

Montgomery Township Schools
ELEMENTS OF GEOMETRY
Grade 10

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I. OVERVIEW

Elements of Geometry is the second course in a three year sequence that will provide students with a modified version of the traditional Algebra I, Geometry, and Algebra II course sequence in the regular college preparatory program in mathematics. All essential elements and topics of plane geometry are covered. In addition, a full treatment of properties and relationships around the essential elements of measurement, congruence, similarity, parallelism, perpendicularity, and transformations is presented. Students are also given the opportunity to develop powers of spatial visualization, strengthen and expand basic algebraic skills, and are introduced to the relationships between shapes and equations through the study of coordinate geometry. Students are exposed to problem solving situations that require both informal and formal reasoning. They are given the opportunity to visualize, analyze and make conjectures about geometric shapes based upon inductive investigations. These investigations in turn lead to the development of deductive reasoning skills and the concept of mathematical proof. Emphasis is placed on deductive reasoning and its applications in developing a logical system of mathematics, problem solving, and writing of original proofs.

Successful completion of Elements of Algebra with a 70% or higher grade is a prerequisite for Elements of Geometry.

II. RATIONALE

Elements of Geometry is the first course in which the logic and structure of mathematics is clearly presented. Knowledge of this logic and structure provides a foundation for the study and application of the more complex concepts students may encounter in later math courses. In addition, geometry demonstrates clearly the strong tie between mathematics and the physical environment. It is a subject in which the student can actually see and feel mathematics through the countless geometric patterns and forms found in nature, art, and architecture, thereby gaining an understanding and awareness of the world around them.

III. STANDARDS AND STRANDS

The Montgomery Township Mathematics Program is aligned with the NJ Core Curriculum Content Standards. These standards and their associated strands are enumerated below:

- 4.1 All students will develop number sense and will perform standard numerical operations and estimations on all types of numbers in a variety of ways. (A,B,C)*
 - A. Number Sense
 - B. Numerical Operations
 - C. Estimation

*These letters represent cross-referencing to the district goals.

- 4.2 All students will develop spatial sense and the ability to use geometric properties, relationships, and measurement to model, describe and analyze phenomena.
 - A. Geometric Properties
 - B. Transforming Shapes
 - C. Coordinate Geometry
 - D. Units of Measurement
 - E. Measuring Geometric Objects
- 4.3 All students will represent and analyze relationships among variable quantities and solve problems involving patterns, functions, and algebraic concepts and processes.
 - A. Patterns and Relationships
 - B. Functions
 - C. Modeling
 - D. Procedures
- 4.4 All students will develop an understanding of the concepts and techniques of data analysis, probability, and discrete mathematics, and will use them to model situations, solve problems, and analyze and draw appropriate inferences from data.
 - A. Data Analysis (Statistics)
 - B. Probability
 - C. Discrete Mathematics—Systematic Listing and Counting
 - D. Discrete Mathematics—Vertex-Edge Graphs and Algorithms
- 4.5 All students will use mathematical processes of problem solving, communication, connections, reasoning, representations, and technology to solve problems and communicate mathematical ideas.
(A,B,C,D,E,N,U)
 - A. Problem Solving
 - B. Communication
 - C. Connections
 - D. Reasoning
 - E. Representations

Workplace Readiness Standards and Strands are enumerated below:

- 8.1 Self – Management
 - A. Self – Management
 - B. Interpersonal Skills
- 8.2 Career Awareness
 - A. Career Awareness
 - B. Employability Skills
- 8.3 Critical Thinking
 - A. Critical thinking
- 8.4 Computer Applications

- A. Specific Applications
- B. Social Aspects
- C. Research
- D. Problem Solving
- 8.5 Technology Education
(Engineering and Technological Design)
 - A. Nature and Impact of Technology
 - B. Design Process and Impact Assessment
 - C. Systems in the Designed World
- 8.6 Career/Technical Arts (Practical Arts)
 - A. Career Planning and Employability Skills
 - B. Consumer/Life Skills
 - C. Safety

IV. STUDENT OUTCOMES

A. KNOWLEDGE (Information and Concepts)

The student will:

