5

Circular Motion

Chapter 5 presents the greatest challenge students have faced so far: constant speed circular motion. To master the concepts of constant speed circular motion we suggest that students start with qualitative dynamics, then proceed to kinematics and after that combine qualitative dynamics and expression for radial acceleration to make Newton's second law for circular motion. The content-based learning goals for constant speed circular motion are listed below.

Students should be able to:

1. Describe experiments that allow us to infer that when an object moves at constant speed in a circular path the sum of the forces exerted on it must point toward the center.
2. Use the velocity method to determine the direction of acceleration of an object moving at constant speed in a circular path.
3. Carry out a graphical derivation of $a_r = \frac{v^2}{r}$, explain why it makes sense and how to test it experimentally.
4. Be able to determine the sum of the forces exerted on an object moving at constant speed in a circle experimentally.
5. Apply Newton's law(s) to processes involving circular motion at constant speed with multiple forces in two dimensions.
6. Be able to demonstrate how to get to Newton's law of universal gravitation from the Moon's data and Newton's second and third laws.
7. Explain why objects are not "weightless" in orbit.

We have broken the chapter into three parts. For each part, we provide examples of activities that can be used in the classroom, common student difficulties, and a brief discussion of the motivation for using these activities.

Chapter subject matter	Related textbook and ALG sections	Textbook videos	ALG videos
Qualitative analysis of constant speed circular motion	5.1, 5.2	OET 5.1 (p119), TET 5.2 (p120)	5.1.4
Quantitative analysis of constant speed circular motion	5.3, 5.4		5.3.5
Newton's universal law of gravitation	5.5		

Nontraditional end-of-chapter questions and problems

Choose answer and explanation (CAE): Q5.3, Q5.19
Choose measuring procedure (MEP): P5.15
Evaluate (reasoning or solution…) (EVA): Q5.3, Q5.16, Q5.17, P5.37
Linearization (LIN): P5.71
Multiple possibility and tell all (MPO): P5.10
Jeopardy (JEO): P5.32, P5.33
Design an experiment (or pose a problem) (DEX): P5.34, P5.36, P5.54, P5.62, P5.63, P5.65
Problem based on real data (that students can collect by themselves) (RED): Q5.15, P5.11, P5.12, P5.70

Brief summary of student difficulties with constant speed circular motion

The biggest difficulty comes from reconciling the everyday life experience of feeling you are being thrown outward when a car is making a turn and the concept that the sum of the forces exerted on an object moving at constant speed in a circle points toward the center of the circle. The second difficulty stems from a seeming conflict between the motion being accelerated and, at the same time, being constant-speed. Finally, common language referring to "microgravity" or "weightlessness" of astronauts in orbit make the students think that objects in orbit are weightless.

I. Qualitative analysis of constant speed circular motion

Students begin by performing and analyzing three experiments in which objects move in horizontal constant speed circular motion (ALG Activity 5.1.1). They analyze the situations with force diagrams and build a pattern in which, in each case, the sum of the forces exerted on the object of interest points toward the center of the circle (ALG Activity 5.1.2). They develop an explanation or hypothesis that the sum of the forces exerted on an object moving in a circle at constant speed *always* points toward the center of the circle (ALG Activity 5.1.3). They then proceed to test this hypothesis in ALG Activity 5.1.4 by predicting what will happen to a ball rolling around inside a metal ring when a piece of the ring is removed. We suggest that students finish working on this section by reading the textbook section or working on ALG Activity 5.1.6.

At this point, students know that the sum of the forces exerted on an object moving at a constant speed in a circle points toward the center. Now the question is: why is such a net force needed? To answer this question, students need to examine the motion more carefully and figure out that, despite moving at a constant speed, the object is accelerating because the direction of its velocity is changing. How to determine the direction of this acceleration? To answer this question and to make the logical connections described above, students need to work with ALG Activity 5.2.1 which at the end sends them to the textbook where they learn the velocity method to estimate the direction of an object's velocity change and therefore its acceleration during 2D motion (this method was developed by Fred Reif and colleagues).

