AP Calculus Exam Prep Assignment #6 KEY

1) $f(x)$ is continuous on $[a,b]$. If this interval is partitioned into n equal subintervals of length Δx, and if x_k is a number in the k^{th} subinterval, then $\lim_{n\to\infty}\sum_{k=1}^{n} f(x_k)\Delta x$ =

C) $\int_a^b f(x)\,dx$

2) If $F'(x) = G'(x)$ for all x, then:

A) $\int_a^b F'(x)dx = \int_a^b G'(x)dx$

3) If $f(x)$ is continuous on $[a,b]$ then there exists at least one number c where $a < c < b$ such that $\int_a^b f(x)dx =$

C) $f(c)(b-a)$ The Mean Value Theorem is used here.

4) If $f(x)$ is continuous on $[a,b]$ and k is a constant, then $\int_a^b kf(x)dx =$

D) $k\int_a^b f(x)dx$

5) $\frac{d}{dt}\int_0^t \sqrt{x^3+1}\,dx =$

A) $\sqrt{t^3+1}$

6) If $F(x) = \int_1^{2x} \frac{1}{1-t^3}dt$, then $F'(x) =$

E) $\frac{2}{1-8x^3}$

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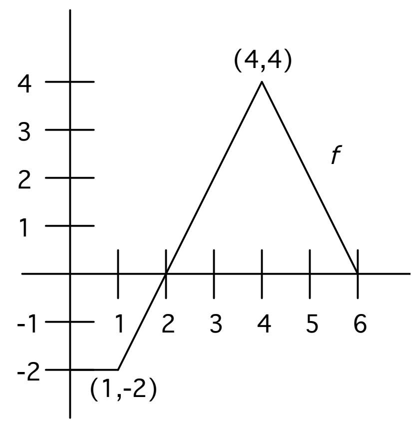
7) Suppose f is continuous on $1 \le x \le 2$, that $f'(x)$ exists on $1 < x < 2$, that $f(1) = 3$, and that $f(2) = 0$. Which of the following is not necessarily true?

C) There exists a number c in [1,2] such that $f'(c) = 3$

8) If $G(2) = 5$ and $G'(x) = \dfrac{10x}{9 - x^2}$ then an estimate of $G(2.2)$ using local linearization is approximately:

C) 5.8 $y = 4x - 3$ is the equation of the tangent at (2,5)

9) Let $H(x) = \int_0^x f(t)\,dt$ where f is the function whose graph appears to the right. The local linearization of $H(x)$ near $x = 3$ is $H(x)$=



B) 2*x* - 4

10) $f(x) = \int_0^{x^2+2} \sqrt{1+\cos t}\,dt$. Then $f'(t) =$

A) $2x\sqrt{1+\cos(x^2+2)}$

11) $\int_0^6 f(x-1)\,dx =$

E) $\int_1^7 f(x)\,dx$

12) The number c satisfying the Mean Value Theorem for $f(x) = \sin x$ on [1,1.5] is:

$$f'(c) = \frac{\sin 1.5 - \sin 1}{1.5 - 1} \approx 0.3120 \quad \cos c \approx 0.3120 \Rightarrow c \approx \cos^{-1}(0.3120) = 1.253$$ **D)**

13) The only function that doesn't satisfy the Mean Value Theorem on the interval specified is:

D) $f(x) = x + \dfrac{1}{x}$ on [-1,1]

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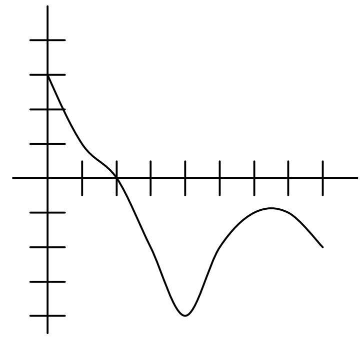
14) **E)** 16



15) If the trapezoidal rule is used with $n = 5$ then $\int_0^1 \frac{dx}{1+x^2}$ = _______(to 3 decimal places)

x	0	0.2	0.4	0.6	0.8	1.0
y	1	0.962	0.862	0.735	0.610	0.5

$0.2\left(0.5\left(1+2(0.962)+2(0.862)+2(0.735)+2(0.610)+0.5\right)\right)$ **A)**

≈ 0.784

16) The table shows the speed of an object in feet per second during a 3-second period. Estimate the distance the object travels, using the trapezoidal method.

Time (sec)	0	1	2	3
Speed (ft/sec)	30	22	12	0

D) 49 ft.

