Circular motion and Gravitation

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Please rename yourself

First name

Level of teaching (high school, college, university)

Country

Eugenia, University, USA

Link to today’s folder and materials

https://drive.google.com/drive/folders/1jvSufTlxaFd3_Oeng2tBCDIbGPdHhsEZ

OALG file

https://docs.google.com/document/d/1HrIN1NAbTkZhso58ar2ifONERz8FWyPe/edit

ALG file

https://docs.google.com/document/d/1O54fzkyXF14rSEc_4hvHAmhi4g7lLetA/edit

What should students know before starting this unit:

1. How to draw motion diagrams, most importantly what delta v arrow means
2. How to add vectors
3. How to determine how many forces are exerted on an object of interest (the system)
4. How do draw a force diagram
5. How to label forces on the force diagram
6. Newton's second law as *a* = sum of *F*/ *m* (vector form and in components)

Reminder on how to draw force diagrams and how to use two subscripts

PHYSICS TOOL BOX 3.1 **Constructing a force diagram**

1. Sketch the situation (a rock sinking into sand).

2. Circle the system (the rock).

3. Identify external interactions:
   - The sand pushes up on the rock.
   - Earth pulls down on the rock.

4. Place a dot at the side of the sketch, representing the system.

5. Draw force arrows to represent the external interactions.

6. Label the forces with a subscript with two elements.

Need to know

https://www.youtube.com/watch?v=dSDb9oKMCRc&t=19s watch all

https://www.youtube.com/watch?v=sOY9p5gFa5Q first 47 seconds

OALG 5.1.1 Observe and find a pattern

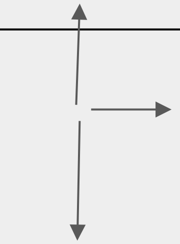
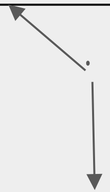
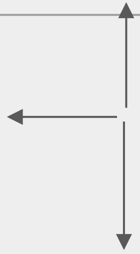
Watch the videos of the following three experiments:

https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-OALG-5-1-1. For each experiment, fill in the blanks in the table that follows. Assume that the frictional forces exerted on all three objects are negligible and that the objects move at constant speed.

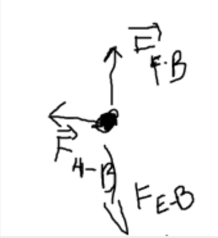
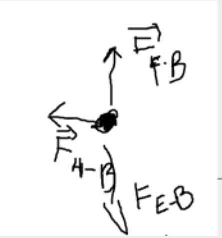
When you are drawing the force diagrams, assume that the center of the circle is horizontally **to the left of the point of interest.**

Do not forget double subscript force notation!

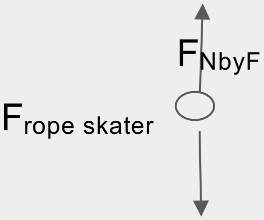
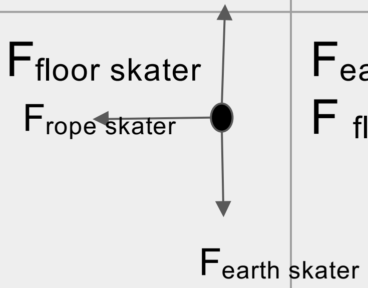
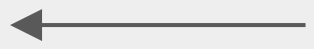
Team 1

Experiment; the circling object is in bold.	**List objects that interact with the circling object.**	**Draw a force diagram for the circling object.**	**List forces or force components that add to zero.**	**Indicate the direction of the sum of the forces exerted on the object.**
a. Tapping a **bowling ball.** So it moves in a circle on the floor.	Mallet, floor		F surface on ball F Earth on ball	Fmallet on ball towards center
b. Swinging a **bucket** in a horizontal circle.	Rope on the bucket gravity			Towards center
c. Pulling a rope attached to a moving **rollerblader** so she moves in a circle.	Rope on roller blader Gravity, normal force		F surface on rollerblader F Earth on rollerblader	Towards center F rope on rollerblader

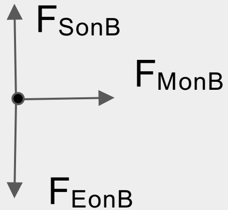
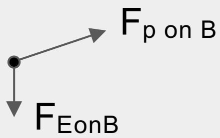
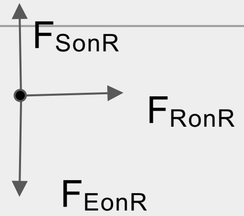
Team 2

Experiment; the circling object is in bold.	**List objects that interact with the circling object.**	**Draw a force diagram for the circling object.**	**List forces or force components that add to zero.**	**Indicate the direction of the sum of the forces exerted on the object.**
a. Tapping a **bowling ball.** So it moves in a circle on the floor.	The hammer, Earth, floor		FF-B- + FE-B = 0 (vertical) FH-B (horizontal) not adding to 0	Towards the center (to the left)
b. Swinging a **bucket** in a horizontal circle.				
c. Pulling a rope attached to a moving	The rope, Earth		FE-E + FF-E	Towards the center (to

Team 3

Experiment; the circling object is in bold.	**List objects that interact with the circling object.**	**Draw a force diagram for the circling object.**	**List forces or force components that add to zero.**	**Indicate the direction of the sum of the forces exerted on the object.**
a. Tapping a **bowling ball.** So it moves in a circle on the floor.	Ball, floor , earth, hammer			
b. Swinging a **bucket** in a horizontal circle.	Bucket, rope, earth			
c. Pulling a rope attached to a moving **rollerblader** so she moves in a circle.	Rope Floor earth		$F_{earth\ skater}$ + F floor skater	

Team 4

Experiment; the circling object is in bold.	**List objects that interact with the circling object.**	**Draw a force diagram for the circling object.**	**List forces or force components that add to zero.**	**Indicate the direction of the sum of the forces exerted on the object.**
a. Tapping a **bowling ball.** So it moves in a circle on the floor.	Surface Earth Mallet		F_{SonB}, F_{EonB}	Towards the center of the circle
b. Swinging a **backpack** in a horizontal circle.	Person Earth		$F_{Eon\ B}$,vert comp of $F_{p\ on\ B}$	Towards the center of the circle
c. Pulling a rope attached to a moving **rollerblader** so she moves in a circle.	Surface Earth Rope		F_{SonR} ,$F_{Eon\ R}$	Towards the center of the circle

OALG 5.1.2 Find a pattern

Review your analysis recorded in the table for Activity 5.1.1. Based on your observations and on the analysis, **find a pattern** for the direction of the sum of the forces exerted on the objects moving at constant speed in a circle in all three experiments.

Summarize your pattern in words and compare to the pattern identified in Observational Experiment Table 5.1 on page 119 in the textbook.

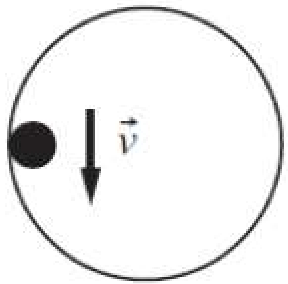
OALG 5.1.3 Test your explanation (ALG 5.1.4)

For the following testing experiment, use the pattern that you formulated in Activity 5.1.2 and Newton's laws to predict the outcome of the experiment. Do not watch the video until you finish part **b** of this activity.

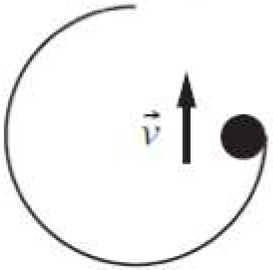
a. Inside a metal ring, a person rolls a small ball or a marble on a smooth horizontal surface. The marble rolls along the ring. Is the motion of the ball consistent with the pattern formulated in Activity 5.1.2? Explain.

b. Use the pattern you found in Activity 5.1.2 (not your intuition) to predict what will happen to the ball if, after the ball rolls for a couple of turns, the person removes a quarter of the ring as shown in the figure. Justify your prediction in words and with a force diagram before you watch the video of the experiment.