The goal of this method is to build on what students have already learned: how to *add* vectors head to tail. Thus it is conceptually easier for them to think of adding a $\Delta\vec{v}$ vector to the $\vec{v}_{\mathrm{i}}$ vector to get a *resultant* $\vec{v}_{\mathrm{f}}$ rather than subtracting $\vec{v}_{\mathrm{i}}$ from $\vec{v}_{\mathrm{f}}$ to get a resultant $\Delta\vec{v}$. However, more advanced students quickly become comfortable with the idea that subtracting a vector is simply equivalent to adding a $-\vec{v}_{\mathrm{i}}$. Our experience is that it pays for students to see both ways and choose whatever method they feel comfortable with. It is important that the students draw velocity vectors for the object equally distant from the point at which they want to determine the direction of acceleration of the object, just before and just after that point. They also need to label the velocity vectors properly. If they use the technique in the book, they then determine what vector needs to be added to the initial velocity vector to produce the final velocity vector. This is the velocity change vector, which also indicates the direction of the acceleration of the object at the point of interest, which is positioned exactly between the points at which they determined initial and final velocity vectors.

After students have gotten acquainted with the method, they should complete ALG Activities 5.2.2 (or Example 5.1 in the textbook). This activity can readily be turned into multiple-choice in-class clicker questions. For example, you can apply the method in Activity 5.2.2 to one point and then ask one-third of the students to apply it to point A, a second third to point B, and the final third to point C. The primary goal of these activities is for students to devise a key pattern: When an object is moving in a circle at a constant speed, its acceleration always points toward the center of the circle. From these activities, students have learned two important lessons. First, even though the speed is constant, there is acceleration because the *direction of the velocity* is changing. Second, for constant speed circular motion, the acceleration points toward the center of the circle. It is worth mentioning at this point that when an object moves in a circular path but not at constant speed, its acceleration will not point toward the center of the circle (building on ALG Activity 5.2.4).

Now students are ready to connect dynamics and kinematics. Both the acceleration and the sum of the forces point toward the center of the circle, just as it should be if Newton's second law were valid for circular motion! This connection is made in the ALG Activity 5.2.3.

Students can either proceed to the application of their new ideas by doing ALG Activity 5.2.7 or if you wish them to actually test these ideas, they can do Activities 5.2.5 and 5.2.6. In these activities, the situation is a little more complicated because the speed of the pendulum is changing, however, if they assume that the speeds right before the pendulum passes the lowest point and right after are the same, they can make a fascinating prediction concerning the reading of the scale – it should be greater! When students see that their prediction matches the outcome, they clap! Note that according to PER research, only a few physics graduate students can reason to the prediction that matches the outcome of this particular experiment. Your students will be very proud of themselves if they can do it!

Now students are ready to read the textbook – ALG Activity 5.2.8. The tip before Conceptual Exercise 5.1 is very important. Students have a tendency to draw very small diagrams, which makes it very difficult for them to determine the velocity change arrow. Ask them to make the diagrams larger, and it will help them a great deal.

Students can then use the graphical velocity technique, force diagrams, and Newton's second law to reason qualitatively about circular motion questions (for example, EOC Questions 1–3, 5, 6, or 9). Do not skip ALG Activity 5.2.9 as many students have this question!

Students often add extra forces in the direction of motion to their diagrams—for example, in a force diagram for a pendulum ball while passing the bottom of its swing. They also have a tendency to add an outward-pointing force because of the sensation of being pushed outward when they themselves are moving in a circular path (such as being in a car taking a turn at high speed). To deal with these issues, insist that students be able to identify the external object exerting each force. They will not be able to give satisfying answers in either case. We also do not ever use the term centripetal force in the book. It is simply a synonym for the radial component of net force, but when it has a special name it becomes confusing for students. Students tend to interpret centripetal force as an additional force exerted on the object and include it on their force diagrams.

II. Quantitative analysis of constant speed circular motion

In Section 5.3, students examine the kinematics of circular motion quantitatively to develop an expression for the magnitude of the radial acceleration of an object moving at constant speed v in a circle of radius R: $a_r = v^2/R$.

We suggest that they start with the ALG Activities 5.3.1 and 5.3.2 and then read the textbook answers to the questions posed in these activities. They analyze experiments (Observational Experiment Tables 5.3 and 5.4). The key to ALG Activity 5.3.1 or Experiments 2 and 3 in the textbook table is understanding that the acceleration of an object is the ratio of the velocity change over the time interval during which this change occurred. Students tend to focus on the velocity change and overlook the time interval. Make sure students read the analysis (the second column) of the experiments in both tables. Also, notice the *Try It Yourself* question after Quantitative Exercise 5.3 that helps students see the importance of choosing correctly the radius of the circle. To test Newton's second law for circular motion, students can do ALG Activities 5.3.3 and 5.3.4. Activity 5.3.3 is a full lab that we have been running for years. It works very well with the students. Do not skip ALG Activity 5.3.4 – it can be a homework exercise. EOC Questions 8, 10, and 24 and Problems 4 – 8 are useful here.

