ISLE progression: From conceptual to quantitative

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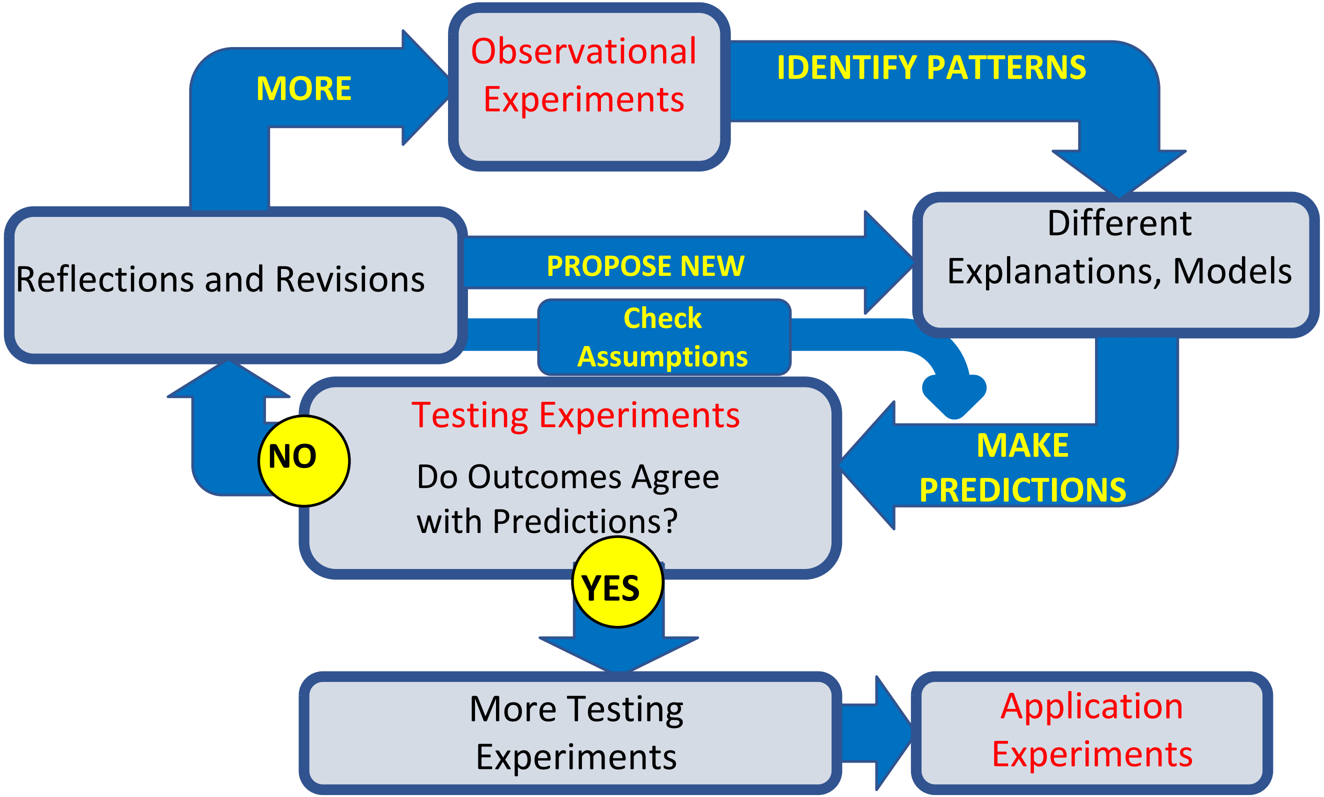
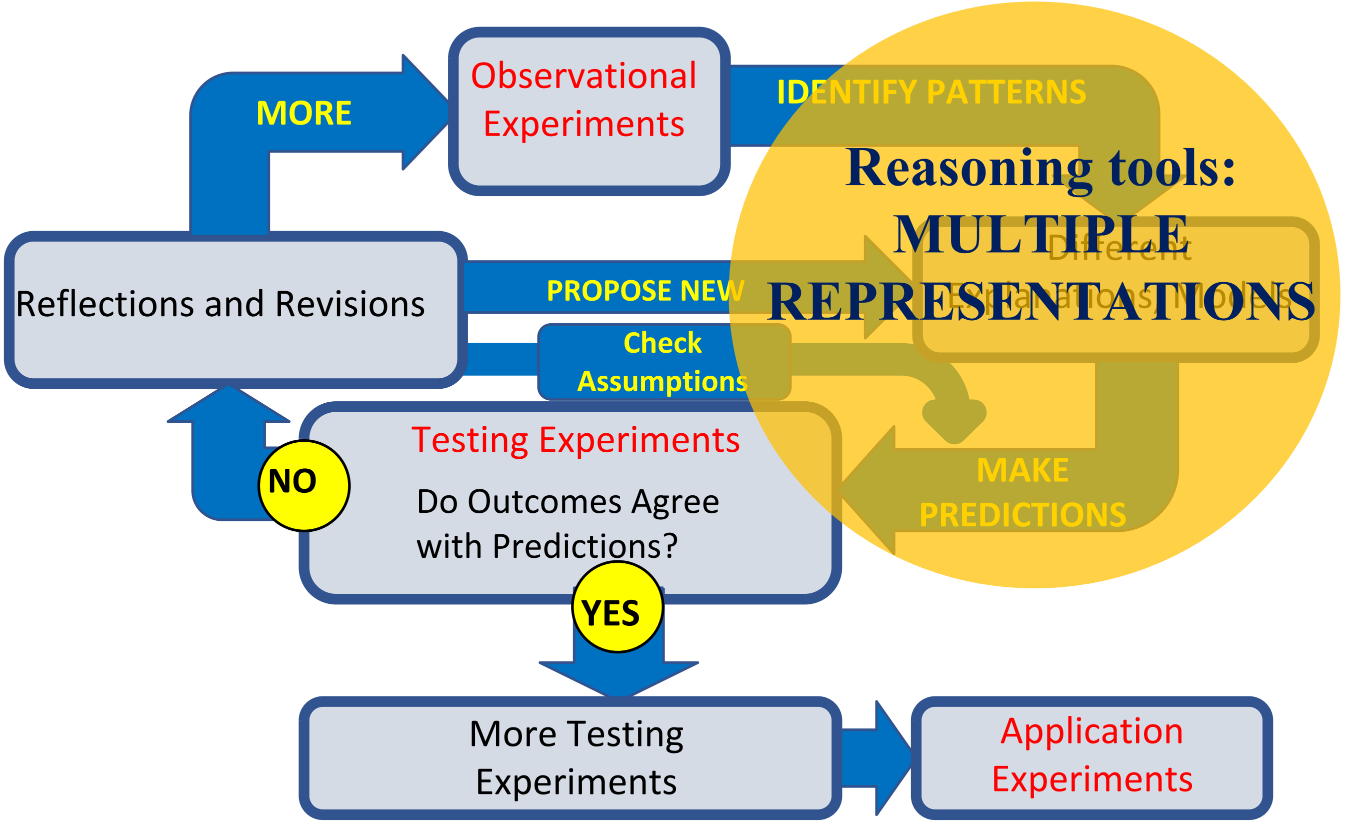
Investigative Science Learning Environment (ISLE) process



