AP Calculus Exam Prep Assignment #11 KEY

1) The equation of the tangent to the curve $2x^2 - y^4 = 1$ at (-1,1) is:

$4x - 4y^3 \dfrac{dy}{dx} = 0 \quad \dfrac{dy}{dx} = \dfrac{x}{y^3} \quad$ At $(-1,1)$ $\dfrac{dy}{dx} = \dfrac{1}{-1} = -1 \quad y - 1 = -1\left(x+1\right) \Rightarrow = -x$ **A)**

2) The tangent to $y = xe^{-x}$ is horizontal when $x =$ ______.

$y' = e^{-x} - xe^{-x} = e^{-x}(1-x) \quad y' = 0 \Rightarrow x = 1$ **B)**

3) An equation for a tangent to $y = \arcsin\left(\dfrac{x}{2}\right)$ at the origin is:

$y' = \dfrac{1}{\sqrt{1-\left(\dfrac{x}{2}\right)^2}}\left(\dfrac{1}{2}\right) = \dfrac{1}{2\sqrt{1-\dfrac{x^2}{4}}} \quad y = \dfrac{1}{2}x \Rightarrow x - 2y = 0$ **A)**

4) The approximate value of $y = \sqrt{4 + \sin x}$ at $x = 0.12$, obtained from the tangent to the graph at $x = 0$, is:

$y' = \dfrac{1}{2}\left(4 + \sin x\right)^{-1/2}\left(\cos x\right) \quad y = \sqrt{4+0} = 2 \quad y - 2 = \dfrac{1}{4}x \Rightarrow y = \dfrac{1}{4}x + 2 \quad y = \dfrac{1}{4}(0.12) + 2 = 2.03$ **B)**

5) If $f(x) = \ln x$, then the average rate of change of *f* on the interval [2,5] is:

$\dfrac{\ln 5 - \ln 2}{5-2} = \dfrac{1}{3}\ln\dfrac{5}{2} \approx 0.305$ **C)**

6) If $f(x) = \sqrt[5]{x^3 - 2x}$, then $\left(\sqrt{3}\right) =$ **D) 1.116**

7) [GC] The slope of the line containing the point (4,0) and tangent to the graph of $y = e^{-x}$ is:

$y' = -e^{-x}$ Let $\left(x, e^{-x}\right)$ be the point of tangency

E)

$-e^{-x} = \dfrac{e^{-x} - 0}{x - 4} \Rightarrow x - 4 = -1 \Rightarrow x = 3,$ so the slope is $-e^{-3} \approx -0.050$

8) If *c* is the number defined by Rolle's Theorem, then for $f(x) = 2x^3 - 6x$ on the interval $0 \le x \le \sqrt{3}$, $c =$:

$f'(x) = 6x^2 - 6 \quad f'(x) = 0 \Rightarrow 6x\left(x-1\right) = 0 \Rightarrow x = 0,1$ Since 0 is an endpoint of the interval, the number is 1.

A)

9) If $f'(x)$ exists on $[a,b]$, it follows that:

E) the Mean Value Theorem applies

10) The Mean Value Theorem guarantees the existence of a special point on the graph of $y = \sqrt{x}$ between (0,0) and (4,2). What are the coordinates of this point?

$f'(c) = \dfrac{f(b) - f(a)}{b - a} = \dfrac{2}{4} \quad f'(x) = \dfrac{1}{2\sqrt{x}} \quad \dfrac{1}{2\sqrt{c}} = \dfrac{1}{2} \Rightarrow \sqrt{c} = 1 \Rightarrow c = 1 \quad (1,1)$ **B)**

11) The function $f(x) = x^{2/3}$ on [-8,8] does not satisfy the conditions of the Mean Value Theorem because:

E) $f'(0)$ does not exist

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Problems

12) Consider the curve given by $xy^2 - x^3y = 6$

A)

$$\frac{d}{dx}\left(xy^2 - x^3y = 6\right) = x\left(2y\frac{dy}{dx}\right) + 1\left(y^2\right) - \left[x^3\left(\frac{dy}{dx}\right) + 3x^2y\right] = 0$$

$$\frac{dy}{dx}\left(2xy - x^3\right) = 3x^2y - y^2$$

$$\frac{dy}{dx} = \frac{3x^2y - y^2}{2xy - x^3}$$

B)

$$(1)y^2 - (1)^3y = 6 \Rightarrow y^2 - y - 6 = 0 \Rightarrow (y-3)(y+2) = 0 \Rightarrow y = 3, -2$$

$(1,3)$ and $(1,-2)$

$$(1,3): \frac{dy}{dx} = \frac{3(1)(3) - 3^2}{2(1)(3) - 1^3} = 0 \quad \text{Tangent equation}: \quad y = 3$$

$$(1,-2): \frac{dy}{dx} = \frac{3(1)(-2) - (-2)^2}{2(1)(-2) - 1^3} = \frac{-10}{-5} = 2 \quad \text{Tangent equation}: \quad y = 2(x-1) - 2$$

C) Find the x-coordinate of each point on the curve where the tangent line is vertical.

Vertical tangent $\Rightarrow \frac{dy}{dx}$ is undefined $\Rightarrow$ denominator $= 0$

$$2xy - x^3 = 0 \Rightarrow x\left(2y - x^2\right) = 0 \Rightarrow x = 0, \pm\sqrt{2y}$$

$$x = \pm\sqrt{2y} \Rightarrow y = \frac{x^2}{2} \quad x\left(\frac{x^2}{2}\right)^2 - x^3\left(\frac{x^2}{2}\right) = 6 \Rightarrow \frac{x^5}{4} - \frac{x^5}{2} = 6 \Rightarrow x^5 - 2x^5 = 24 \Rightarrow x = \sqrt[5]{-24}$$

13) Let h be a function defined for all $x \neq 0$ such that $h(4) = -3$ and the derivative of h is given by $h'(x) = \frac{x^2 - 2}{x}$ for all $x \neq 0$.

A) Find all values of x for which the graph of h has a horizontal tangent, and determine whether h has a local maximum, a local minimum, or neither at each of these values. Justify your answers.

$$h'(x) = 0 \Rightarrow x^2 - 2 = 0 \Rightarrow x = \pm\sqrt{2}$$

$$h''(x) = \frac{x(2x) - (x^2 - 2)}{x^2} = \frac{x^2 + 2}{x^2} \quad h''(x) = 0 \Rightarrow x^2 + 2 = 0, \text{ which is impossible.}$$

$h''\left(\pm\sqrt{2}\right) = 2,$ so local minimum at $x = \pm\sqrt{2}$

B) On what intervals, if any, is the graph of h concave up? Justify your answer.

Concave up when $h''(x) > 0$. As $h''(x) > 0$ for all $x \neq 0$, the graph is concave up for the entire domain.

C) Write an equation for the line tangent to the graph of h at $x = 4$.

$h'(4) = \frac{7}{2}$. $y = \frac{7}{2}(x-4) - 3$

D) Does the line tangent to the graph of h at $x = 4$ lie above or below the graph of h for $x > 4$? Why?

Below, as the second derivative of h indicates that the graph is concave upward, thus the tangent line at any point on the graph will lie below the graph.

14) Consider the curve by $-8x^2 + 5xy + y^3 = -149$

A) Find dy/dx.

$$-16x + 5x\frac{dy}{dx} + 5y + 3y^2\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{16x - 5y}{5x + 3y^2}$$

B) Write an equation for the line tangent to the curve at the point (4,-1).

$$\frac{dy}{dx} = \frac{64+5}{20+3} = 3 \quad y = 3(x-4) - 1$$

C) There is a number k so that the point (4.2,k) is on the curve. Using the tangent line found in part (B), approximate the value of k.

$k = 3(4.2 - 4) - 1 = -0.4$

D) Write an equation that can be solved to find the actual value of k so that the point (4.2,k) is on the curve. Do not solve the equation at this point.

$$-8(4.2)^2 + 5(4.2)k + k^3 = -149$$

E) Solve the equation found in part (D) for the value of k. (Calculator) -0.373