Top view



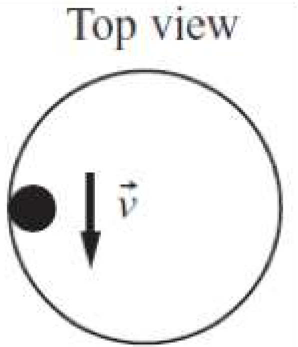
Later when loop is open

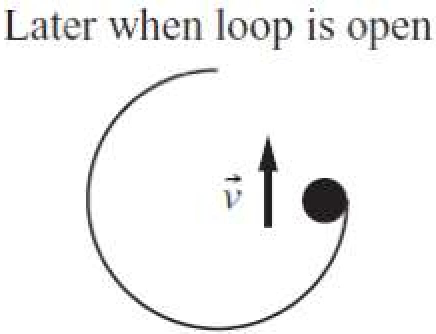


OALG 5.1.3 Test your explanation (ALG 5.1.4)

c. After you make your prediction, watch the video, and compare the outcome to your prediction. What judgment can you make about the idea that you're testing? Does the outcome support, prove, or disprove the idea you're testing?

https://youtu.be/BOkX_BnNKzU





OALG 5.2.1 Represent and reason

In the activities in the previous section, you learned that the sum of the forces exerted on an object moving in a circle at constant speed points toward the center of the circle. Why is that? Think of the motion of the object. The speed is constant, but is the velocity constant? How can you find the direction of the velocity of an object undergoing this type of motion at any instant?

a. Observe the following experiment. https://youtu.be/YCfb5mnb4yM. Use the direction of the stick that Eugenia is holding to draw the velocity vectors for such an object at four different points on the circle. What is the direction of the velocity vector? What is its magnitude?

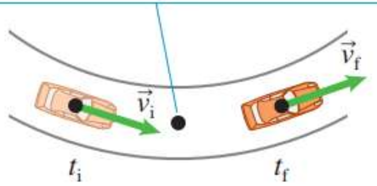
b. What can you say about the motion of the object? Is it motion with constant velocity? If not, how can you determine the acceleration at each point during the motion? Think of the definition of acceleration ( ) and how you determined the direction of the acceleration in Chapter 2 for objects moving in a straight line.

c. Read and interrogate Section 5.2, especially Physics Toolbox 5.1 in the textbook (page 121) to learn the technique for determining the direction of acceleration of an object that is not moving along a straight line.

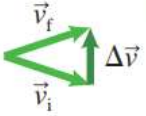
How do you interrogate a figure?

PHYSICS TOOL BOX 5.1 **Estimating the direction of acceleration during two-dimensional motion**

1. Choose the point at which you want to determine the direction of acceleration and draw velocity vectors at equal distances before and after the point.



2. Place the $\vec{v}_i$ and $\vec{v}_f$ arrows tail to tail. Draw a $\Delta\vec{v}$ arrow from the head of $\vec{v}_i$ to the head of $\vec{v}_f$.



$$\vec{v}_i + \Delta\vec{v} = \vec{v}_f$$
or
$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

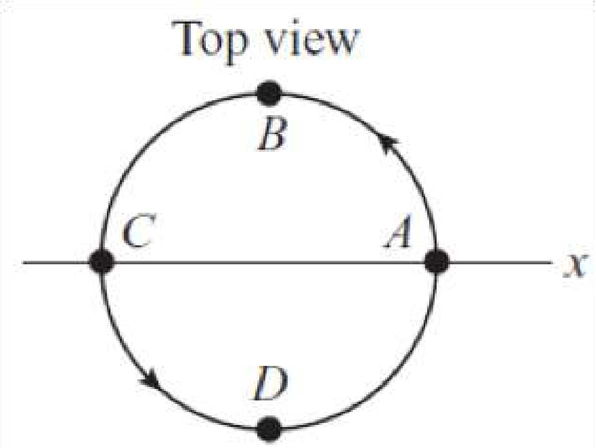
3. The acceleration arrow $\vec{a}$ is in the direction of $\Delta\vec{v}$.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

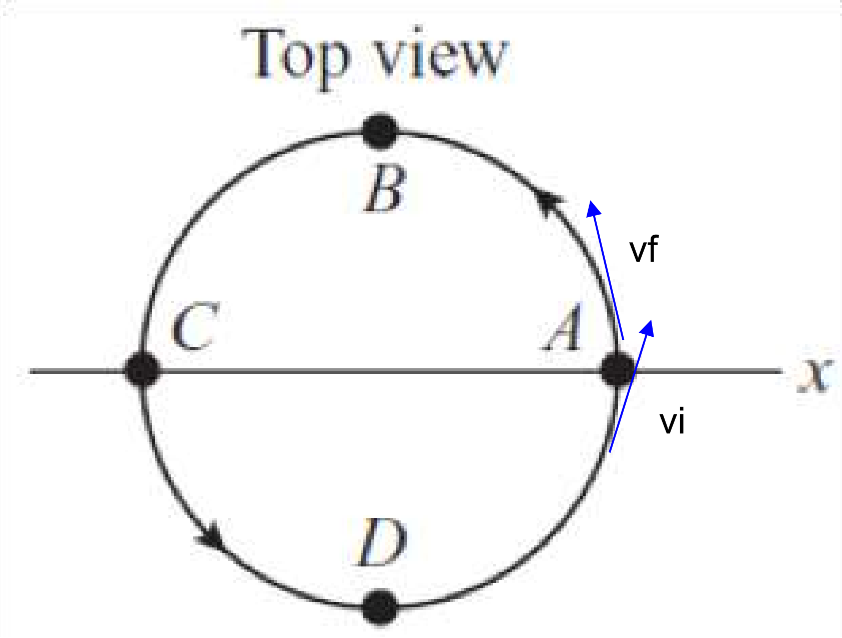
OALG 5.2.2 Represent and reason

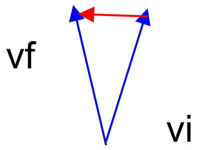
An object moves at constant speed in a circle.

a. Determine the direction of its acceleration at a designated point? for your team position shown in the illustration. Use what you learned in Physics Toolbox 5.1. *Make sure you take a point right before the point of interest and right after, and use a ruler to make sure the lengths of the velocity vectors remain the same and their directions are tangent to the circle.*