1. construct and judge the validity of a logical argument consisting of a set of premises and conclusions. [4.1A,B; 4.2A.3, 4; 4.3A.3, D.8.4; 4.5A,B,C,D,E,F, 8.3A, 8.4A, C]** This will include:
 - identifying and formulating the converse, inverse, and contrapositive of a conditional statement
 - translating a logical argument into symbolic form
 - differentiating between conclusions reached intuitively, inductively, and deductively
 - using valid forms of deductive reasoning, including the law of syllogism law of detachment, and recognizing logical errors in faulty arguments.
2. use physical models and proper symbols to present basic geometric concepts. [4.1A.3,C.1; 4.2A,D,E; 4.3D.3, 4.5 B, D, E.2]
This will include:
 - the postulates expressing the relationships between undefined terms (points, lines, planes)
 - development of basic geometric definitions and the development and application of theorems relating to these terms
 - ability to select and use tools of measurement
3. solve practical problems involving angles and their measures. [4.1A.1, 2, C.1; 4.2A.1, 3,D.2; 4.5A.1, B.1-4, C.1, 2; 8.3A, 8.4A]
4. use the relationships between angles formed by two lines cut by a transversal to determine if two lines are parallel and verify algebraically as well as through writing deductive proof. [4.2A.3, 4]

**These numbers represent cross-referencing to the NJ Core Curriculum Standards building upon knowledge and skills in preceding grades that will be attained by the end of Grade 12.

5. investigate and identify relationships between triangles or other geometric figures with respect to congruency. Given information in the form of a diagram or statement, the student will use algebraic, coordinate and deductive proof to prove two triangles or other geometric figures are congruent. [4.4.6A.3; 4.5A.3, 5, B.1-4, C.1, D.1-6, F.1, 4, 6]
6. use inductive and deductive methods to develop and identify properties of quadrilaterals; these properties will be used in algebraic, coordinate and deductive proofs and to solve practical problems. [4.2A.3, 4, C.1, E.1; 4.5A.1, 2, 3, 5, B.1, 3]
7. investigate and identify similarity relationships between two triangles or between other geometric figures; use deductive reasoning to verify similarity given information in the form of a diagram or statement; use proportional reasoning to solve problems involving similar figures, including relationship of perimeter and area. [4.2.8A.4, 4.2.E.1; 4.5A.1, 2, 5, B.1-3; 8.3A]
8. apply the Triangle Inequality Theorem. Given information regarding the side and/or angle measures, the student will determine the order of the sides and angles. These concepts will be considered in the context of practical applications. [4.2E1; 4.5A1, 2, 3, B1, 2, 3, C5, D1, 2, F3, 4, 5]
9. use transformations on the coordinate plane to describe relationships between figures and to create tessellations and tilings. [4.2B1, 3, 4, C2; 4.5A1, 3, 4, C1, 3, F3, 5]
10. estimate square roots to the nearest tenth, and simplify radical expressions involving addition, subtraction, and multiplication. [4.1.8B3; 4.1A1,2, C; 4.5A1, 3, 4, B, C, E, F]
11. use the Pythagorean Theorem, its converse and properties of special right triangles to solve triangles. [4.1A1, 2, C1; 4.2A1, 2, E1; 4.5A, B, C, F4, 5; 8.3A]
12. develop and use the properties of angles, arcs, chords, tangents and secants to solve problems involving circles and use properties to find area of sectors and solve real problems. [4.2A,C,E, 4.3B, 4.5A,B,C,D,E,F]
13. use formulas to find perimeter, area, surface area, and volume of geometric shapes and three-dimensional objects. [4.1B,C; 4.2A,D,E; 4.3A,B; 4.4A,B,D; 4.5A,B,C,E,F; 8.3]
14. represent geometric relationships through the use of:
 - constructions
 - diagrams involving relationships in a plane and 2 and 3 dimensional diagrams involving relationships in space including using shadows, projections, and perspectives
 - coordinate geometry as a tool in indirect measurement[4.1A,B, 4.2A,E, 4.3B, 4.4A,B,C,D, 4.5A,B,C,E, 8.3A,8.4A]
15. use discrete methods to model geometric problems. [4.1A, B, 4.2A, 4.4C,D, 4.5A - F]

B. ATTITUDES

The student will:

1. develop a desire to pursue the study of mathematics in the future. [CPI: 4.5C, F, 8.2A, B]***
2. develop an understanding and appreciation for the application of mathematics in real life situations and its relationship to other disciplines. [CPI: 4.5C, 4.5F, 8.4A, B, D]
3. develop a desire to persist and solve mathematical problems from beginning to end. [CPI: 4.5D, 8.1A, 8.3A]
4. develop an appreciation for the history of mathematics. [CPI: 4.5C]
5. develop an appreciation for mathematics as an integrated whole. [CPI: 4.5C, 8.3A]

C. SKILLS AND BEHAVIORS

The student will:

