AP Calculus Exam Prep Assignment #4   Name ______________________

Multiple Choice: No calculator is allowed.

1)

$$u = 2y - 3y^2 \quad du = (2-6y)dy \Rightarrow (1-3y)dy = \frac{1}{2}du$$

$$\frac{1}{2}\int \frac{du}{\sqrt{u}} = \frac{1}{2}\left(\frac{\sqrt{u}}{2}\right) + C = \frac{\sqrt{2y-3y^2}}{4} + C$$

C)

2) $\int \frac{dy}{\sqrt{y}\left(1-\sqrt{y}\right)} =$

$$u = 1 - \sqrt{y} \quad du = -\frac{1}{2\sqrt{y}}dy \Rightarrow \frac{1}{\sqrt{y}}dy = -2du$$

$$\int \frac{-2du}{u} = -2\ln|u| + C = -2\ln\left|1-\sqrt{y}\right| + C$$

E)

3)

$$u = (2t)^2 = 4t^2 \quad du = 8t\,dt \Rightarrow t\,dt = \frac{1}{8}du$$

$$\int \frac{1}{8}\cos u\,du = \frac{1}{8}\sin u + C = \frac{1}{8}\sin 4t^2 + C$$

A)

4)

$$u = 2y - 3 \quad du = 2dy \Rightarrow dy = \frac{1}{2}du$$

$$\int_1^3 \frac{du}{2u} = \left[\frac{1}{2}\ln u\right]_1^3 = \frac{1}{2}\ln 3 - \frac{1}{2}\ln 1 = \frac{1}{2}\ln 3$$

D)

5)

$$\int_1^2 \left(\frac{1}{x} - \frac{4}{x^2}\right)dx = \left[\ln x + \frac{4}{x}\right]_1^2 = (\ln 2 + 2) - (\ln 1 + 4) = \ln 2 - 2$$

B)

6) The area bounded by $y = e^{2x}$, the x-axis, the y-axis, and the line $x = 2$ is:

$$\int_0^2 e^{2x}dx = \frac{1}{2}\left[e^{2x}\right]_0^2 = \frac{1}{2}\left[e^4 - 1\right]$$

C)

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7) $\int_0^{\pi/4} \pi(\sec x)^2\,dx = \pi[\tan x]_0^{\pi/4} = \pi[1-0] = \pi$ **C)**

8) $u = 5x \quad du = 5dx \Rightarrow dx = \frac{du}{5} \qquad \int_5^{10} \frac{\sqrt{u}\,du}{5} = \frac{1}{5}\left[\frac{2u^{3/2}}{3}\right]_5^{10} = \frac{2}{15}\left[10^{3/2} - 5^{3/2}\right] \approx 2.726$ **A**

9) The region in the first quadrant enclosed by the y-axis and the graphs of $y = \cos x$ and $y = x$ is rotated about the x-axis. The volume of the solid generated is:

$V = \pi\int_0^{0.739}\left(\cos^2 x - x^2\right)dx \approx 1.520$ **C)**

10) $u = x^2 \quad du = 2x\,dx \Rightarrow x\,dx = \frac{du}{2} \qquad \int \frac{e^u du}{2} = \frac{1}{2}e^u + C = \frac{1}{2}e^{x^2} + C$ **A)**

11)

$u = x^2 \quad dv = e^x dx \quad du = 2x\,dx \quad v = e^x$

$x^2e^x - \int 2xe^x dx$

$u = 2x \quad dv = e^x dx \quad du = 2dx \quad v = e^x$

$x^2e^x - \left(2xe^x - \int 2e^x dx\right) = x^2e^x - 2xe^x + 2e^x + C$ **C)**

12) $u = \ln v \quad du = \frac{1}{v}dv \quad \int u\,du = \frac{u^2}{2} + C = \frac{1}{2}(\ln v)^2 + C$ **C)**

13)

$u = \ln\sqrt{x} \quad du = \frac{1}{\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right)dx = \frac{1}{2x}dx \Rightarrow \frac{1}{x}dx = 2du$

$2\int u\,du = u^2 + C = \left(\ln\sqrt{x}\right)^2 + C = \left(\ln x^{1/2}\right)\left(\ln x^{1/2}\right) = \frac{1}{4}(\ln x)^2 + C$ **E)**

14)

$\frac{x^2}{x^2-1} = 1 + \frac{1}{x^2-1} \qquad \int\left(1 + \frac{1}{x^2-1}\right)dx = x + \frac{1}{2}\int\left(\frac{1}{x-1}\right)dx - \frac{1}{2}\int\left(\frac{1}{x+1}\right)$

$x + \frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C = x + \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C$ **A)**