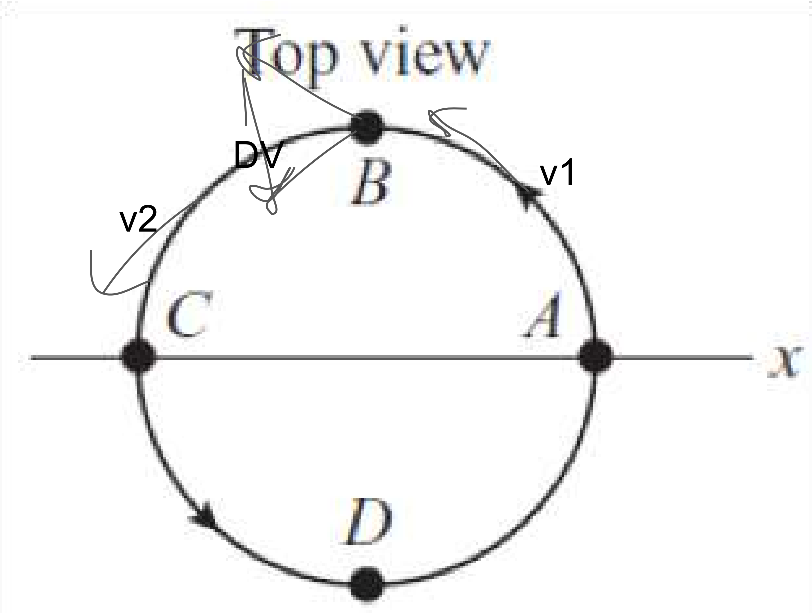


Team 1 Point A

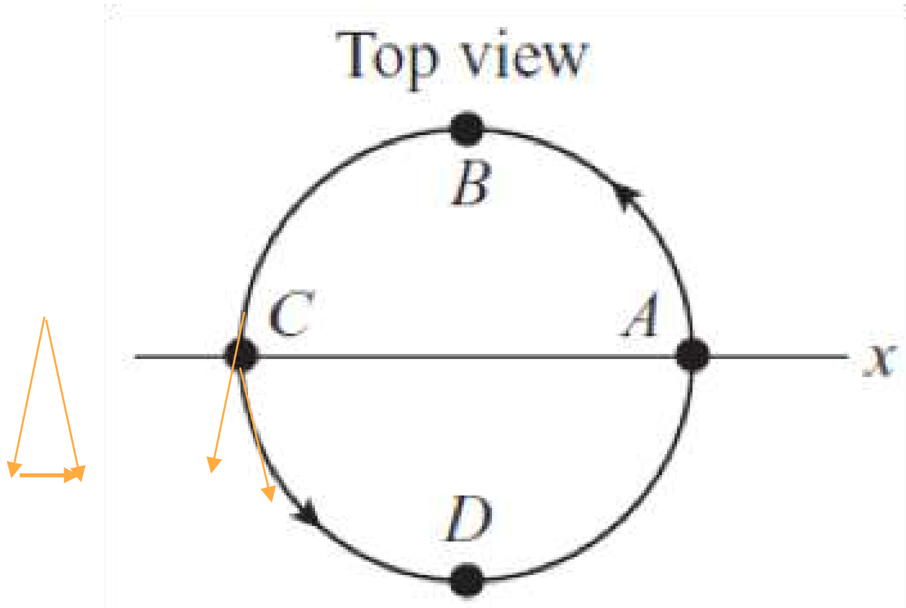




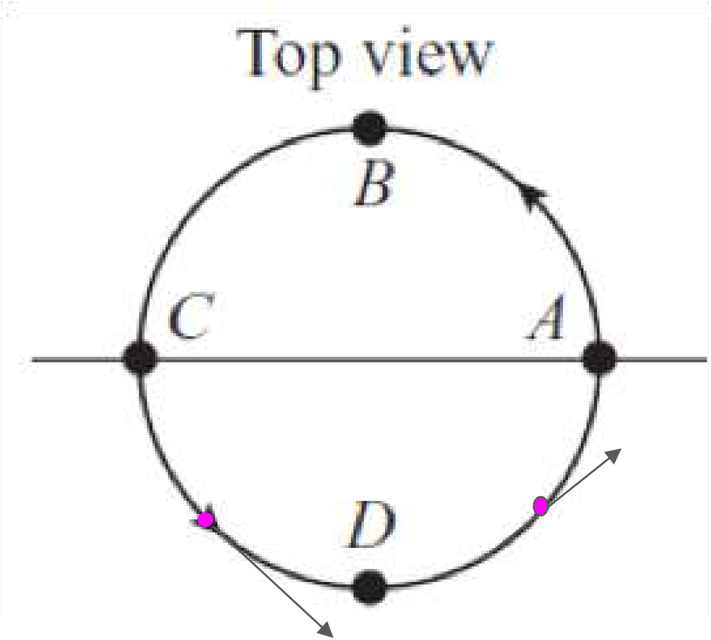
Team 2 Point B

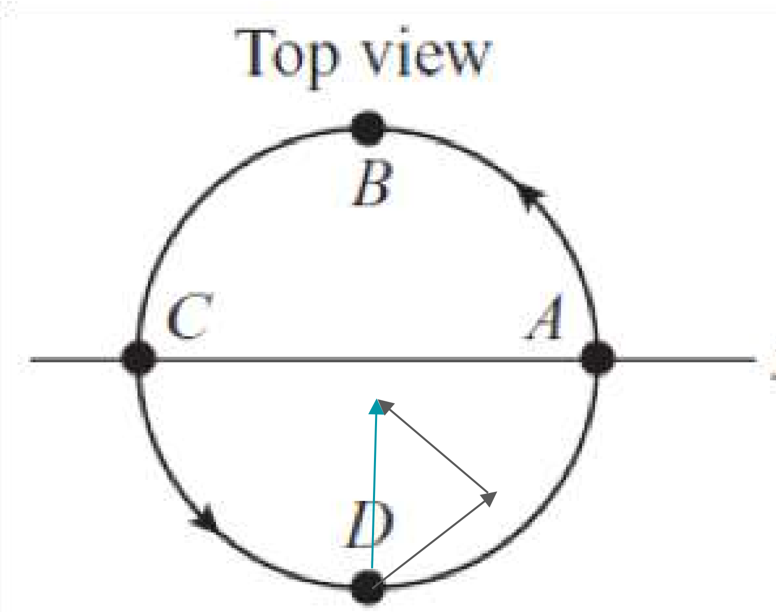


Team 3 Point C



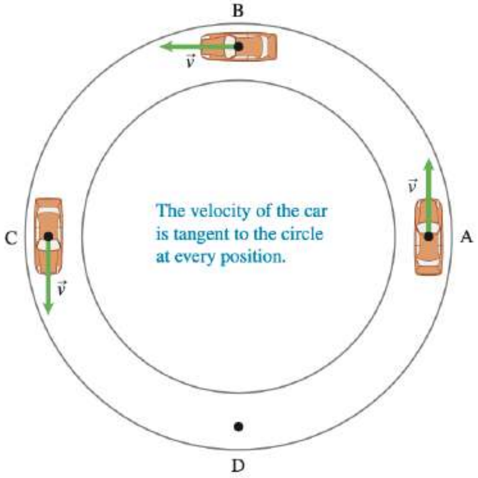
Team 4 Point D

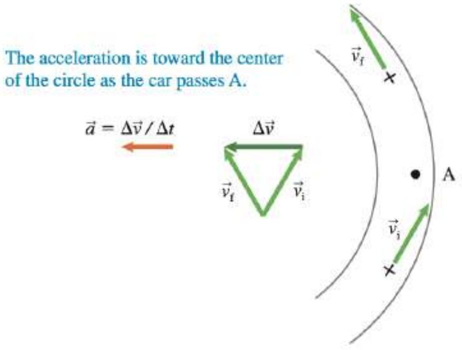


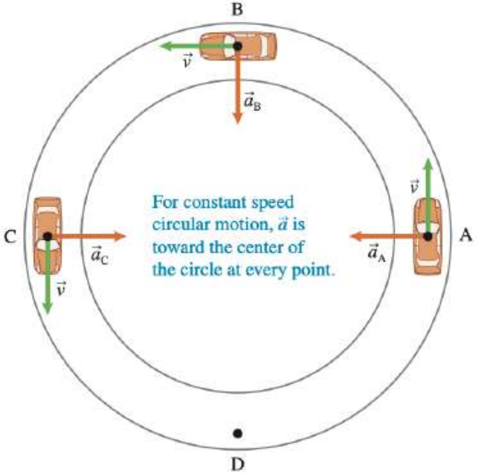


CONCEPTUAL EXERCISE 5.1 Direction of a racecar's acceleration

Determine the direction of the racecar's acceleration at points A, B, and C in the figure below as the car travels at constant speed on the circular path.



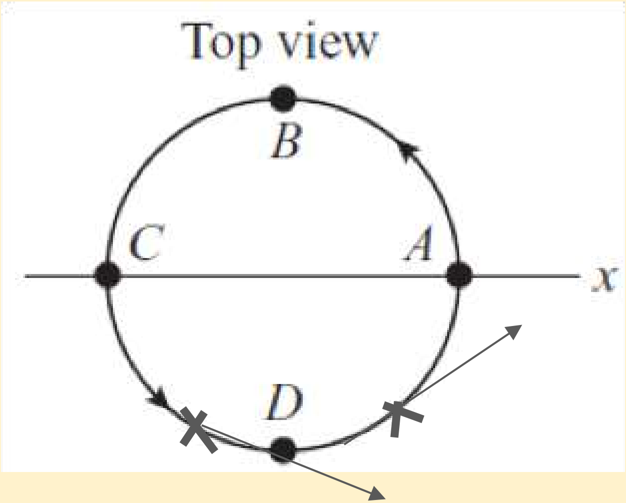


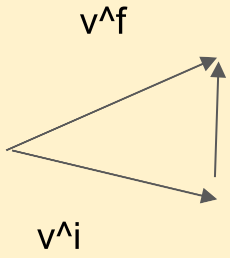


Sketch and translate A top view of the car's path is shown above. We are interested in the car's acceleration as it passes points A, B, and C.

Simplify and diagram To find the direction of the car's acceleration at each point, we use the velocity change method (shown for point A above, right). When done for all three points, notice that a pattern emerges: the acceleration at different points along the car's path has a different direction, but in every case it points toward the center of the circular path.

Example Point D





b. Do you see a pattern in the directions of the acceleration vectors? If so, what is it? Summarize your pattern and compare it with the pattern in Conceptual Exercise 5.1 on page 122 in the textbook.

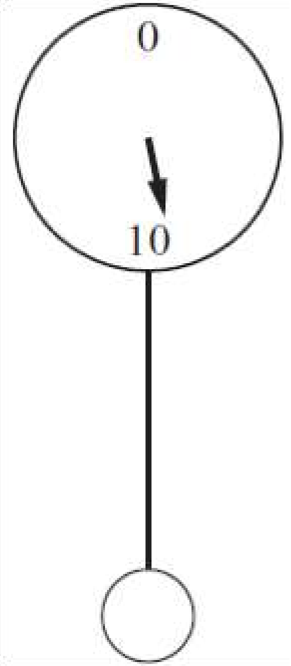
OALG 5.2.3 Explain

Explain how the pattern you found in Activity 5.2.2 is connected with the pattern you found in Activity 5.1.2. Does the relationship between these two patterns make sense?

