AP Calculus Exam Prep Assignment #12 KEY

1) $\cos x = e^y \frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{\cos x}{e^y} = \frac{\cos x}{\sin x} = \cot x$ **C)**

2)

$$\frac{dy}{dx} = \frac{dy}{du}\cdot\frac{du}{dv}\cdot\frac{dv}{dx} = \left(\sec^2 u\right)\left(1+\frac{1}{v^2}\right)\left(\frac{1}{x}\right) = \left(\sec^2\left(v-\frac{1}{v}\right)\right)\left(1+\frac{1}{\left(\ln|x|\right)^2}\right)\left(\frac{1}{x}\right)$$

$$= \left(\sec^2\left(\ln|x| - \frac{1}{\ln|x|}\right)\right)\left(1+\frac{1}{\left(\ln|x|\right)^2}\right)\left(\frac{1}{x}\right)$$ When $x = e$, $\frac{dy}{dx} = \left(\sec^2\left(\ln|e| - \frac{1}{\ln|e|}\right)\right)\left(1+\frac{1}{\left(\ln|e|\right)^2}\right)\left(\frac{1}{e}\right)$ **D)**

$$= \left(\sec^2(0)\right)(2)\left(\frac{1}{e}\right) = \frac{2}{e}$$

3) $\frac{1}{\sqrt{1-(2x)^2}}(2) = \frac{2}{\sqrt{1-4x^2}}$ **D)**

4) **A)** $f'(x_0)$

5) If $y = x^{\ln x}$ then $y' =$

$$\ln y = (\ln x)(\ln x) \Rightarrow \frac{1}{y}\frac{dy}{dx} = 2\ln x\left(\frac{1}{x}\right) \Rightarrow \frac{dy}{dx} = y\left(\frac{2\ln x}{x}\right) = \frac{2x^{\ln x}\ln x}{x}$$ **C)**

6) Which has a graph that is symmetric with respect to the origin? **[in other words, an odd function]**

B) $y = -x^5 + 3x$

7) $\lim_{x\to\infty}\frac{20x^2-13x+5}{5-4x^3} = \lim_{x\to\infty}\frac{\frac{20}{x}-\frac{13}{x^2}+\frac{5}{x^3}}{\frac{5}{x^3}-4} = \frac{0}{-4} = 0$ **C)** -5 [yes, you could also use L'Hôpital's Rule]

8) $\lim_{x\to\frac{\pi}{2}}\left(\frac{\cos x}{x-\frac{\pi}{2}}\right) = \lim_{x\to\frac{\pi}{2}}\left(\frac{-\sin x}{1}\right) = -1$ **A)**

9) Let $h = \frac{1}{x}$ $\lim_{x\to\infty}\left(x\sin\frac{1}{x}\right) = \lim_{h\to 0}\left(\left(\frac{1}{h}\right)\sinh\right) = \lim_{h\to 0}\left(\frac{\sin h}{h}\right) = 1$ **E)**

10) $\lim_{x\to 0}\left(\frac{\sin^2\frac{x}{2}}{x^2}\right) = \lim_{x\to 0}\left(\frac{2\sin\frac{x}{2}\left(\cos\frac{x}{2}\right)\left(\frac{1}{2}\right)}{2x}\right) = \lim_{x\to 0}\left(\frac{\sin\frac{x}{2}\cos\frac{x}{2}}{2x}\right) = \lim_{x\to 0}\left(\frac{\frac{1}{2}\cos^2\frac{x}{2}-\frac{1}{2}\sin^2\frac{x}{2}}{2}\right) = \frac{1}{4}$ **C) 0.25**

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11) $\lim_{x\to 0}\frac{\sin 2x}{x}=\lim_{x\to 0}\frac{2\cos 2x}{1}=2$ **B)**

12) $\lim_{x\to 0}\left(x\sin\frac{1}{x}\right)$ = ______.

B) 0

13) $\lim_{x\to\infty}\frac{\sqrt{x}-4}{4-3\sqrt{x}}=\lim_{x\to\infty}\frac{\frac{1}{2\sqrt{x}}}{\frac{-3}{2\sqrt{x}}}=-\frac{1}{3}$ **A)**

14) $\lim_{x\to 0}\frac{e^{2x}-1}{\tan x}=\lim_{x\to 0}\frac{2e^{2x}}{\sec^2 x}=\lim_{x\to 0}\left(2e^{2x}\cos^2 x\right)=2$ **D)**

15) $\lim_{h\to 0}\frac{1}{h}\ln\left(\frac{2+h}{2}\right)=\lim_{h\to 0}\frac{\ln\left(\frac{2+h}{2}\right)}{h}=\lim_{h\to 0}\frac{\left(\frac{1}{1+\frac{h}{2}}\right)\frac{1}{2}}{1}=\frac{1}{2}$ **C)**

16) $\lim_{x\to\infty}\frac{e^x}{x^{50}}=\lim_{x\to\infty}\frac{e^x}{50!}=\infty$ **D)**

17) $\lim_{a\to\infty}a^{\left[\frac{1}{2a}\right]}=$

$u=a^{\left[\frac{1}{2a}\right]}\Rightarrow \ln u=\ln a^{\left[\frac{1}{2a}\right]}=\frac{1}{2a}\ln a=\frac{\ln a}{2a}$

A)

$\lim_{a\to\infty}\frac{\ln a}{2a}=\lim_{a\to\infty}\frac{\frac{1}{a}}{2}=0 \quad u=e^0=1$

Problems

18) Find the limit: $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x$

$$u=\left(1+\frac{2}{x}\right)^x \quad \ln u=\ln\left(1+\frac{2}{x}\right)^x=x\ln\left(1+\frac{2}{x}\right)=\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}$$

$$\lim_{x\to\infty}\left[\frac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{x}}\right]=\lim_{x\to\infty}\left[\frac{\frac{1}{1+\frac{2}{x}}\left(\frac{-2}{x^2}\right)}{\frac{-1}{x^2}}\right]=2 \quad e^2$$

19) Find the maximum volume of a box that can be made by cutting off squares from the corners of an 8-inch by 15-inch rectangular sheet of cardboard and folding up the sides. Justify your answer.

$V=x(15-2x)(8-2x)=120x-46x^2+4x^3$

$V'=120-92x+12x^2=0\Rightarrow 4(3x^2-23x+30)=0\Rightarrow 4(3x-5)(x-6)=0$

$x=6,\frac{5}{3}$

Eliminate 6 (extraneous)

Max Volume $=\left(\frac{5}{3}\right)\left(\frac{35}{3}\right)\left(\frac{14}{3}\right)=\frac{2450}{27}$ cubic inches