Etkina and Van Heuvelen, 2001, 2007; Etkina, 2015

The Investigative Science Learning Environment (ISLE) approach



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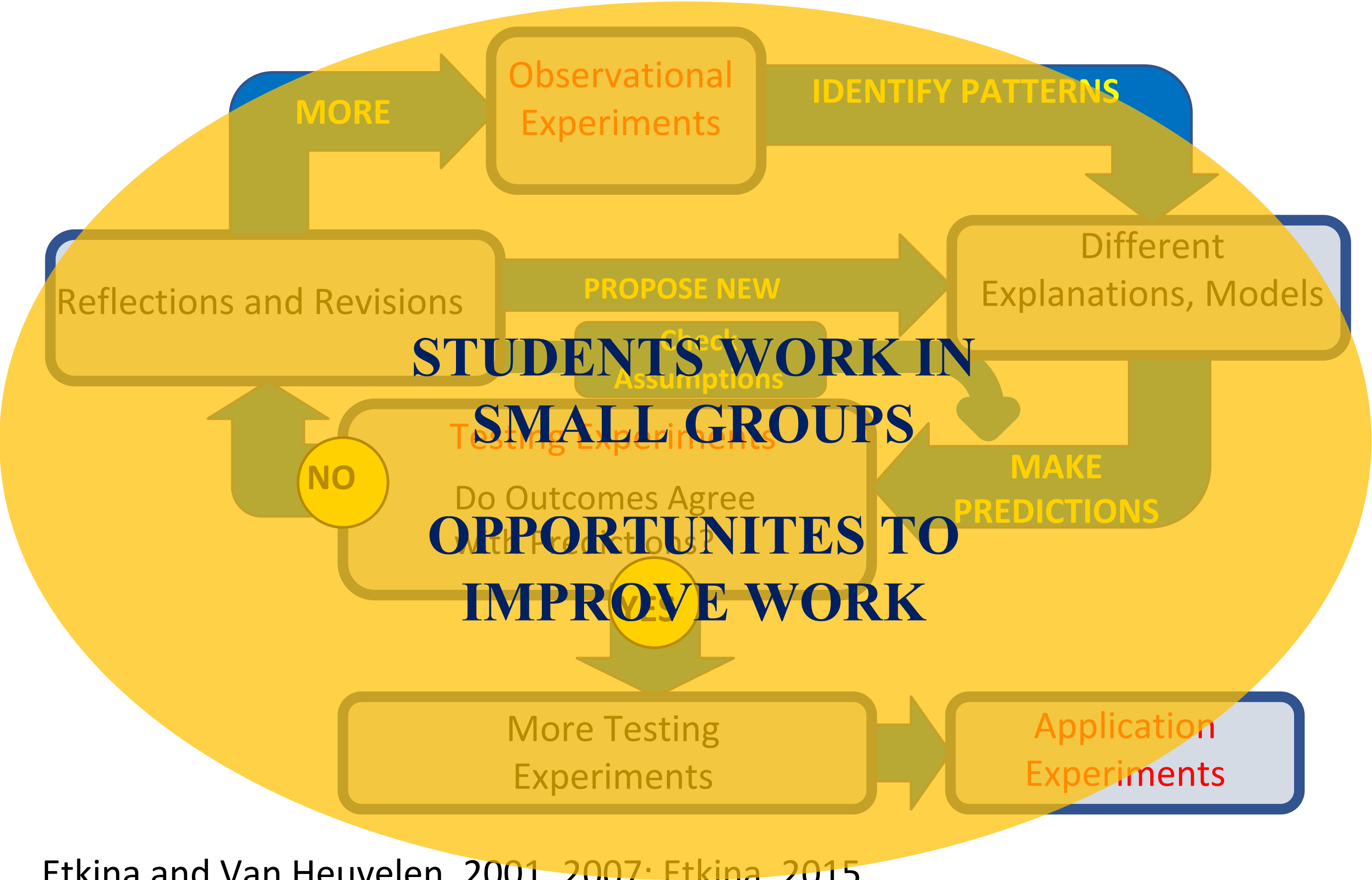
The Investigative Science Learning Environment (ISLE) approach



Etkina and Van Heuvelen, 2001, 2007; Etkina, 2015

Examine slides 6- 10 below. What do you notice about the development of a concept?