1. develop and apply various strategies to solve problems from everyday and mathematical situations as well as career-based problems. [CPI: 4.5A, 8.2A, B]
2. develop the ability to communicate mathematically through a variety of forms of expression including oral, written, and visual. [CPI: 4.5B, E, 8.1B]
3. use physical models and manipulatives to model and investigate problem situations and mathematical concepts. [CPI: 4.3C, E]
4. use calculators and other appropriate forms of technology to gather, analyze, and display mathematical data, and to facilitate and enhance their mathematical thinking, understanding, and power. [CPI: 4.4A, 4.5F, 8.5B, C, 8.6B]
5. demonstrate organization by keeping a complete and detailed notebook. [CPI: 8.2B]
6. work independently and in cooperative groups to enhance mathematical thinking. [CPI: 4.5B, 8.1B, 8.2B]
7. recognize that there may be multiple ways to solve a problem, weigh their relative merits, and select and use appropriate problem solving strategies. [CPI: 4.5A, D, E, 8.1B, 8.2B]
8. reflect on and clarify their thinking so as to present convincing arguments for their conclusions. [CPI: 4.5A, B, C, 8.1B]

***CPI refers to Cumulative Progress Indicators as outlined by the NJ Core Curriculum Standards

V. STRATEGIES

This course will place emphasis on the logical development of geometry as a mathematical system using deductive reasoning. Students will be given the opportunity to move from an intuitive and inductive approach in reasoning to one based on mathematical proof.

This movement in implementing a deductive approach to drawing conclusions will be accomplished by allowing each student to be an investigator/explorer in generating logical thought patterns.

To help in implementing a deductive approach to drawing conclusions students will:

- formulate a hypotheses;
- develop and perform mathematically based experiments to test their hypothesis;
- gather and organize data from the experiments;
- formulate conclusions and write mathematical proofs to verify the conclusions using: two-column proof; indirect proof; paragraph proof; key-step proof; and flow proof.

The students will participate actively in discovering geometric relationships using the following strategies:

- note-taking, writing ideas, brainstorming
- using manipulatives/measurement tools
- using calculators/computers.

Since the focus of all strategies and methods will be to foster the development of logical thinking and clear communication of ideas, appropriate classroom time will be planned to allow students to:

- work independently, work in pairs, and work in cooperative groups
- present original work to other students; receive critiques of their work from other students in a classroom setting; and present a clearly expressed critique of the works of others.

VI. EVALUATION

Students will be evaluated by multiple criteria, which may include:

- Chapter/Unit Tests & Quizzes; these will consist of recall questions, short constructed response questions, and open-ended questions requiring students to explain their thinking in arriving at their solution/conclusion.
- Notebooks; specific criteria will be determined by the teacher and will include note taking and homework.
- Out-of-class graded assignments such as enrichment problems
- Research and/or enrichment projects
- Oral presentations
 - informal - participation in class discussions;
 - formal - presentation of special assignments/projects.
- Class participation as determined by teacher

The marking period grades for the course will be determined as follows:

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| ▪ Formal assessment (tests/quizzes) | 70% |
| ▪ Homework (method determined by teacher) | 20% |
| ▪ Other - notebook, project, class participation, assignments (as determined by teacher) | 10% |

The number of and frequency of tests/quizzes and other assessments will be determined by the teacher.

The final grade for Elements of Geometry will be determined by the following:

Four Quarter Grades each 20%	80%
One Midterm Exam	10%
One Final Exam	10%

Midterm and final exams are departmental tests and will consist of multiple choice, short constructed response, and open-ended questions.

VII. REQUIRED RESOURCES

1. The text for this course is: Boswell, L., Larson, R., Kanold, T., and Stiff, L, McDougal Littell Geometry, Evanston, IL 60204.
2. Other primary resources:
 - a. Teachers' Resources, Geometry, Glencoe.
 - b. Jacobs, Harold R., Geometry, W.H. Freeman & Co., San Francisco, CA, 1974.
 - c. Bennett, Dan, Exploring Geometry with Geometer's Sketchpad, Key Curriculum Press, Berkeley, CA, 1999.
 - d. NJ Mathematics Curriculum Framework, NJ Department of Education, 1996.
3. Other materials:
Protractors, compasses, rulers, pentominoes, tangrams, Geoboards, scientific calculators, TI – 92 Plus graphing calculators
4. Recommended Software: The Geometer's Sketchpad 3.0 by Key Curriculum Press

