AP Calculus Exam Prep Assignment #8 KEY

1) $\frac{dy}{dx} = \frac{1}{x\sqrt{x^2+1}}\left(\sqrt{x^2+1} + \frac{x^2}{\sqrt{x^2+1}}\right) = \frac{1}{x\sqrt{x^2+1}}\left(\frac{2x^2+1}{\sqrt{x^2+1}}\right) = \frac{2x^2+1}{x(x^2+1)}$ **D)**

2)

$x = \sqrt{u^2+1} \Rightarrow x^2 - 1 = u^2 \quad \frac{d}{du}\sin u^2 = 2u\cos u^2$ or $\frac{dy}{du} = \frac{dy}{dx}\cdot\frac{dx}{du} = 2x\cos(x^2-1)\left[\frac{u}{\sqrt{u^2+1}}\right]$ **D)**

$= 2\sqrt{u^2+1}\left(\cos u^2\right)\left[\frac{u}{\sqrt{u^2+1}}\right] = 2u\cos u^2$

3) $dy = \left[\sqrt{1+x^2} + \frac{x^2}{\sqrt{1+x^2}}\right]dx \Rightarrow dy = (1+0)2 = 2$ **E)**

4) $\sec^2(xy)\left[y + x\frac{dy}{dx}\right] = 1 \Rightarrow y + x\frac{dy}{dx} = \cos^2(xy) \Rightarrow \frac{dy}{dx} = \frac{\cos^2(xy) - y}{x}$ **E)**

5) $\frac{dy}{dx} = \frac{\left(e^x + e^{-x}\right)\left(e^x + e^{-x}\right) - \left(e^x - e^{-x}\right)\left(e^x - e^{-x}\right)}{\left(e^x + e^{-x}\right)^2} = \frac{e^{2x} + 2 + e^{-2x} - \left(e^{2x} - 2 + e^{-2x}\right)}{\left(e^x + e^{-x}\right)^2} = \frac{4}{\left(e^x + e^{-x}\right)^2}$ **C)**

6)

$\frac{dy}{dx} = \frac{1}{x^2+y^2}\left[2x + 2y\frac{dy}{dx}\right] \Rightarrow \frac{dy}{dx}\left(x^2+y^2\right) - 2y\frac{dy}{dx} = 2x \Rightarrow \frac{dy}{dx} = \frac{2x}{x^2 - 2y + y^2}$ **D)**

at (1,0) $\frac{dy}{dx} = \frac{2}{1-0-0} = 2$

7) $\frac{ds}{dt} = 4(t-2)^3 \quad 4(t-2)^3 = 0 \Rightarrow t = 2$ **B) 1**

8)

$s' = 3t^2 - 12t + 9 \quad s' = 0 \Rightarrow t^2 - 4t + 3 = 0 \Rightarrow t = 1, 3$

$s' < 0$ for $1 < t < 3$, so s is increasing on $-\infty < t < 1$ and $3 < t < \infty$ **E)**

9) $v = 3t^2 + 3 \quad a = 6t$ The speed is decreasing when $a < 0$. This occurs when $t < 0$ **C) $t < 0$**

10)

$v(t) = e^{-t}(4\cos 4t) + -e^{-t}\sin 4t$

Distance $= \int_0^1 |v(t)|dt = \int_0^1 \left|e^{-t}(4\cos 4t) + -e^{-t}\sin 4t\right|dt \approx 1.671$ **E)**

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Problems

11) A particle moves along a line so that at any time t its position is given by $x(t) = 2\pi t + \cos 2\pi t$.

A) Find the velocity at time t.

$v(t) = 2\pi - 2\pi \sin 2\pi t$

B) Find the acceleration at time t.

$a(t) = -4\pi^2 \cos 2\pi t$

C) What are all values of t, $0 \le t \le 3$, for which the particle is at rest?

$v(t) = 0 \Rightarrow \sin 2\pi t = 1 \Rightarrow 2\pi t = \frac{\pi}{2} + 2n\pi \Rightarrow t = \frac{1}{4} + n$

$t = \frac{1}{4}, \frac{5}{4}, \frac{9}{4}$

D) What is the maximum velocity?

$a(t) = 0 \Rightarrow \cos 2\pi t = 0 \Rightarrow 2\pi t = \frac{\pi}{2}$ or $\frac{3\pi}{2} \Rightarrow t = \frac{1}{4}, \frac{3}{4}$

$v\left(\frac{1}{4}\right) = 0, v\left(\frac{3}{4}\right) = 4\pi$

12) (1995 AB2) A particle moves along the y-axis so that its velocity at any time $t \ge 0$ is given by $v(t) = t\cos t$. At time $t = 0$, the position of the particle is $y = 3$.

A) For what values of t, $0 \le t \le 5$, is the particle moving upward?

$t\cos t > 0 \Rightarrow 0 < t < \frac{\pi}{2}$ and $\frac{3\pi}{2} < t \le 5$ on $[0,5]$

B) Write an expression for the acceleration of the particle in terms of t.

$a(t) = \cos t - t\sin t$

C) Write an expression for the position $y(t)$ of the particle.

$y(t) = \int t\cos t\, dt$ $\quad u = t \quad dv = \cos t\, dt$ $\quad du = dt \quad v = \sin t$

$y(t) = t\sin t - \int \sin t\, dt = t\sin t + \cos t + C$

$3 = 0 + \cos 0 + C \Rightarrow C = 2 \quad y(t) = t\sin t + \cos t + 2$

D) For $t > 0$, find the position of the particle the first time the velocity of the particle is zero.

$t\cos t = 0 \Rightarrow \cos t = 0 \Rightarrow t = \frac{\pi}{2}$

$y\left(\frac{\pi}{2}\right) = \frac{\pi}{2}\sin\frac{\pi}{2} + \cos\frac{\pi}{2} + 2 = \frac{\pi}{2} + 2$