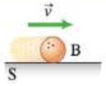
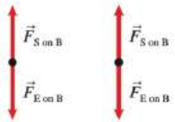
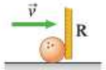
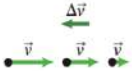
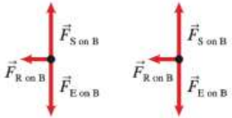
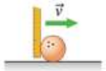
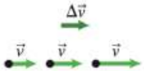
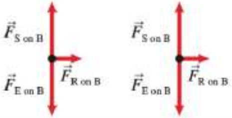
In your team outline the progression from qualitative to quantitative and make a list of reasons why qualitative should precede quantitative.

Note: I skipped testing experiments.

Video at: https://mediaplayer.pearsoncmg.com/assets/_frames.true/secs-experiment-video-2

OBSERVATIONAL EXPERIMENT TABLE 3.1 How are motion and forces related?

VIDEO OET 3.1

Observational experiment		Analysis: Motion diagram	Analysis: Force diagrams for first and third positions
Experiment 1. A bowling ball B rolls on a very hard, smooth surface S without slowing down.			
Experiment 2. A ruler R lightly pushes the rolling bowling ball opposite the ball's direction of motion. The ball continues to move in the same direction, but slows down.			
Experiment 3. A ruler R lightly pushes the rolling bowling ball in the direction of its motion. The ball speeds up.			

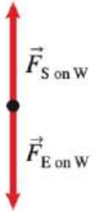
Pattern

- In all the experiments, the vertical forces add to zero and cancel each other. We consider only forces exerted on the ball in the horizontal direction.
- In the first experiment, the sum of the forces exerted on the ball is zero; the ball's velocity remains constant.
- In the second and third experiments, when the ruler pushes the ball, the velocity change arrow ($\Delta\vec{v}$ arrow) points in the same direction as the sum of the forces.

Summary: The $\Delta\vec{v}$ arrow in all experiments is in the same direction as the sum of the forces. Notice that there is no pattern relating the *direction* of the velocity $\vec{v}$ to the direction of the sum of the forces. In Experiment 2, the velocity and the sum of the forces are in opposite directions, but in Experiment 3, they are in the same direction.

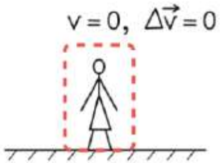
CONCEPTUAL EXERCISE 3.2 **Diagram Jeopardy**

The force diagram shown here describes the forces that external objects (the surface and Earth) exert on a woman (in this scenario, the force diagram does not change with time). Describe three different types of motion of the woman that are consistent with the force diagram.

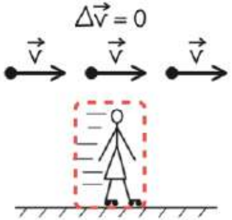


Sketch and translate Two equal-magnitude, oppositely directed forces are being exerted on the woman ($\Sigma\vec{F} = 0$). Thus, a motion diagram for the woman must have a zero velocity change ($\Delta\vec{v} = 0$).

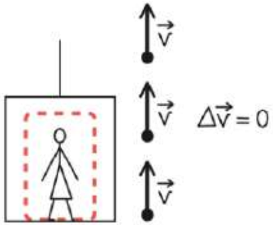
Simplify and diagram Three possible motions consistent with this idea are shown below.



1. She stands at rest on a horizontal surface.



2. She glides at constant velocity on Rollerblades on a smooth horizontal surface.



3. She stands on the floor of an elevator that moves up or down at constant velocity.

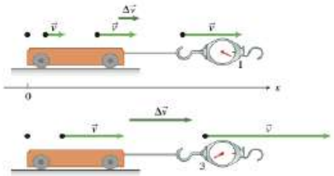
Note that in all three of the above, the velocity change arrow is zero. This is consistent with the sum of the forces being zero.

OBSERVATIONAL EXPERIMENT TABLE 3.4 Forces and resulting acceleration

VIDEO OET 3.4

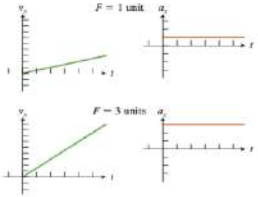
Observational experiment

Experiment 1. A cart starts at rest on a smooth horizontal track. A spring scale continuously exerts one unit of force in the positive direction. The experiment is repeated four more times. Each time, the force probe exerts one additional unit of force on the cart (up to three units). We record the value of the force, and use a motion detector on the track to record the cart's speed and acceleration.