VIII. SCOPE AND SEQUENCE

CHAPTER

TIME

1. ESSENTIALS OF GEOMETRY

14 days

1-1 Identify Points, Lines, and Planes

1-2 Use Segments and Congruence

1-3 Use Midpoint and Distance Formulas

- Modeling Mathematics: Locating the Midpoint of a Segment

- 1-4 Measure and Classify Angles
1-5 Describe Angle Pair Relationships
- 2. REASONING AND PROOF 15 days**
2-1 Use Inductive Reasoning
2-2 Analyze Conditional Statements
 - Investigating Geometry Activity: Logic Puzzles2-3 Apply Deductive Reasoning
2-4 Use Postulates and Diagrams
2-5 Reason Using Properties from Algebra
2-6 Prove Statements about Segments and Angles
 - Using Technology: Angles and Intersecting Lines2-7 Prove Angle Pair Relationships
- 3. PARALLEL AND PERPENDICULAR LINES 14 days**
3-1 Identify Pairs of Lines and Angles
 - Using Technology: Parallel Lines and Angles3-2 Use Parallel Lines and Transversals
3-3 Prove Lines are Parallel
3-4 Find and Use Slopes of Lines
 - Using Technology: Investigate Slopes3-5 Write and Graph Equations of Lines
- 4. CONGRUENT TRIANGLES 18 days**
4-1 Apply Triangle Sum Properties
 - Modeling Mathematics: Angle Sums in Triangles Activity4-2 Apply Congruence and Triangles
4-3 Prove Triangles Congruent by SSS
4-4 Prove Triangles Congruent by SAS and HL
4-5 Prove Triangles Congruent by ASA and AAS
4-7 Use Isosceles and Equilateral Triangles
4-8 Perform Congruence Transformations
 - Modeling Mathematics: Investigate Slides and Flips
- 5. RELATIONSHIPS WITHIN TRIANGLES 15 days**
5-1 Midsegment Theorem and Coordinate Proof
 - Modeling Mathematics: Investigate Segments in Triangles5-2 Use Perpendicular Bisectors
5-3 Use Angle Bisectors of Triangles
5-4 Use Medians and Altitudes
 - Using Technology: Investigate Points of Concurrency5-5 Use Inequalities in a Triangle
5-6 Inequalities in Two Triangles and Indirect Proof
- 6. SIMILARITY 16 days**
6-1 Ratios, Proportions, and the Geometric Mean

- 6-2 Use Proportions to Solve Geometry Problems
 - 6-3 Use Similar Polygons
 - Modeling Mathematics: Similar Polygons
 - 6-4 Prove Triangles Similar by AA
 - 6-5 Prove Triangles Similar by SSS and SAS
 - 6-6 Use Proportionality Theorems
 - Using Technology: Investigate Proportionality
 - 6-6A Extension: Fractals
 - 6-7 Perform Similarity Transformations
- 7. RIGHT TRIANGLES AND TRIGONOMETRY 16 days**
- 7-1 Apply the Pythagorean Theorem
 - 7-2 Use the Converse of the Pythagorean Theorem
 - Similar Right Triangles
 - 7-3 Use Similar Right Triangles
 - 7-4 Special Right Triangles
 - 7-5 Apply the Tangent Ratio
 - 7-6 Apply the Sine and Cosine Ratios
- 8. QUADRILATERALS 14 days**
- 1-6 Classify Polygons
 - 8-1 Find Angle Measures in Polygons
 - 8-2A Using Technology: Investigate Parallelograms
 - 8-2 Use Properties of Parallelograms
 - 8-3 Show that a Quadrilateral is a Parallelogram
 - 8-4 Properties of Rhombuses, Rectangles, and Squares
 - 8-5A Using Technology: Midsegment of a Trapezoid
 - 8-5 Use Properties of Trapezoids and Kites
- 10. PROPERTIES of CIRCLES 18 days**
- 10-1A Modeling Mathematics: Explore Tangent Segments
 - 10-1 Use Properties of Tangents
 - 10-2 Find Arc Measures
 - 10-3 Apply Properties of Chords
 - Modeling Mathematics: Explore Inscribed Angles
 - 10-4 Use Inscribed Angles and Polygons
 - 10-5 Apply Other Angle Relationships in Circles
 - Modeling Mathematics: Investigate Segment Lengths
 - 10-6 Find Segment Lengths in Circles
- 11. Measuring Length and Area 14 days**
- 11-1 Areas of Triangles and Parallelograms
 - 11-2A Modeling Mathematics: Areas of Trapezoids and Kites

11-2 Areas of Trapezoids, Rhombuses, and Kites
11-3 Perimeter and Area of Similar Figures
11-4 Circumference and Arc Length
11-5 Areas of Circles and Sectors
11-6 Areas of Regular Polygons
11-7 Geometric Probability

12. SURFACE AREA AND VOLUME OF SOLIDS

10 days

12-2 Surface Area of Prisms and Cylinders
12-3 Surface Area of Pyramids and Cones
12-4 Volume of Prisms and Cylinders
12-5 Volume of Pyramids and Cones
 • Using Technology: Minimize Surface Area
12-6 Surface Area and Volume of Spheres
12-7A Modeling Mathematics: Investigate Similar Solids
12-7 Explore Similar Solids

**MIDTERMS, FINALS, STANDARDIZED TESTING DAYS,
AND OTHER SPECIAL SCHEDULES**

16 days