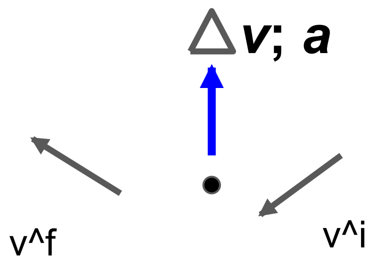
OALG 5.2.6 Test your ideas

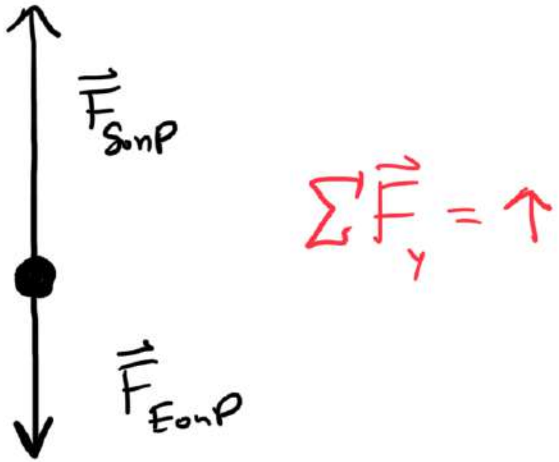
Now that you have figured out how to determine the direction of the acceleration of the pendulum bob at the bottom of its swing, use everything you have learned about circular motion so far to make a prediction of the outcome of the following experiment:

a. A 0.1-kg ball hangs from a 1.0-m long string. The other end of the string is attached to a Newton force measuring scale. The string pulls up on the ball, exerting about a 1.0-N force. The string and ball, in turn, pull down on the scale, exerting a 1.0-N force—the scale reads about 1.0 N. Imagine that you pull the ball to the side and release it so that the ball swings like a pendulum at the end of the string. Predict the scale reading as the ball passes directly under the scale (i.e., is it more than, less than, or equal to 1.0 N?).



IF the direction of the sum of the forces points toward the center of the circle AND, we release a ball on a string so it can swing freely, THEN the scale reading at the bottom of the arc will be greater than 1 N, BECAUSE the force exerted by the string on the ball must be greater for than the force exerted by Earth in order for its direction of motion to change.





b. Watch the video of the experiment Pendulum Suspended from Spring Scale: Testing Experiment; record the outcome, and compare it to your prediction. Did the outcome support the pattern?

Time for telling

ASummarize what we have learned about circular motion so far.

Acceleration is pointing towards the centre, so as the net force

There is a resultant force towards the centre

The velocity is tangential to the circle

Force towards center

- Sum Forces towards center, acceleration towards center

Sum of the Forces is towards the center

Change in velocity is perpendicular to the velocity

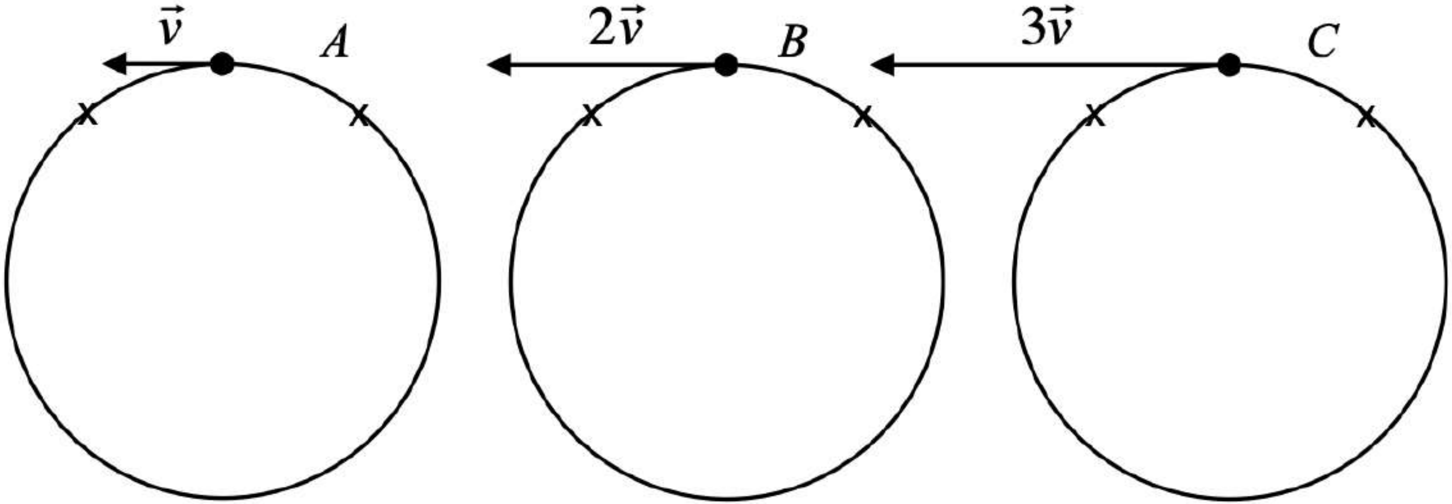
I saw that the scale measured higher values showing the change in acceleration

Net force is always towards the center of the circle.

Please do the next two activities together and then report to the group

OALG 5.3.1

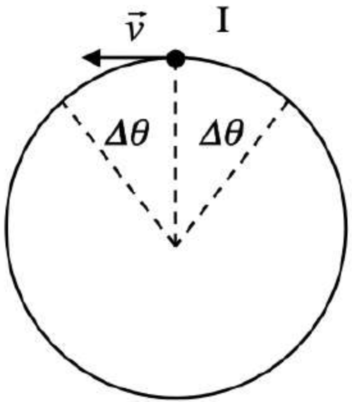
Imagine three small toy cars travel at constant speed in identical-radii horizontal circular paths (a top view is shown below). Car *A* moves at speed v, car *B* at speed $2v$, and car *C* at speed $3v$. Use the velocity technique (Physics Toolbox 5.1 in the textbook) to determine how the magnitude of the acceleration of the cars depends on their speeds. Remember that acceleration is $\Delta\vec{v}/\Delta t$ and that you need to compare the velocity change $\Delta\vec{v}$ vectors for the three speeds and also the time interval Δt needed for the velocity changes in each of the three cases.

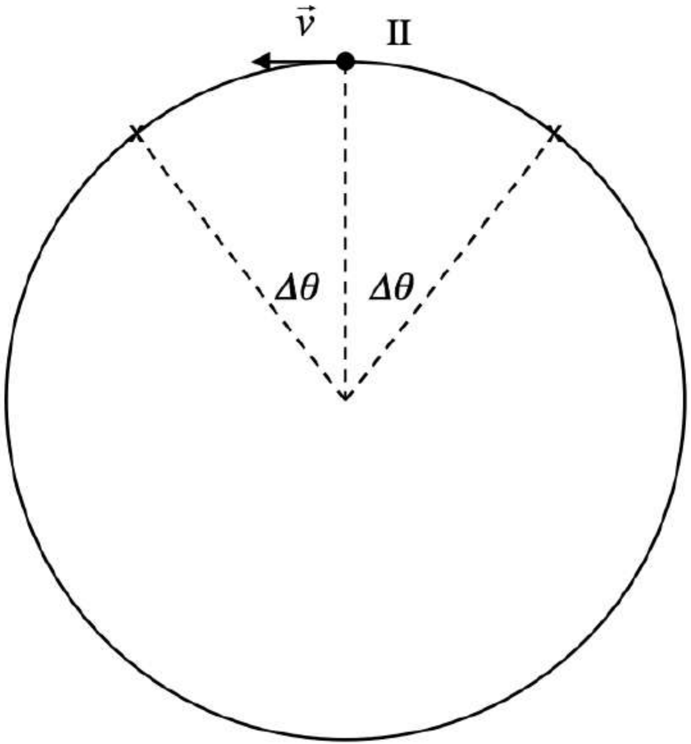


OALG 5.3.2

Two small toy cars travel at the same constant speed in horizontal circular paths (a top view is shown below). Car I moves in a circle of radius r and car II in a circle of radius $2r$.

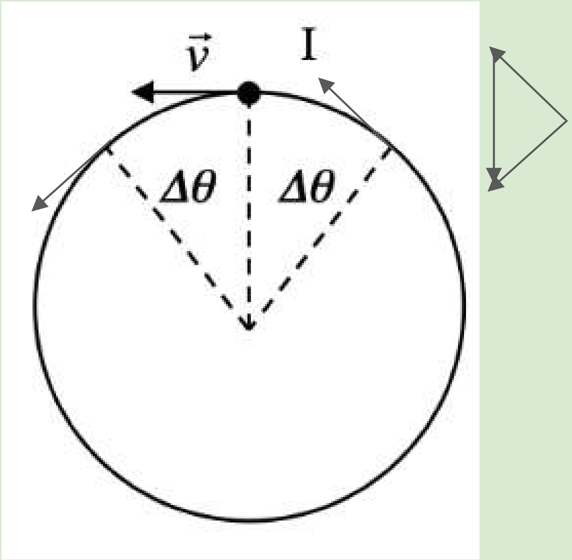
a. Use the velocity technique (Physics Toolbox 5.1 in the textbook) to determine how the magnitudes of the accelerations of the cars depend on the radii of the circles. Do not forget to consider the time intervals needed for the velocity changes.