Analysis

Using this information, we create velocity-versus-time and acceleration-versus-time graphs for two of the five different magnitudes of force. Note that the greater the force, the greater the acceleration.



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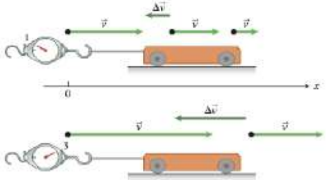
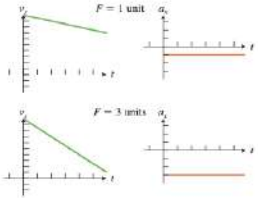
Observational experiment

Experiment 2. We repeat the same five experiments, only this time the cart is moving in the positive direction, and the probe pulls back on the cart in the negative direction so that the cart slows down.



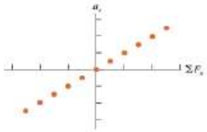
Analysis

We create velocity-versus-time and acceleration-versus-time graphs for the cart when forces of two different magnitudes oppose the cart's motion.



Pattern

- When the sum of the forces exerted on the cart is constant, its acceleration is constant—the cart's speed increases at a constant rate.
- When we plot acceleration versus force using the five positive and five negative values of the force, we obtain the graph at the right. The eleventh point is (0,0), which we know from previous experiments.
- The acceleration is directly proportional to the force exerted by the spring scale (in this case, it is the sum of all forces) and points in the direction of the force.

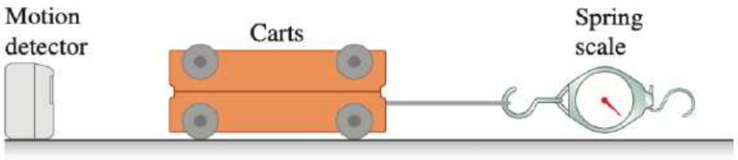


OBSERVATIONAL EXPERIMENT TABLE 3.5 Amount of matter and acceleration

Observational experiment

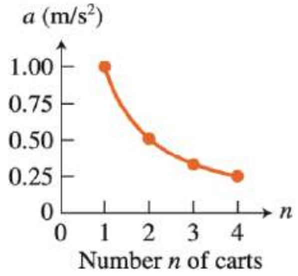
We pull the indicated number of stacked carts using an identical pulling force and measure the acceleration with a motion detector.

Number n of carts	Acceleration (m/s^2)
1	1.00
2	0.49
3	0.34
4	0.25

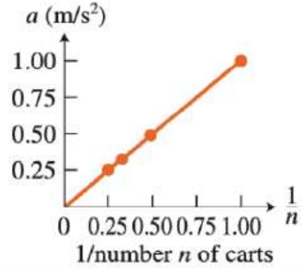


Analysis

We graph the acceleration versus number of carts for constant pulling force.



From the graph, we see that increasing the number of carts decreases the acceleration. To check whether this relationship is inversely proportional, we plot a versus $\frac{1}{n}$.



Pattern

Since the graph a versus $\frac{1}{n}$ is a straight line, we conclude that a is inversely proportional to n, which we write as $a \propto \frac{1}{n}$.

TIP Notice that the "vector sum of the forces" mentioned in the definition at right does not mean the sum of their magnitudes. Vectors are not added as numbers, their directions affect the magnitude of the vector sum.

Newton's second law The acceleration $\vec{a}_S$ of a system is proportional to the vector sum of all forces being exerted on the system and inversely proportional to the mass m of the system:

$$\vec{a}_S = \frac{\Sigma \vec{F}_{\text{on S}}}{m_S} = \frac{\vec{F}_{1 \text{ on S}} + \vec{F}_{2 \text{ on S}} + \cdots}{m_S} \tag{3.6}$$

Here the subscripts 1 and 2 stand for the objects exerting forces on the system.

