AP Calculus Exam Prep Assignment #7 KEY

1) $\int_{-1}^{2}\frac{|x|}{x}dx = \int_{-1}^{0}\frac{|x|}{x}dx + \int_{0}^{2}\frac{|x|}{x}dx = \lim_{b\to 0}\int_{-1}^{b}\frac{|x|}{x}dx + \lim_{c\to 0}\int_{c}^{2}\frac{|x|}{x}dx$

$= \lim_{b\to 0}\int_{-1}^{b}\frac{-x}{x}dx + \lim_{c\to 0}\int_{c}^{2}\frac{x}{x}dx = \lim_{b\to 0}\left[-x\right]_{-1}^{b} + \lim_{c\to 0}\left[x\right]_{c}^{2}$

$= \lim_{b\to o}\left[-b-(1)\right] + \lim_{c\to 0}\left[2-c\right] = -1+2 = 1$ **B)**

2) $v = t^2 - t = t(t-1)$ This equals zero at $t = 0$ and $t = 1$.
v is negative on (0,1) and positive on (1,2)

$s = -\int_0^1 \left(t^2 - t\right)dt + \int_1^2 \left(t^2 - t\right)dt$ $\qquad s = -\left[\frac{t^3}{3} - \frac{t^2}{2}\right]_0^1 + \left[\frac{t^3}{3} - \frac{t^2}{2}\right]_1^2$

$s = -\left[\frac{1}{3} - \frac{1}{2}\right] + \left[\left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - \frac{1}{2}\right)\right] = 1$ **C)**

3) $s(4) - s(2) = \int_2^4 v(t)dt$ $\qquad v(t) = \int \alpha dt = \int (8 - 6t)dt = 8t - 3t^2 + C$

$25 = 8(1) - 3(1)^2 + C \Rightarrow C = 20$ $\qquad v(t) = 8t - 3t^2 + 20$

$s(4) - s(2) = \int_2^4 \left(8t - 3t^2 + 20\right)dt = \left[4t - t^3 = 20t\right]_2^4 = 32$ **D)**

4) $v = 8 - 6t = 0 \Rightarrow t = \frac{4}{3}$ $\qquad v > 0$ on $\left(0, \frac{4}{3}\right)$ $\quad v < 0$ on $\left(\frac{4}{3}, 1\right)$

$s = \int_1^{4/3}(8 - 6t)dt + -\int_{4/3}^{2}(8 - 6t)dt$ $\qquad s = \left[8t - 3t^2\right]_1^{4/3} - \left[8t - 3t^2\right]_{4/3}^{2} = \frac{5}{3}$ **C)**

5) $t = 8$ **D)**

6) $t = 10$ (remember, speed is the absolute value of velocity) **E)**

7) **E** (look at the symmetry above and below the x-axis in the others)

8) A) $a(t) = kt$ $\qquad v(t) = \int a(t)dt = \int kt\, dt = \frac{k}{2}t^2 + C$

$300 = \frac{k(0)^2}{2} + C \Rightarrow C = 300$ $\qquad v(t) = \frac{k}{2}t^2 + 300$

$0 = \frac{k}{2}(10)^2 + 300 \Rightarrow k = -6$ $\qquad v(t) = -3t^2 + 300$

B) $s = \int_0^{10}\left(-3t^2 + 300\right)dt = \left[-t^3 + 300t\right]_0^{10} = 2{,}000m$

9) A) $v(t) = \int 10e^{2t}dt = 5e^{2t} + C$ $\qquad 5 = 5e^{2(0)} + C \Rightarrow C = 0$

$v(t) = 5e^{2t}$

B) $5 = 5e^{2t} \Rightarrow e^{2t} = 1 \Rightarrow 2t = \ln 1 \Rightarrow t = 0$

$15 = 5e^{2t} \Rightarrow e^{2t} = 3 \Rightarrow 2t = \ln 3 \Rightarrow t = \frac{\ln 3}{2}$

$s = \int_0^{(\ln 3)/2} 5e^{2t}\,dt = \frac{5}{2}e^{2t}\Big|_0^{(\ln 3)/2} = \frac{5}{2}e^{\ln 3} - \frac{5}{2} = 5$ units

C) $s(t) = \int v(t)dt = \int 5e^{2t}dt = \frac{5}{2}e^{2t} + C \qquad 0 = \frac{5}{2}e^{0} + C \Rightarrow C = -\frac{5}{2}$

$s(t) = \frac{5}{2}e^{2t} - \frac{5}{2}$

10) A) $v(t) = 3t^2 - 12t + 9 \qquad a(t) = 6t - 12 \qquad a'(t) = 6$

Since the derivative of acceleration is constant and positive, the mimimum value occurs at the left endpoint, so the minimum acceleration is $a(0) = -12$.

B) Velocity = 0 at $t = 1$ and $t = 3$. Testing values between shows that v > 0 on (0,1) and (3,5) and v < 0 on (1,3). So:

$s = \int_0^1 (3t^2 - 12t + 9)dt - \int_1^3 (3t^2 - 12t + 9)dt + \int_3^5 (3t^2 - 12t + 9)dt = 28$

C) 28/5 = 5.6 average SPEED

Average velocity = $\frac{1}{5-0}\int_0^5 v(t)\,dt = 4$