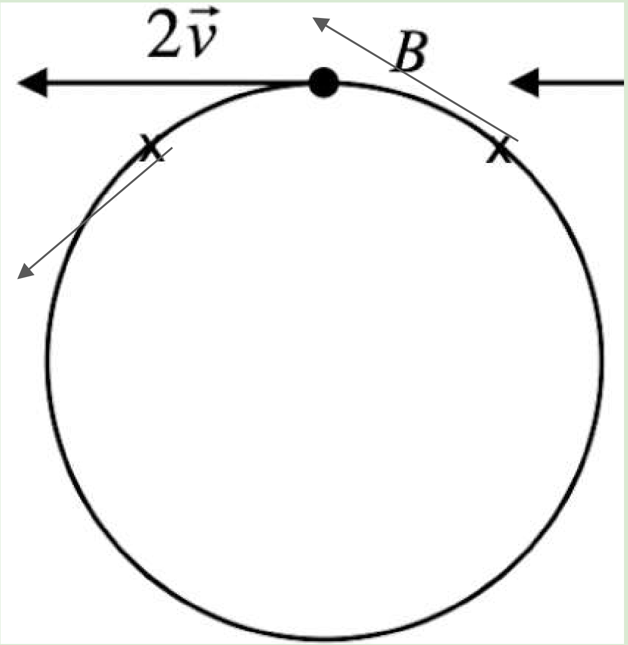


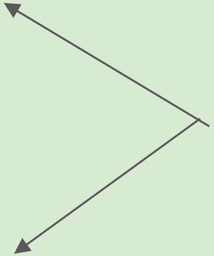


b. Combine the results from Activities 5.3.1 and 5.3.2a to write a general expression for the magnitude of the acceleration during constant-speed circular motion.

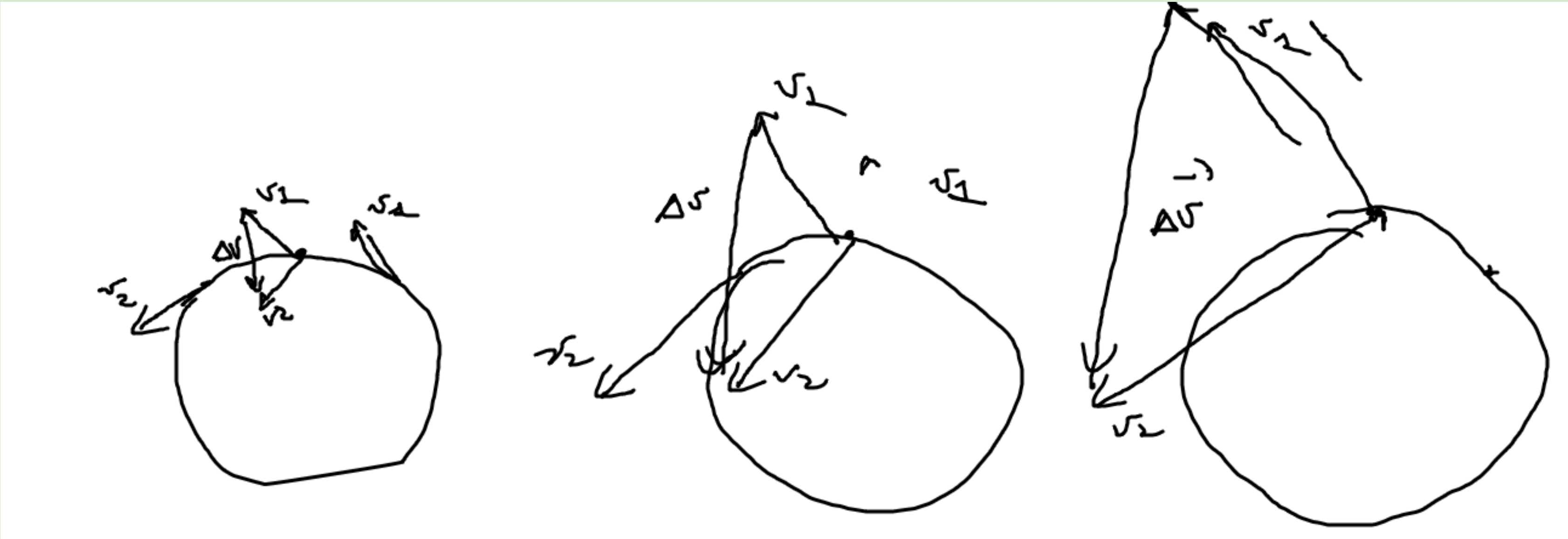
Team 1 OALG 5.3.1 and 5.3.2







Team 2 OALG 5.3.1 and 5.3.2



Team 3 OALG 5.3.1 and 5.3.2

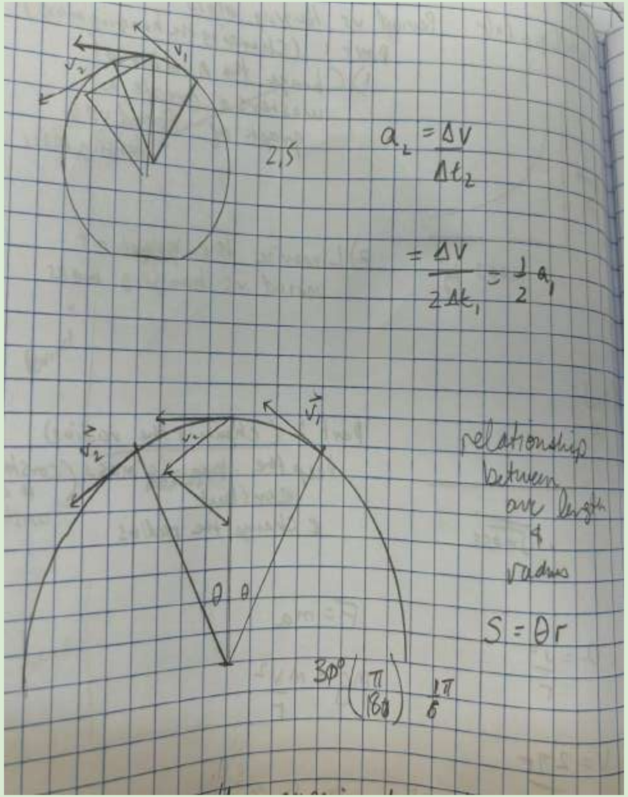
Delta V doubles then triples
Delta t halves then thirds

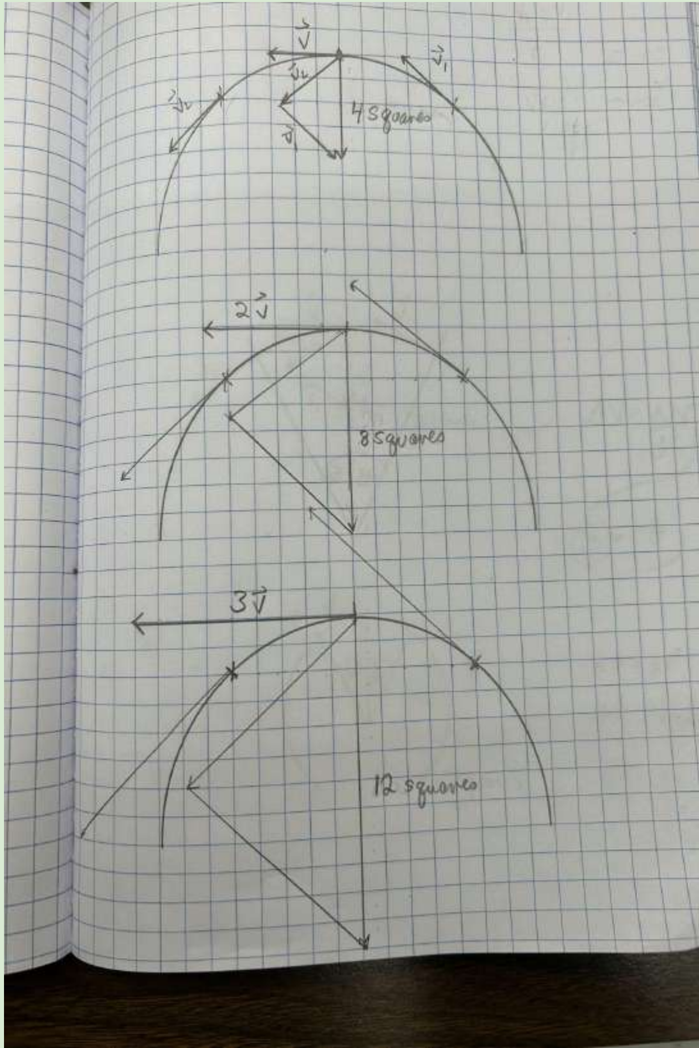
So…. a ~ v^2

Double the radius, half the delta v

So a ~ v^2/r

Team 4 OALG 5.3.1 and 5.3.2





a is proportional to v^2

a is inversely proportional to r

a is prop to v^2 and to 1/r

ALG 5.3.3 Test the relation

***PIVOTAL* Lab:** *Equipment per group:* whiteboard and markers, ruler, string, objects, ring stand, arms and rods, clamp, digital scale, stopwatch, protractor.

