Geometry

Students explore several geometry topics they have been developing over the years such as angles, area, surface area, and volume in the most challenging form students have experienced yet. This module assumes students understand the basics; the goal is to build a fluency in these difficult problems. The remaining topics (i.e., working on constructing triangles and taking slices (or cross sections) of three-dimensional figures) are new to students.

What Came Before this Module:

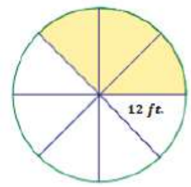
Students began their study of probability and learned how to interpret and compute probabilities in simple settings. They also learned how to estimate probabilities empirically. Students built on their knowledge of data distributions that they studied in Grade 6, compared data distributions of two or more populations, and were introduced to the idea of drawing informal inferences based on data from random samples.

This is the last module in Grade 7.

How can you help at home?

Every day, ask your child what they learned in school and ask them to show you an example.

Ask your child to find the area of the shaded region of the circle below. Have your child explain how they determined the area.



Solution:

$A = \frac{3}{8}\pi r^2$

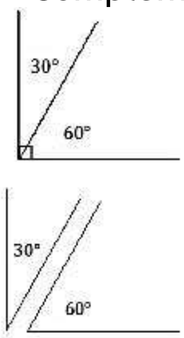
$A = \frac{3}{8}(\pi)(12)^2\ ft^2$

$A = 54\pi\ ft^2$

$A \approx 169.65\ ft^2$

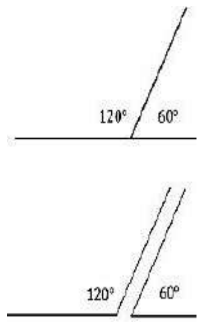
The shaded area of the circle is approximately $169.65\ ft^2$.

Complementary Angles



When two angles are complementary, the measurements have a sum of $90°$.

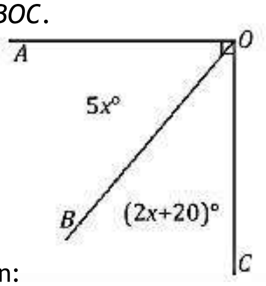
Supplementary Angles



When two angles are supplementary, the measurements have a sum of $180°$.

Geometry & Algebra

In a complete sentence, describe the relevant angle relationships in the diagram. Set up and solve an equation to find the value of x. Find the measurements of $\angle AOB$ and $\angle BOC$.



Solution:

$\angle AOB$ and $\angle BOC$ are complementary and sum to 90°.

$5x + (2x + 20) = 90$ *complementary* $\angle s$

$7x + 20 = 90$

$7x + 20 - 20 = 90 - 20$

$7x = 70$

$x = 10$

$\angle AOB = 5(10°) = 50°$

$\angle BOC = 2(10°) + 20° = 40°$

Key Common Core Standards:

Draw, construct, and describe geometrical figures and describe the relationships between them.

- Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

- Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
- Solve real-world and mathematical problems involving area, volume, and surface area of two and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Key Words

Correspondence
A correspondence between two triangles is a pairing of each vertex of one triangle with one and only one vertex of the other triangle. A triangle correspondence also induces a correspondence between the angles of the triangles and the sides of the triangles.

Identical Triangles
Two triangles are said to be identical if there is a triangle correspondence that pairs angles with angles of equal measure and sides with sides of equal length.

Unique Triangle
A set of conditions for two triangles is said to determine a unique triangle if whenever the conditions are satisfied, the triangles are identical.

Three sides condition
Two triangles satisfy the three sides condition if there is a triangle correspondence that pairs all three sides of one triangle with sides of equal length. The three sides condition determines a unique triangle.

Two angles and the included side condition
Two triangles satisfy the two angles and the included side condition if there is a triangle correspondence that pairs two angles and the included side of one triangle with angles of equal measure and a side of equal length. This condition determines a unique triangle.

Two angles and the side opposite a given angle condition
Two triangles satisfy the two angles and the side opposite a given angle condition if there is a triangle correspondence that pairs two angles and a side opposite one of the angles with angles of equal measure and a side of equal length. The two angles and the side opposite a given angle condition determines a unique triangle.

Two sides and the included angle condition
Two triangles satisfy the two sides and the included angle condition if there is a triangle correspondence that pairs two sides and the included angle with sides of equal length and an angle of equal measure. The two sides and the included angle condition determines a unique triangle.

Two sides and a non-included angle condition
Two triangles satisfy the two sides and a nonincluded angle condition if there is a triangle correspondence that pairs two sides and a non-included angle with sides of equal length and an angle of equal measure. The two sides and a non-included angle condition determines a unique triangle if the non-included angle measures 90˚ or greater. If the non-included angle is acute, the triangles are identical with one of two non-identical triangles.

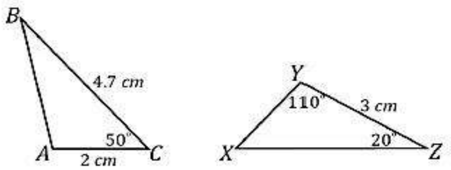
Right rectangular pyramid
Given a rectangular region B in a plane E, and a point V not in E, the rectangular pyramid with base B and vertex V is the union of all segments VP for any point P in B. It can be shown that the planar region defined by a side of the base B and the vertex V is a triangular region, called a lateral face. If the vertex lies on the line perpendicular to the base at its center (the intersection of the rectangle's diagonals), the pyramid is called a right rectangular pyramid.

Surface of a pyramid
The surface of a pyramid is the union of its base region and its lateral faces.

Lesson 5: Identical Triangles

*Sample problem from the curriculum.

The triangles on the right are identical and have the correspondence △ABC↔△YZX. Find the measurements for each of the following sides and angles.



$AB =$ ______
$BC =$ ______
______ $= XY$

$\angle A =$ ______
$\angle B =$ ______
______ $= \angle X$

Solution:

$\mathbf{AB = 3\ cm}$
$\mathbf{XZ = 4.7\ cm}$
$\mathbf{2\ cm = XY}$

$\mathbf{\angle A = 110°}$
$\mathbf{\angle B = 20°}$
$\mathbf{50° = \angle X}$

Exploring Triangles!

Try making a triangle with segments that are 4cm, 10cm, and 5cm in length. What happens? What does this tell us about the length of the sides of a triangle?