15)

$$\frac{3x+5}{(x-3)(x+4)} = \frac{A}{x-3} + \frac{B}{x+4} \Rightarrow \begin{cases} A+B=3 \\ 4A-3B=5 \end{cases} \Rightarrow A=2,\ B=1$$

$$2\int_4^5 \left(\frac{1}{x-3}\right)dx + \int_4^5 \left(\frac{1}{x+4}\right)dx = \Big[2\ln|x-3| + \ln|x+4|\Big]_4^5$$

D)

$$= (2\ln 2 + \ln 9) - (2\ln 1 + \ln 8) = \ln 36 - \ln 8 = \ln\frac{9}{2}$$

16) Which of these indefinite integral requires the method of REPEATED integration by parts?

D) $\int e^{2x}\cos 3x\,dx$ \

17)

$$\int \frac{x}{\sqrt{4-x^2}}dx \quad + \quad \int \frac{2}{\sqrt{4-x^2}}dx \qquad \int \frac{2}{\sqrt{4-x^2}}dx = \int \frac{2}{\sqrt{4\left(1-\frac{x^2}{4}\right)}}dx = \int \frac{2}{2\sqrt{\left(1-\frac{x^2}{4}\right)}}dx$$

$u = 4-x^2,\ du = -2x\,dx$

B)

$$-\frac{1}{2}\int \frac{du}{\sqrt{u}} + \int \frac{1}{\sqrt{1-\left(\frac{x}{2}\right)^2}}dx = -\frac{1}{2}\left(2\sqrt{u}\right) + 2\sin^{-1}\left(\frac{x}{2}\right) + C = -\sqrt{4-x^2} + 2\sin^{-1}\left(\frac{x}{2}\right) + C$$

18) $\int_{-\infty}^{\infty} \frac{dx}{x^2+1} =$ **C)**

A) 0 B) π/2 C) π D) 2π E) N.O.T.

19) $\int_2^4 \frac{dx}{(x-3)^{2/3}}$

$$\left.\frac{(x-3)^{1/3}}{1/3}\right|_2^4 = 3-(-3) = 6$$

A)

20) Which of the following improper integrals diverges? **D)**

$$\lim_{a\to 0}\int_{-1}^{a}\frac{dx}{x^2} + \lim_{b\to 0}\int_{b}^{1}\frac{dx}{x^2}$$

$$\lim_{a\to 0^-}\left[-\frac{1}{x}\right]_{-1}^{a} + \lim_{b\to 0^+}\left[-\frac{1}{x}\right]_{b}^{1} = \lim_{a\to 0^-}\left[-\frac{1}{a}+1\right]_{-1}^{a} + \lim_{b\to 0^+}\left[-1+\frac{1}{b}\right]_{b}^{1} = \infty + \infty = \infty$$

23) C) $\pi \int_{-2}^{2} \left[\left(k - x^2\right)^2 - \left(k - 4\right)^2 \right] dx$

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21) Let R be the region in the first quadrant under the graph of $y = \dfrac{1}{\sqrt{x}}$ for $4 \le x \le 9$.

A) Find the area of R.

$$\int_4^4 \frac{1}{\sqrt{x}}\,dx = \left[2\sqrt{x}\right]_4^9 = 6 - 4 = 2 \text{ square units.}$$

B) If the line $x = k$ divides the region R into two regions of equal area, what is the value of k?

$$\int_4^k \frac{1}{\sqrt{x}}\,dx = \int_k^9 \frac{1}{\sqrt{x}}\,dx$$

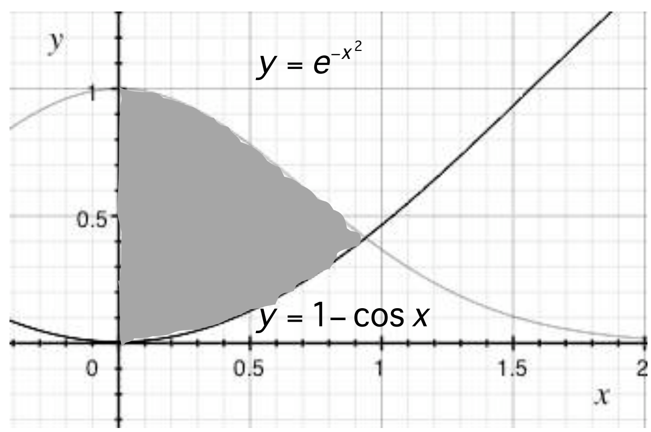
$$\left[2\sqrt{x}\right]_4^k = \left[2\sqrt{x}\right]_k^9$$

$$2\sqrt{k} - 4 = 6 - 2\sqrt{k} \Rightarrow 4\sqrt{k} = 10 \Rightarrow k = \frac{25}{4}$$

C) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x-axis are squares.

$$V = \int_4^9 \left(\frac{1}{\sqrt{x}}\right)^2 dx = \int_4^9 \frac{1}{x}\,dx = \left[\ln x\right]_4^9 = \ln\frac{9}{4}$$

22) [GC] Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure.