Learning goal: Use the equipment to construct a conical pendulum and use that set-up to test Newton's second law for circular motion, namely v^2/r=sumof F/m or to test that in v^2/r the coefficient in front of v^2/r is 1..

a. First brainstorm with your group members. What physical quantities can you measure? What physical quantity could you predict with the equation in order to test it? Describe how you will model the objects, interactions, and processes you will use in your mathematical model. Construct force diagrams as appropriate.

b. Describe your experimental procedure. Include a sketch of your experimental design. Explain what steps you will take to minimize experimental uncertainty.

c. Decide what assumptions about the objects, interactions, and processes you need to make to solve the problem. How might these assumptions affect the result? Be specific.

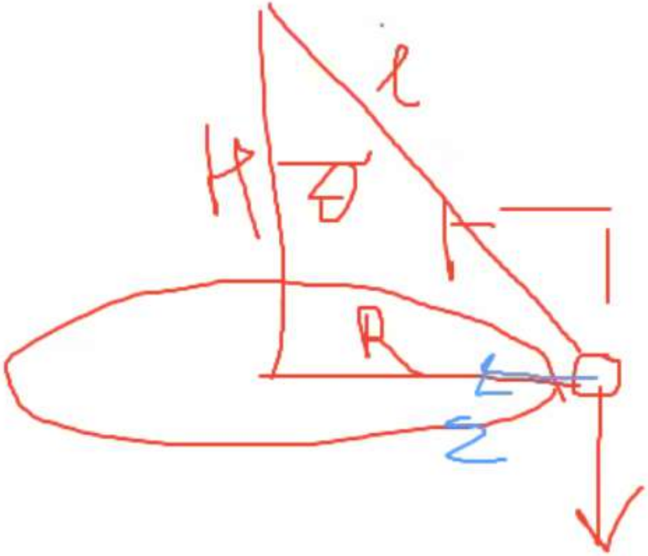
d. What are the sources of experimental uncertainty? Which measurement is the most uncertain? How did you decide?

e. Make a numerical prediction. Be sure to show your mathematical procedure. Show your work to an instructor.

f. Perform the experiment. Record your results in an appropriate format. What is the outcome of the experiment?

g. Make sure to compare your experimentally measured and predicted values. Taking into account experimental uncertainties and the assumptions you made, decide if these two values are consistent or not. If they are not consistent, explain possible reasons for how this could have happened.

Te



$$a = \frac{v^2}{R} \qquad T, R$$

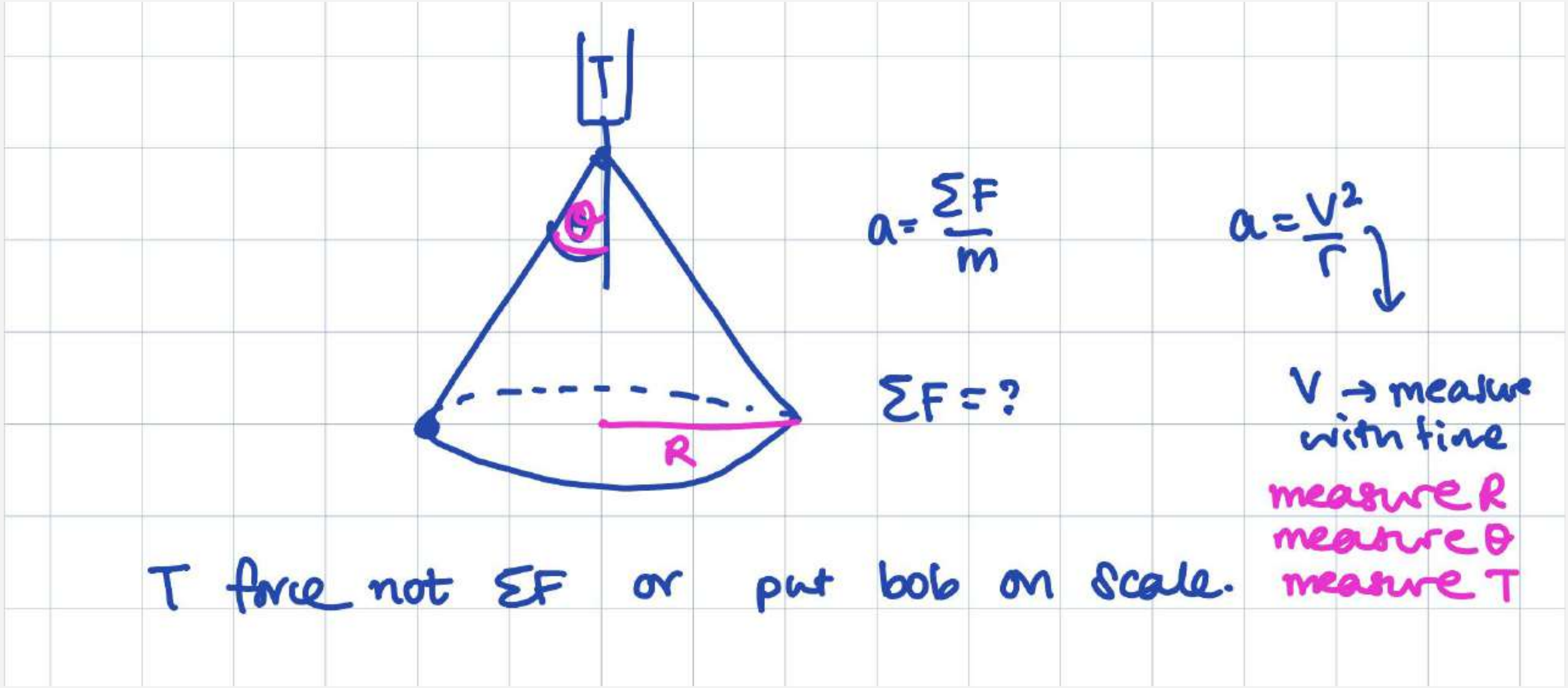
$$a = \frac{\sum F}{m}$$

$$F_{grav} = mg$$

Team 2 Ideas what to do and what to measure

Team 3 Ideas what to do and what to measure

Team 4 Ideas what to do and what to measure



OALG 5.3.3 Test the relation - another option for the testing experiment

Equipment: a ruler, a protractor.

Use the video https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-OALG-5-3-3 to test whether Newton's second law works for circular motion, namely . For this experiment, you will need a ruler and a protractor.

a. Watch two spheres A and B of different masses move in a circle (m_A=29 g and m_B=89 g). What do you notice about their motion? Write all your observations below (do not measure anything). Draw force diagrams to explain the tilt of the strings qualitatively.

b. Use the data in the video for sphere A to determine whether the angle is consistent with the equation under test. What other quantities do you need to determine to make this judgment? Describe how you will model the objects, interactions, and processes that you will use in your mathematical model. Construct force diagrams as appropriate.

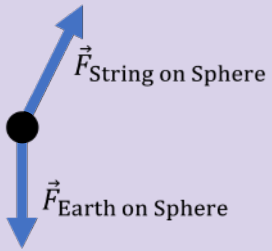
c. Consider the uncertainties in your data. How do they affect your judgment?

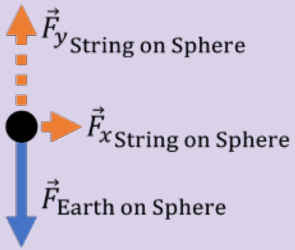
d. How can you explain that the tilt of the string for sphere B is the same as for sphere A although sphere B has about three times larger mass?

Model solution for the OALG lab

a.Both spheres swing outwards so that their strings tilt at an angle to the vertical. Both strings seem to form the same angle with the horizontal, even though the spheres have different masses.

The string is angled so that the the vertical components of both forces cancel out, leaving the sum of the forces to point in the horizontal direction.





This is qualitatively consistent with what we’ve learned since that would leave the sum of forces pointing towards the center of the circle through which the sphere is moving.

Model solution

b. & c.

