AP Calculus Exam Prep Assignment #10 KEY

1) At $x = 0$, which of the following is true of the function $f(x) = x^2 + e^{-2x}$?

$f'(x) = 2x - 2e^{-2x}$ $\quad f'(0) = -1$ $\quad$ **B) *f* is decreasing**

2) Given $f(x) = 3x^5 - 20x^3$, find all values of x for which the graph is concave up.

$f'(x) = 15x^4 - 60x^2$ $\quad f''(x) = 60x^3 - 120x$ $\quad 60x^3 - 120x = 0 \Rightarrow 60x\left(x^2 - 2\right) = 0$ $\quad$ **E)**

$x = 0, \pm\sqrt{2}$ $\quad f(x)$ is concave up when $f''(x) > 0$, which is on $(-\sqrt{2}, 0)$ and $(\sqrt{2}, \infty)$

3) Which function has a graph that is asymptotic to the y-axis?

C) $y = x - \dfrac{2}{x}$

4) [GC] The absolute minimum value of $f(x) = 100e^{-x}\sin x$ for $x \geq 0$ is nearest to:

$f'(x) = 100e^{-x}\cos x - 100e^{-x}\sin x$ $\quad 100e^{-x}\cos x - 100e^{-x}\sin x = 0 \Rightarrow 100e^{-x}\cos x = 100e^{-x}\sin x$

$\Rightarrow \cos x = \sin x \Rightarrow x = \dfrac{\pi}{4}, \dfrac{5\pi}{4}, etc.$ $\quad f\left(\dfrac{\pi}{4}\right) = 100e^{-\pi/4}\left(\sin\left(\dfrac{\pi}{4}\right)\right) \approx 32.240$ $\quad$ **D) -1**

$f\left(\dfrac{5\pi}{4}\right) = 100e^{-\pi/4}\left(\sin\left(\dfrac{5\pi}{4}\right)\right) \approx -1.393$

5) [GC] The number of inflection points for the graph of $y = x + \cos x^2$ on $[0,5]$ is:

$y' = 1 - 2x\sin x^2$ $\quad y'' = -2x\left(\cos x^2\right)\left(2x\right) - 2\sin x^2 = -4x^2\cos x^2 - 2\sin x^2$ $\quad$ **D) 9**

$y'' = 0 \Rightarrow -4x^2\cos x^2 - 2\sin x^2 = 0 \Rightarrow x^2 = -\dfrac{1}{2}\tan x^2$

Problems

6) Let f be the function defined by $f(x) = \ln(2 + \sin x)$ for $\pi \leq x \leq 2\pi$.

A)

$f'(x) = \dfrac{\cos x}{2 + \sin x}$ $\quad f'(x) = 0 \Rightarrow \cos x = 0 \Rightarrow x = \dfrac{3\pi}{2}$ on $[\pi, 2\pi]$

$f\left(\pi\right) = \ln 2$ $\quad f\left(\dfrac{3\pi}{2}\right) = \ln 1 = 0$ $\quad f\left(2\pi\right) = \ln 2$

The absolute minimum value of f is 0, the absolute maximum value is ln2.

B) Find the x-coordinate of each inflection point on the graph of f. Justify your answer.

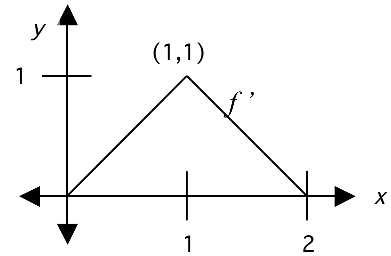
$$f''(x) = \frac{\left(2 + \sin x\right)\left(-\sin x\right) - \cos x\left(\cos x\right)}{\left(2 + \sin x\right)^2} = \frac{-2\sin x - \sin^2 x - \cos^2 x}{\left(2 + \sin x\right)^2} = \frac{-2\sin x - 1}{\left(2 + \sin x\right)^2}$$

$f''(x) = 0 \Rightarrow -2\sin x - 1 = 0 \Rightarrow \sin x = -\dfrac{1}{2} \Rightarrow x = \dfrac{7\pi}{6}, \dfrac{11\pi}{6}$

7) The figure shows the graph of f', the derivative of f. The domain of f is the set of all x such that $0 < x < 2$.

A) Write an expression for $f'(x)$ in terms of x.

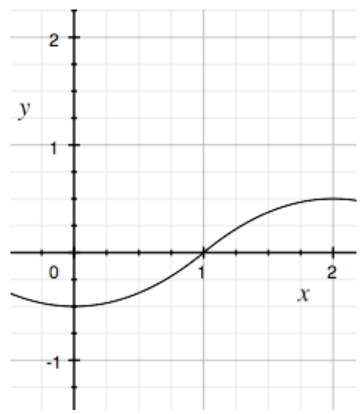
$f' = 1 - |x-1|,\ 0 < x < 2$



B) Given that $f(1) = 0$, write an expression for $f(x)$ in terms of x.

$$f(x) = \begin{cases} \dfrac{x^2}{2} - \dfrac{1}{2} & 0 < x \le 1 \\ 2x - \dfrac{x^2}{2} - \dfrac{3}{2} & 1 < x < 2 \end{cases}$$

C) Sketch the graph of $y = f(x)$.



8) The figure below shows the graph of f', the derivative of f. The domain of f is the set of all x such that $-10 \le x \le 10$.

A) For what values of x does the graph of f have a horizontal tangent?

$x = -7, -1, 4, 8$

B) For what values of x in the interval $(-10,10)$ does f have a relative maximum? Justify.

$x = -1, 8$ as the slope changes from positive to negative.

C) For what values of x is the graph of f concave downward?

f is concave downward when the slope of f ' is negative, so on $(-3,2)$ and $(7,10)$