17) If $M(4)$ is used to approximate $\int_0^1 \sqrt{1+x^3}\,dx$, then the definite integral is equal, to two decimal places, to:

x	0.125	0.375	0.625	0.875
y	1.001	1.026	1.115	1.292

$\int_0^1 \sqrt{1+x^3}\,dx \approx 0.25(1.001+1.026+1.115+1.292) \approx 1.1085 \approx 1.11$ **B)**

18) If $\int_0^1 \sqrt{1+x^3}\,dx$ is approximated by Riemann sums and the same number of subdivisions, and if *L, R, M,* and *T* denote respectively Left, Right, Midpoint, and Trapezoidal sums, then:

$$y=\sqrt{1+x^3} \Rightarrow y'=\frac{3x}{2\sqrt{1+x^3}} \Rightarrow y''=\frac{6\sqrt{1+x^3}-3x\left(\frac{3x}{\sqrt{1+x^3}}\right)}{4(1+x^3)}=\frac{\frac{6(1+x^3)-9x^2}{\sqrt{1+x^3}}}{4(1+x^3)}=\frac{6-9x^2+6x^3}{4(1+x^3)\sqrt{1+x^3}}$$

$y'' > 0$ on $[0,1]$ so

D) $\boldsymbol{L \le M \le T \le R}$

Problems

19) [1994 AB6] Let $F(x) = \int_0^x \sin\left(t^2\right)dt$ for $0 \le x \le 3$.

A)

$$T = \frac{1/4}{2}\left(\sin\left(0^2\right) + 2\left(\sin\left(.25^2\right) + \sin\left(.5^2\right) + \sin\left(.75^2\right)\right) + \sin\left(1\right)\right)$$
$$\approx 0.316$$

B)

Increases when $F'(x) > 0$. $F'(x) = \sin\left(x^2\right)$. $\sin\left(x^2\right) > 0$

$\Rightarrow 0 < x^2 < \pi$ and $2\pi < x^2 < 3\pi \Rightarrow 0 < x < 1.772$ and $2.507 < x < 3.070$

Increasing on $(0,1.772)\cup(2.507,3)$

C) If the average rate of change of *F* on the closed interval [1,3] is *k*, find $\int_1^3 \sin(t^2)\,dt$ in terms of *k*.

$\frac{1}{3\ \ 1}\int_1^3 F(x)dx = k \qquad \int_1^3 F(x)dx = 2k$

20) [GC]The temperature, in degrees Celsius (°C), of the water in a pond is a differentiable function W of time t. The table shows the water temperature as recorded every 3 days over a 15-day period

t (days)	$W(t)$ (°C)
0	20
3	31
6	28
9	24
12	22
15	21

A) Use data from the table to find an approximation for $W'(12)$. Show the computations that lead to your answer. Indicate units of measure.

$$W'(12) \approx \frac{21-24}{15-9} \approx -\frac{1}{2} \text{ °C/day}$$

B) Approximate the average temperature, in degrees Celsius, of the water over the time interval $0 \le t \le 15$ by using a trapezoidal approximation with subintervals of length $\Delta t = 3$ days.

Average temp (in °C) =

$$\frac{3\left(\frac{1}{2}\right)(31+20)+3\left(\frac{1}{2}\right)(28+31)+3\left(\frac{1}{2}\right)(24+28)+3\left(\frac{1}{2}\right)(22+24)+3\left(\frac{1}{2}\right)(21+22)}{15} = \frac{376.5}{15} \approx 25.1$$

C) A student proposes the function P, given by $P(t) = 20 + 10te^{(-t/3)}$, as a model for the temperature of the water in the pond at time t, where t is measured in days and $P(t)$ is measured in degrees Celsius. Find $P'(12)$. Using appropriate units, explain the meaning of your answer in terms of water temperature.

$$P'(t) = 10t\left(-\frac{1}{3}e^{-t/3}\right) + 10e^{-t/3}$$

$$P'(12) = 10(12)\left(-\frac{1}{3}e^{-4}\right) + 10e^{-4} = -\frac{30}{e^4} \approx -0.549$$

The would show that the temperature of the water is decreasing at a rate of approximately 0.549°C per day on the 12th day.

D) Use the function P defined in part (C) to find the average value, in degrees Celsius, of $P(t)$ over the time interval $0 \le t \le 15$ days.

Average value = $\frac{1}{15-0}\int_0^{15}\left(20+10te^{-t/3}\right)dt = \frac{1}{15}\left[20t\right]_0^{15} + \frac{10}{15}\int_0^{15} te^{-t/3}dt$

$u = t \quad dv = e^{-t/3}dt$

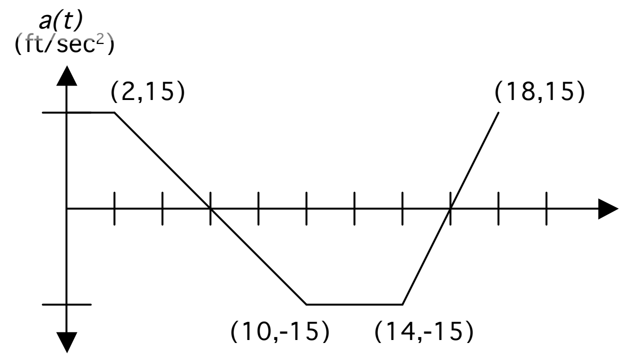
$du = dt \quad v = -3e^{-t/3}$

$$= \frac{1}{15}\left[300-0\right] + \frac{2}{3}\left[-3te^{-t/3}\right]_0^{15} - \frac{2}{3}\int_0^{15} -3e^{-t/3}dt = 20 + \frac{2}{3}\left[-45e^{-5}-0\right] + 2\left[-3e^{-t/3}\right]_0^{15}$$

$$= 20 - 30e^{-5} + 2\left[-3e^{-5} - (-3)\right] = 26 - \frac{36}{e^5} \approx 25.757°C$$

21) A car is traveling on a straight road with velocity 55 ft/sec at time $t = 0$. For $0 \le t \le 18$ seconds, the car's acceleration *a(t)*, in ft/sec^2, is the piecewise linear function defined by the graph at the right.