Hypothesis: Newton's 2nd Law applies to circular motion and the coefficient is 1 ($a = \frac{v^2}{r} = \frac{\Sigma F}{m}$)

Prediction: We can predict the angle the string will make with the vertical if we measure the period and radius of the sphere's swinging:

> **Period** is measured to be 40 frames (the spheres go around from 340 to 380) or 1.33s. The uncertainty in this measurement is 1 frame (1/30 s)
>
> **Radius** of the circle was measured to be 27cm by counting pixels. The uncertainty in this measurement is about 1 cm.

Model solution

b.

Apply Newton's 2nd Law in the x- and y- directions:

$$a_x = \frac{\Sigma F_x}{m} = \frac{v^2}{r}$$

$$\frac{F_{x\,\text{String on Sphere}}}{m} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$\frac{F_{\text{String on Sphere}} \sin\theta}{m} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$\sin\theta = \frac{m\left(\frac{2\pi r}{T}\right)^2}{F_{\text{String on Sphere}} \cdot r}$$

$$a_y = \frac{\Sigma F_y}{m} = 0$$

$$F_{y\,\text{String on Sphere}} - F_{\text{Earth on Sphere}} = 0$$

$$F_{\text{String on Sphere}} \cos\theta = mg$$

$$\cos\theta = \frac{mg}{F_{\text{String on Sphere}}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{2\pi r}{T}\right)^2}{gr}$$

$$\theta = \tan^{-1}\left(\frac{\left(\frac{2\pi(.27\text{m})}{1.33\text{s}}\right)^2}{\left(9.8\frac{\text{N}}{\text{kg}}\right)(.27\text{m})}\right) = 32° \pm 2°$$

Model solution

b. & c.

Outcome & Judgment: The angle of the string, as measured in the video, is 33°. This is within the bounds of our prediction, 32°±2°, so from this experiment we cannot reject our hypothesis that Newton's 2nd Law applies to objects moving in circles.

Model solution

d.

The angle is the same for both spheres for the same reason the freefall acceleration is the same for all objects regardless of mass: All forces exerted on the spheres in this experiment depend linearly on their mass, so the mass cancels out when using Newton’s 2nd Law.

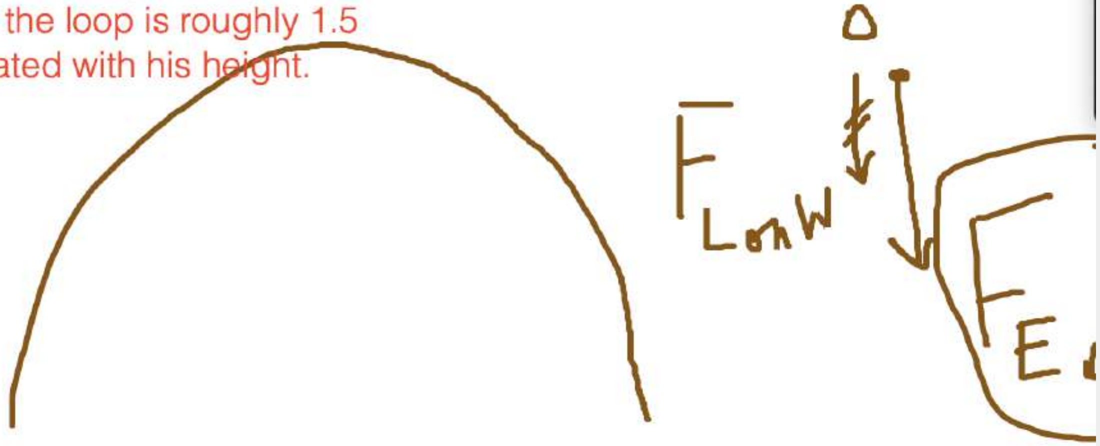
Let’s go back to the “need to know”

Let’s go back to Damien Walters! In your teams…

1. Useful data: Damien Walters has a **height of 5’11” (1.80 m)** and the **radius of the loop is roughly 1.5 m**, estimated with his height.
2. Draw a force diagram for Damien running **upside down at the top of the loop**.
3. Represent your force diagram mathematically using Newton’s 2nd Law and what you now know about objects moving in uniform circular motion.
4. Can Damien run the loop at any speed? Or there is some minimum speed that he needs to stay in contact with the loop? Why?
5. Predict the minimum speed Damien will need to run at the top to make it through the loop without falling.
6. State any assumptions you make.

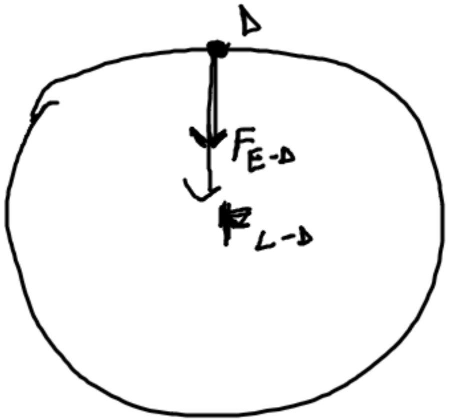
Team 1

Useful data: Damien Walters has a height of 5’11” (1.80 m) and the radius of the loop is roughly 1.5 m, estimated with his height.



$$v = \sqrt{gr} \qquad \frac{\cancel{m}v^2}{r} = \cancel{m}g$$

Team 2



"NORMAL FORCE"

$$F_{E-D} + F_{L-D} = m\frac{v^2}{r}$$

$$mg + F_{L-D} = m\frac{v^2}{r}$$

The minimum velocity when $F_{L-D} = 0$

$$v_{MIN}^2 = gr$$

Team 3

Free body diagram —>

Minimum force of track on Damien is zero

F_{eD} = mg = mv^2/r

So v = sqrt(gr)=3.83 m/s

Team 4

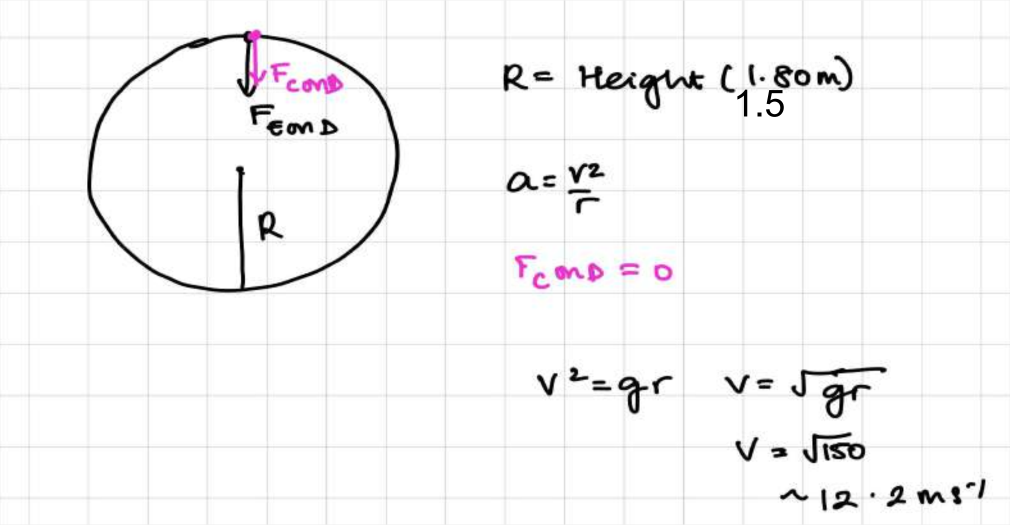
F_{EonD}

$F_{EonD} = mg$

$mg = mv^2/r$

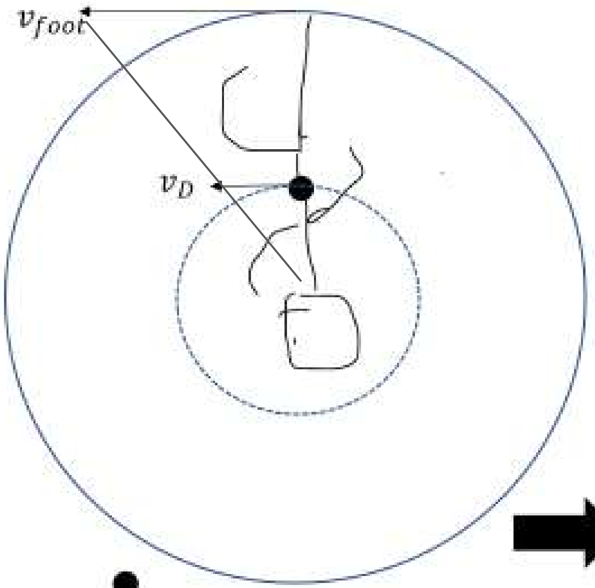
$gr = v^2$

Use the radius of the circle, but account for Damien's center of mass



Will it be possible? - Start at 01:24

Solution: Damien (D) Walters



$$a_D = \frac{\sum F_{on\ D}}{m_D} = \frac{F_{E\ on\ D}+F_{T\ on\ D}}{m_D} \geq \frac{F_{E\ on\ D}}{m_D}$$

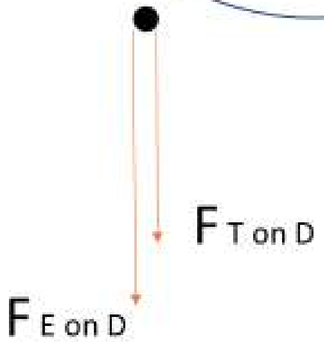
Also known,
for a uniform circular motion:

$$a_D = \frac{v_D^2}{r_c}$$

Where r_c is radius of the circle that Damien's center of mass is going around.

$$\frac{v_D^2}{r_c} = \frac{F_{E\ on\ D}+F_{T\ on\ D}}{m_D} \geq \frac{F_{E\ on\ D}}{m_D} = g$$

r_c is ~0.6 m, estimated by the given height of Damien.