A) Find the area of R.

First find the point of intersection using the GC:

$$A = \int_0^{.942} \left(e^{-x^2} - (1 - \cos x)\right) dx \approx 0.591$$

B) Find the volume of the solid generated when the region R is revolved about the x-axis.

$$V = \pi \int_0^{.942} \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2\right) dx \approx 1.747$$

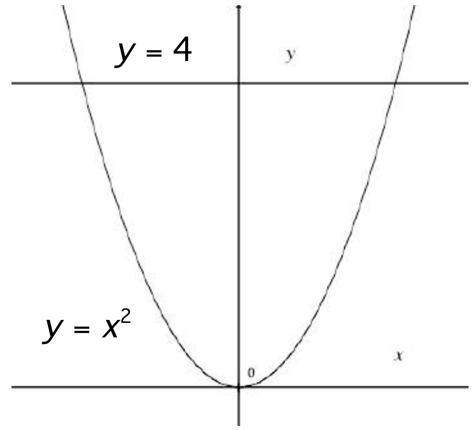
C) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of the solid.

$$V = \int_0^{.942} \left(\left(e^{-x^2}\right) - (1 - \cos x)\right)^2 dx \approx 0.461$$

23) The shaded region, R, is bounded by the graph $y = x^2$ and the line $y = 4$, as shown in the figure.

A) Find the area of R.

$$A = \int_{-2}^{2} \left(4 - x^2\right) dx \approx \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = \left(8 - \frac{8}{3}\right) - \left(-8 + \frac{8}{3}\right) = \frac{32}{3}$$



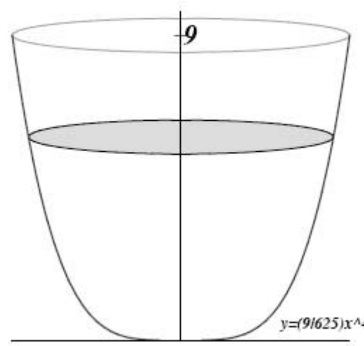
B) Find the volume of the solid generated when the region R is revolved about the x-axis.

$$V = \int_{-2}^{2} \left(4^2 - \left(x^2\right)^2\right) dx \approx \left[16x - \frac{x^5}{5}\right]_{-2}^{2} = \left(32 - \frac{32}{5}\right) - \left(-32 + \frac{32}{5}\right) = \frac{256}{5}$$

C) There exists a number, k, $k > 4$, such that when R is revolved about the line $y = k$, the resulting solid has the same volume as the solid in part (B). Write, but do not solve, an equation involving an integral expression that can be used to find the value of k.

24) An oil storage tank has the shape shown below, obtained by revolving the curve $y = \frac{9}{625}x^4$ from $x = 0$ to $x = 5$ about the y-axis, where x and y are measured in feet. Oil weighing 50 pounds per cubic foot flowed into an initially empty tank at a constant rate of 8 cubic feet per minute. When the depth of the oil reached 6 feet, the flow stopped.

A) Let h be the depth, in feet, of oil in the tank. How fast was the depth of the oil in the tank increasing when $h = 4$? Indicate units of measure.



$$y = \frac{9}{625}x^4 \Rightarrow x = \sqrt[4]{\frac{625}{9}y}$$

$$V = \pi \int_0^h \left(\sqrt[4]{\frac{625}{9}y}\right)^2 dy = \pi \int_0^h \sqrt{\frac{625}{9}y}\, dy = \pi \int_0^h \frac{25}{3}\sqrt{y}\, dy$$

$$\frac{dV}{dt} = \pi\left(\frac{25}{3}\sqrt{h}\right)\frac{dh}{dt}$$

$$8 = \pi\left(\frac{25}{3}\sqrt{4}\right)\frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{12}{25\pi} \text{ ft/min}$$

B) Find, to the nearest foot-pound, the amount of work required to empty the tank by pumping all of the oil back to the top of the tank.

$$W = 50\pi \int_0^6 \left(\frac{25}{3}\sqrt{y}\right)(9 - y)\, dy$$

$$= 50\pi \int_0^6 \left(75\sqrt{y} - \frac{25}{3}y^{3/2}\right) dy = 50\pi\left[50y^{3/2} - \frac{10}{3}y^{5/2}\right]_0^6$$

$$= 9000\pi\sqrt{6} \text{ ft - lbs}$$