\frac{2}{27}$$

$$y - 0 = -\frac{2}{27}(x-k) \Rightarrow \frac{1}{9} = -\frac{2}{27}(3-k) \Rightarrow -\frac{3}{2} = 3 - k \Rightarrow k = \frac{9}{2}$$

B) For all $w > 0$, find k in terms of w.

$$\frac{1}{w^2} - 0 = -\frac{2}{w^3}(w-k) \Rightarrow -\frac{w}{2} = w - k \Rightarrow k = \frac{3w}{2}$$

C) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of k with respect to time?

$$\frac{dw}{dt} = 7 \quad k = \frac{3w}{2} \Rightarrow \frac{dk}{dt} = \frac{3}{2}\frac{dw}{dt} \quad \frac{dk}{dt} = \frac{3}{2}(7) = 10.5 \text{ units per second.}$$

D) Suppose that w is increasing at the constant rate of 7 units per second. When $w = 5$, what is the rate of change in area of ΔPQR with respect to time? Determine whether the area is increasing or decreasing at this instant.

$$A = \frac{1}{2}(k - w)\left(\frac{1}{w^2}\right) = \frac{k - w}{2w^2}$$

$$\frac{dA}{dt} = \frac{2w^2\left(\frac{dk}{dt} - \frac{dw}{dt}\right) - 4w\frac{dw}{dt}(k - w)}{4w^4} \qquad w = 5,\ k = \frac{15}{2},\ \frac{dw}{dt} = 7,\ \frac{dk}{dt} = \frac{21}{2}$$

$$\frac{dA}{dt} = \frac{2(25)\left(\frac{7}{2}\right) - 20(7)\left(\frac{5}{2}\right)}{4(625)} = \frac{175 - 350}{4(625)} = \frac{-175}{4(625)} = -\frac{7}{100}$$

Alternate solution: path taken by 2 students (2008 BC)

$$A = \frac{1}{2}(k - w)\left(\frac{1}{w^2}\right) = \frac{1}{2}\left(\frac{3w}{2} - w\right)\left(\frac{1}{w^2}\right) = \frac{1}{4w}$$

$$\frac{dA}{dt} = -\frac{1}{4w^2}\left(\frac{dw}{dt}\right) \qquad w = 5,\ \frac{dw}{dt} = 7$$

$$\frac{dA}{dt} = -\frac{1}{100}(7) = -\frac{7}{100}$$