Downward as positive direction

The minimum speed of Damien's center of mass $v_D^{mini} = \sqrt{g \times r_c} \approx 2.4$ m/s

Team 1 OALG 5.5.1 OALG Chapter 5 Final.docx

$V = 2Pi\ R/\ T$

$A = v\ ^2 / R$

$A = 4\ pi^2\ R$

Team 2 OALG 5.5.1 OALG Chapter 5 Final.docx

$$T = 2{,}358 \cdot 10^{6}\ s$$

$$v = \frac{2\pi r}{T} \simeq 1011\ m/s$$

$$a = \frac{v^2}{r} = \frac{(1011)^2}{r} = 0{,}0026\ \frac{m}{s^2}$$

Team 3 OALG 5.5.1 OALG Chapter 5 Final.docx

Team 4 OALG 5.5.1 OALG Chapter 5 Final.docx

OALG 5.5.1 [OALG Chapter 5 Final.docx]

$A = v^2/r$

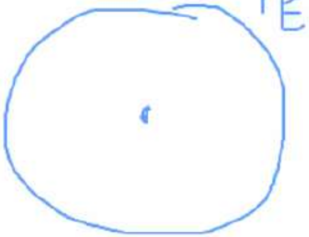
$A = (2* \;\; *r/T)^2/r = 4* \;\; {}^2*r/T^2$

$= 4*3.14^2*3.8x10^8/(27.3*24*60*60)^2$

$=0.002696\ m/s^2=2.7\ x\ 10^{-3} m/s^2$

$$\frac{g}{a_m} = \frac{9.8\,m/s^2}{2.7 \cdot 10^{-3}\,m} = 3600$$

$$\frac{r_m}{r_E}\ \frac{3.8 \cdot 10^8}{6.4 \cdot 10^6} = 60$$



$$F_{E\ on\ M} \sim m_m \qquad F \sim \frac{m_1 m_2}{r^2}$$

$$F_{E\ on\ M} = F_{M\ on\ E} \sim m_E$$

$$F_{E\ on\ M} \sim \frac{1}{r^2} \sim m_M \sim m_E$$

All objects fall with same acceleration $F \propto m$

Force that the Earth exerts on the moon $\propto M_m$

Newton : conceptual leap

$F_{EonM} = F_{MonE}$ (N3L) $\therefore$ if they are equal $F \sim M_E$

$$\left.\begin{aligned} F_{EonM} &\sim \frac{1}{r^2} \\ &\sim M_m \\ &\sim M_E \end{aligned}\right\} \quad F_{EonM} \propto \frac{M_m M_E}{r^2}$$

Newton: to generalise to all objects, he looked at planetary data

$$\frac{M_m v^2}{r} = k \cdot \frac{M_m M_E}{r^2} \qquad T = \frac{2\pi r}{v}$$

$$\frac{\cancel{M_m} \cdot 4\pi r^{\cancel{2}}}{T^2 \cdot \cancel{r}} = k \cdot \frac{\cancel{M_m} M_E}{r^2} \qquad v^2 = \frac{(2\pi r)^2}{T^2}$$

Missing pi square

$$\frac{4\pi r}{T^2} = k \cdot \frac{M_E}{r^2}$$

$$4\pi r^3 = k \cdot M_E \cdot T^2$$

Solution for 5.5.1

second law to determine the gravitational force. He knew that the Moon was separated from Earth by about $r_M = 3.8 \times 10^8$ m, roughly 60 Earth radii ($60R_E$). With this distance and the Moon's orbital period of 27.3 days, Newton was able to use Eq. (5.5) to determine the Moon's radial acceleration:

$$a_{r\text{ at }R=60R_E} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(3.8 \times 10^8\text{ m})}{[(27.3\text{ days})(86{,}400\text{ s/day})]^2}$$
$$= \frac{4\pi^2(3.8 \times 10^8\text{ m})}{(2.36 \times 10^6\text{ s})^2} = 2.69 \times 10^{-3}\text{ m/s}^2$$

He then did a similar analysis for a second situation. He performed a thought experiment in which he imagined what would happen if the Moon were condensed to a small point-like object (while keeping its mass the same) located near Earth's surface. This would place the "Moon particle" at a distance of one Earth radius ($1\ R_E$) from Earth's center. If the Moon only interacted with Earth, it would have the same free-fall acceleration as any object near Earth's surface (9.8 m/s^2), since this acceleration is independent of mass. Now Newton knew the Moon's acceleration at two different distances from Earth's center.

Newton then used this information to determine how the gravitational force that one object exerts on another depends on the separation of the objects, assuming that the force causing the acceleration of the Moon is the gravitational force exerted on it by Earth and that this force changes with the separation of the objects. He took the ratio of these two accelerations using his second law:

$$\frac{a_{r\text{ at }R=60R_E}}{a_{r\text{ at }R=1R_E}} = \frac{F_{\text{E on Moon at }R=60R_E}/m_{\text{Moon}}}{F_{\text{E on Moon at }R=1R_E}/m_{\text{Moon}}} = \frac{F_{\text{E on Moon at }R=60R_E}}{F_{\text{E on Moon at }R=1R_E}}$$

Notice that the Moon's mass cancels when taking this ratio. Substituting the two accelerations he had observed, Newton made the following comparison between the forces:

$$\frac{F_{\text{E on Moon at }R=60R_E}}{F_{\text{E on Moon at }R=1R_E}} = \frac{a_{r\text{ at }R=60R_E}}{a_{r\text{ at }R=1R_E}} = \frac{2.69 \times 10^{-3}\text{ m/s}^2}{9.8\text{ m/s}^2} = \frac{1}{3600} = \frac{1}{60^2}$$

$$\Rightarrow F_{\text{E on Moon at }R=60R_E} = \frac{1}{60^2} F_{\text{E on Moon at }R=1R_E}$$

The force exerted on the Moon when the Moon is at a distance of 60 times Earth's radius is $1/60^2$ times the force exerted on the Moon when near Earth's surface (about 1 times Earth's radius). It appears that as the distance from Earth's center to the Moon *increases* by a factor of 60, the force decreases by a factor of $1/60^2 = 1/3600$. This suggests that the gravitational force that Earth exerts on the Moon depends on the inverse square of its distance from the center of Earth:

$$F_{\text{E on Moon at }r} \propto \frac{1}{r^2} \tag{5.10}$$

All together 5.5.2 [OALG Chapter 5 Final.docx]

Team 2 5.5.5 [OALG Chapter 5 Final.docx]

7:30-7:40

Team 3 5.5.5 [OALG Chapter 5 Final.docx]

Team 4 5.5.5 OALG Chapter 5 Final.docx

5.5.5 [OALG Chapter 5 Final.docx]

$F = GmM/r^2$, and sum of $F = ma = mv^2/r$

So $GmM/r^2 = mv^2/r$

$GmM/r^2 = m(2* \quad *r/T)^2/r = 4 \quad {}^2r/T^2$

$T^2/r^3 = 4* \quad {}^2/(GM) = \text{constant}$

What will I take with me from today’s workshop?

I learned how Newton discovered the law of gravity.

I learned how the equations can be derived and not just accepted

We can go from circular motion to kepler’s laws

Careful construction can be used easily to derive a~v2/r

To test the proportionality between a and v^2 and r. We did not need angular angular velocity.

Story of Newton

I have to read again the book!, also use a Feond acronyms for Forces, teach vectors drawing,I learned a new experiment for circular motion, a great way to introduce the vector add to show change in velocity, transition to go to Universal gravitation and planets, a good story from Newton, Einstein, application to go to from the use and application of Newton and the use of Einstein relativity

The conical pendulum experiment connects F=ma and F=mv^2/r

The power of stories - how physics is done in a real world context