Section 5.4 applies the component form of Newton's second law, kinematics, and our qualitative reasoning abilities to circular motion problems. Students must learn to use a coordinate system with a radial axis (toward the center of the circle) and for some problems, a vertical y-axis. Before doing traditional problems, it is helpful for students to practice drawing force diagrams and writing Newton's second law without solving for anything. ALG Activity 5.4.1 serves this purpose. Again, it is a good idea to use graph paper that students can rotate to assist them in the choice of the coordinate system when they are drawing force diagrams. A problem-solving box outlines the multiple representation strategy for circular motion problems (ALG Activity 5.4.2. Finally all of the ALG activities in the section are suitable for class work. Activity 5.4.9 is another lab that will help your students apply Newton's second law to circular motion. We suggest that you choose between this lab and the lab in Activity 5.3.3. Activity 5.4.10 is an excellent opportunity for the students to practice their evaluation skills. The wrong solution has mistakes that students often make.

Note the wealth of non-traditional EOC problems in this chapter as shown in the table at the beginning here: make sure you assign most of them so that students develop a variety of reasoning skills and science practices.

At the end of Section 5.4 there is a subsection entitled "Conceptual Difficulties with Circular Motion" that helps students reconcile their everyday knowledge with what they just learned (if the students did ALG Activity 5.2.9 they already discussed this issue). Students are invited to analyze the situation of a passenger in a turning car from the point of the view of the observer on the ground or from the point of view of the observer in the car. The latter cannot explain why she/he starts sliding toward the door when the car turns. This means that the car is not an inertial reference frame. This is an excellent opportunity to revisit the circumstances under which Newton's second law holds.

You may want to refer to this section much earlier in the development of circular motion (that is why this activity comes earlier in the ALG). From early on, more curious students will recognize that the feeling of being "thrown to the left" when you turn right (or vice versa) in a car does not seem to be consistent with the idea that the sum of the forces on the person in the car point toward the center of the circle. This is a great opportunity to turn a lecture into an interactive discussion. You could invite students to turn to their neighbors and discuss how they could reconcile the feeling of being thrown away from the center of the circle with the idea that the net force points toward the center of the circle. It is an opportunity for students to integrate their knowledge of Newton's first law with the idea that even though the speed of an object moving in a circle may not be changing, the *direction* of its motion is changing.

Excellent EOC Questions to ponder: 15, 19, and 24. EOC problems to assign for class and homework: 9, 11, 12, 15, 23, 28, 29 32, and 33. Note that in Problem 11 students encounter the explanation of rolling as a combination of circular and translational motion.

II. Newton's law of universal gravitation and large-scale circular motion

One of the goals of the textbook is to help students see the origins of the concepts and relations that comprise the body of introductory physics. The approach we took to help students learn Newton's law of universal gravitation is consistent with this goal. Following Newton's own reasoning, students analyze the data for the circular motion of the Moon and compare the Moon's acceleration to the free-fall acceleration of the objects on Earth. Then they apply Newton's second law to explain why objects on Earth fall with the same acceleration (independent of their mass) and thus reason that the force that Earth exerts on an object is proportional to the object's mass. Finally, they use Newton's third law to come up with the idea that the force that Earth exerts on an object should be proportional to the mass of Earth as well. Students at this level can follow this reasoning leading to the force being proportional to $1/r^2$ and the product of the two masses involved Mm. The proportionality constant G was determined after Newton's time. As a testing experiment, students then can use Newton's laws plus the law of universal gravitation to derive Kepler's third law, well known empirically at that time. This logical progression is reflected in ALG Activities 5.5.1–5.5.4. You may want students to work on those activities in groups before the whole-class discussion.

An important issue here is the one of weightlessness of astronauts in orbit (ALG Activity 5.5.8). Activity 5.5.6 came from a conversation with a student –it addresses another common difficulty often not recognized by teachers. The rest of the activities in this chapter of the ALG are very engaging – make sure your students have a chance to debate the doomsday scenario in Activity 5.5.12! EOC Questions 12–16 and Problems 37–45 are useful here.

Finally, there is a myriad of general problems that will help your students strengthen their conceptual understanding and develop reasoning skills. We recommend; 54, 55, 57–60, 66, 70 and 71 (this is a linearization problem).