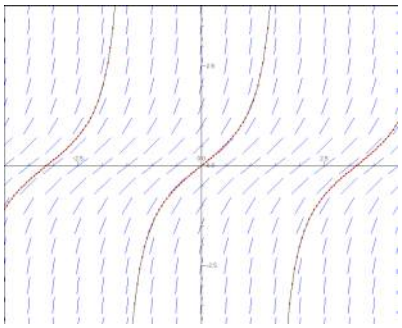
AP Calculus Exam Prep Assignment #5 Name _____

1) A solution curve has been superimposed on the slope fields at the right. The solution is for the differential equation and initial condition:

E) $y' = 1 + y^2;\ y(0) = 0$



2)

$$\frac{dy}{dx} = y \Rightarrow \frac{dy}{y} = dx \Rightarrow \int \frac{dy}{y} = \int dx \Rightarrow \ln y = x + C \Rightarrow \ln 3 = 2 + C \Rightarrow C = \ln 3 - 2$$

C)

$$\ln y = x + \ln 3 - 2 \Rightarrow y = e^{x + \ln 3 - 2} = \frac{3e^x}{e^2}$$

3)

$$\frac{dy}{y} = \frac{dx}{2\sqrt{x}} \Rightarrow\Rightarrow \int \frac{dy}{y} = \int \frac{dx}{2\sqrt{x}} \Rightarrow \ln y = \sqrt{x} + C \Rightarrow \ln 1 = \sqrt{4} + C \Rightarrow C = -2$$

E)

$$\ln y = \sqrt{x} - 2 \Rightarrow y = e^{\sqrt{x} - 2}$$

4) $\frac{dy}{dx} = e^y \Rightarrow \frac{dy}{e^y} = dx \Rightarrow \int \frac{dy}{e^y} = \int dx \Rightarrow -\frac{1}{e^y} = x + C \Rightarrow -1 = 1 + C \Rightarrow C = -2 \quad -\frac{1}{e^y} = x - 2$ **C)**

5)

$$\frac{dy}{dx} = \frac{x}{\sqrt{9 + x^2}} \Rightarrow dy = \frac{x}{\sqrt{9 + x^2}} dx \Rightarrow \int dy = \int \frac{x}{\sqrt{9 + x^2}} dx \quad u = 9 + x^2,\ du = 2x\,dx \Rightarrow x\,dx = \frac{du}{2}$$

B)

$$y = \frac{1}{2}\int \frac{1}{\sqrt{u}} du \Rightarrow y = \sqrt{u} + C \Rightarrow y = \sqrt{9 + x^2} + C \quad 5 = \sqrt{9 + 4^2} + C \Rightarrow C = 0 \quad y = \sqrt{9 + x^2}$$

6) Suppose we plot a particular solution of $\frac{dy}{dx} = 4y$ from initial point (0,1) using Euler's method. After one step of size $\Delta x = 0.1$, how big is the error?

$$\frac{dy}{dx} = 4y \Rightarrow \frac{dy}{y} = 4dx \Rightarrow \int \frac{dy}{y} = 4\int dx \Rightarrow \ln y = 4x + C \quad \ln(1) = 4(0) + C \Rightarrow C = 0$$

A)

$$\ln y = 4x \Rightarrow y = e^{4x} \quad y = e^{4(0.1)} \approx 1.4918 \quad \Delta y \approx 4y\Delta x \approx 4(0.1) = 0.4 \quad \text{Error} \approx 0.0918$$

7) $A = Pe^{rt}$ $40000 = Pe^{.08(35)} \Rightarrow P = \dfrac{40000}{e^{.08(35)}} = \$2432.41$ **C)**

8)

$$\frac{dy}{dt} = -0.11\left(y - 68\right) \Rightarrow \frac{dy}{y-68} = -0.11dx \Rightarrow \int \frac{dy}{y-68} = \int -0.11dx \Rightarrow \ln\left|y-68\right| = -0.11x + C$$

$$\ln\left|180-68\right| = -0.11(0) + C \Rightarrow C = \ln 112 \quad y = e^{-0.11x + \ln 112} + 68 = 112e^{-0.11x} + 68$$

$$y(10) = 112e^{-1.1} + 68 \approx 105.28$$

C)

9)

$$\frac{dy}{dt} = ky \Rightarrow \frac{dy}{y}kdt \Rightarrow \int \frac{dy}{y} = \int kdt \Rightarrow \ln y = kt + C \quad \ln 40 = k(0) + C \Rightarrow C = \ln 40$$

$$y = e^{kt + \ln 40} = 40e^{kt} \quad 10 = 40e^{k(2)} \Rightarrow e^{2k} = \frac{1}{4} \Rightarrow 2k = \ln\left(\frac{1}{4}\right) \Rightarrow k = \frac{\ln\left(1/4\right)}{2} = \ln\left(4^{-1}\right)^{1/2} = -\ln 2$$

A)

For problems 10-15, use the slope fields provided on the next page.

10) Which slope field is for the differential equation $y' = y$? **C) III**

11) Which slope field is for the differential equation $y' = \dfrac{-x}{y}$? **D) IV**

12) Which slope field is for the differential equation $y' = \sin x$? **B) II**

13) Which slope field is for the differential equation $y' = 2x$? **E) V**

14) Which slope field is for the differential equation $y' = e^{-x^2}$? **A) I**

15) A particular solution curve of a differential equation whose slope field is shown in II passes through the point (0,-1). Its equation is:

$$\frac{dy}{dx} = \sin x \Rightarrow dy = \sin x\, dx \Rightarrow \int dy = \int \sin x\, dx \Rightarrow y = -\cos x + C$$

$$-1 = -\cos(0) + C \Rightarrow C = 0 \quad y = -\cos x$$

D)

Problems: Solve the following WITHOUT the use of a calculator.

16) A certain rumor spreads through a community at the rate of $\frac{dy}{dt} = 2y(1-y)$, where y is the proportion of the population that has heard the rumor at time t.

A) What proportion of the population has heard the rumor when it is spreading the fastest?

B) If at time $t = 0$, ten percent of the people have heard the rumor, find y as a function of t.

C) At what time t is the rumor spreading the fastest?

17) Let $v(t)$ be the velocity, in feet per second, of a skydiver at time t seconds, $t \geq 0$. After her parachute opens, her velocity satisfies the differential equation $\frac{dv}{dt} = -2v - 32$ with initial condition $v(0) = -50$.

A) Use separation of variables to find an expression for v in terms of t, where t is measured in seconds.

B) Terminal velocity is defined as $\lim_{t\to\infty} v(t)$. Find the terminal velocity of the skydiver to the nearest foot per second.

C) It is safe to land when her speed is 20 feet per second. At what time t does she reach this speed?

18) The function f is differentiable for all real numbers. The point $\left(3, \frac{1}{4}\right)$ is on the graph of $y = f(x)$, and the slope at each point (x,y) on the graph is given by $\frac{dy}{dx} = y^2(6-2x)$

A) Find $\frac{d^2y}{dx^2}$ and evaluate it at the point $\left(3, \frac{1}{4}\right)$.

B) Find $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = y^2(6-2x)$ with the initial condition $f(3) = \frac{1}{4}$.