A) Is the velocity of the car increasing at $t = 2$ seconds? Why or why not?



The velocity of the car is increasing at $t = 2$, as the acceleration is positive from $t = 0$ to $t = 2$ and the velocity was positive at $t = 0$.

B) At what time in the interval $0 \le t \le 18$, other than $t = 0$, is the velocity of the car 55 ft/sec? Why?

The velocity of the car is 55 ft/sec at $t = 12$. The area between $a(t)$ and the x-axis from $t = 0$ to $t = 6$ (which would be a positive quantity) represents the total (positive) acceleration, and would equal the area between $a(t)$ and the x-axis from $t = 10$ to $t = 12$ (which would be a negative quantity), which would represent the total (negative) acceleration, or deceleration. Thus the velocity would again be 55 ft/sec at $t = 12$.

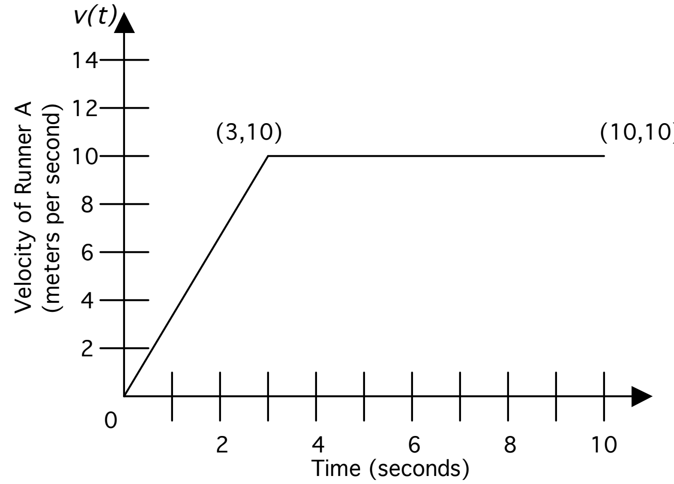
C) On the time interval $0 \le t \le 18$, what is the car's absolute maximum velocity, in ft/sec, and at what time does it occur? Justify.

Maximum velocity occurs when its derivative, acceleration, equals 0. This occurs at $t = 6$ and $t = 16$.
Since at $t = 6$ the acceleration passes from positive to negative, this time is when the maximum velocity occurs.

D) At what times in the interval $0 \le t \le 18$, if any, is the car's velocity equal to zero? Justify.

$\int_{12}^{16} a(t)\,dt = 15(2) + \frac{1}{2}(15)(2) = 45$. This represents the total negative acceleration from 55ft/sec. Therefore, at $t = 16$, the velocity of the car is $55 - 45 = 10$ ft/sec. From $t = 16$ to $t = 18$, the acceleration is positive, so the velocity will be increasing. Thus the car's velocity will never equal zero.

21) Two runners, A and B, run on a straight racetrack $0 \le t \le 10$ seconds. The graph, which consists of two line segments, shows the velocity, in meters per second, of Runner A. The velocity, in meters per second, of Runner B is given by the function v defined by $v(t) = \dfrac{24t}{2t+3}$.



A) Find the velocity of each runner at time $t = 2$ seconds. Indicate units of measure.

Runner A: $v(t) - 0 = \dfrac{10}{3}(t-0) \Rightarrow v(t) = \dfrac{10}{3}t$

At $t = 2$, Runner A's velocity is 20/3 meters per second.

Runner B: $v(2) = \dfrac{24(2)}{2(2)+3} = \dfrac{48}{7}$ meters per second

B) Find the acceleration of each runner at time $t = 2$ seconds. Indicate units of measure.

Runner A: Acceleration is the slope of velocity, so at $t = 2$ Runner A's acceleration is 10/3 m/sec^2.

Runner B: $a(t) = v'(t) = \dfrac{(2t+3)24 - 24t(2)}{(2t+3)^2}$ $\quad a(2) = \dfrac{(2(2)+3)24 - 24(2)(2)}{(2(2)+3)^2} = \dfrac{168-96}{49} = \dfrac{72}{49}$ m/sec^2.

C) Find the total distance run by each runner over the time interval $0 \le t \le 10$ seconds. Indicate units of measure.

Runner A: $\int_0^{10} v(t)\,dt = \frac{1}{2}(3)(10) + 10(7) = 85$ meters.

Runner B: $\int_0^{10} \dfrac{24t}{2t+3}\,dt \approx 83.336$ m