The acceleration of the system points in the same direction as the vector sum of the forces.

Does this new equation make sense? For example, does it work in extreme cases? First, imagine an object with an infinitely large mass. According to the law, it will have zero acceleration for any process in which the sum of the forces exerted on it is finite:

$$\vec{a}_S = \frac{\Sigma \vec{F}_{\text{on S}}}{\infty} = 0$$

This seems reasonable, as an infinitely massive object would not change motion due to finite forces exerted on it. On the other hand, an object with a zero mass will have an infinitely large acceleration when a finite magnitude force is exerted on it:

$$\vec{a}_S = \frac{\Sigma \vec{F}_{\text{on S}}}{0} = \infty$$

TIP In the cause-effect relationship, the quantities on the right side of the equation can be varied independently. In our case, these are the sum of the forces and the mass. Each can be changed independently of the other to affect acceleration. In the operational definition, the quantities on the right side of the equation cannot be varied independently. The velocity change depends on the time interval during which it occurs.

Both extreme cases make sense. Newton's second law is a so-called *cause-effect relationship*. The right side of the equation (the sum of the forces being exerted divided by the mass of the system) is the cause of the effect (the acceleration) on the left side.

$$\text{Effect} \rightarrow \vec{a}_S = \frac{\Sigma \vec{F}_{\text{on S}}}{m_S} \leftarrow \text{Cause}$$

On the other hand, $\vec{a} = \Delta\vec{v}/\Delta t$ is called an *operational definition* of acceleration. It tells us how to determine the quantity acceleration but does not tell us *why* it has a particular value. For example, suppose that an elevator's speed changes from 2 m/s to 5 m/s in 3 s as it moves vertically along a straight line in the positive y-direction. The elevator's acceleration (using the definition of acceleration) is

$$a_y = \frac{5\text{ m/s} - 2\text{ m/s}}{3\text{ s}} = +1\text{ m/s}^2$$

This operational definition does not tell you the reason for the acceleration. If you know that the mass of the elevator is 500 kg and that Earth exerts a 5000-N downward

force on the accelerating elevator and the cable exerts a 5500-N upward force on it, then using the cause-effect relationship of Newton's second law:

$$\frac{5500\text{ N} + (-5000\text{ N})}{500\text{ kg}} = +1\text{ m/s}^2$$

Thus, you obtain the same number using two different methods—one from kinematics (the part of physics that *describes* motion) and the other from dynamics (the part of physics that *explains* motion).

Team 1 Using slides 6-10 Outline the progression from qualitative to quantitative and make a list of reasons why qualitative should precede quantitative

1. Link between motion diagram and force diagram: the direction of the velocity change vector is in the same direction as the net force.
2. Given a force diagram, come up with possible motions for that force diagram.
3. Type of relationship between strength and direction of force and the resulting acceleration of a cart (proportional).
4. Math relationship between mass of cart and resulting acceleration (inversely proportional).
5. Combines #3 and #4 into the equation for Newton's Second Law ($a = F/m$). Contrasts the difference between the operational definition of acceleration and the acceleration in Newton's Second Law equation.

Reasoning: Intuition, observations, and representations are very important -- get students reasoning first without math, then build in the math later. Allows for multiple ways to observe/describe the same phenomena.

Team 2 Using slides 6-10 Outline the progression from qualitative to quantitative and make a list of reasons why qualitative should precede quantitative

Slide 6: Students need to describe 3 experiments with words and then translate them into motion diagrams, and force diagrams. Students will find qualitative patterns as to what aspect of the motion and force diagrams are similar to one another (this ends up being the direction of the net force in comparison to the direction of the change in velocity vector). Only qualitative.

Slide 7: Shows how a force diagram can represent completely different motion “stories”. This helps students to understand that force diagrams alone do not tell us the full story of an object’s motion. Helps students to build on the knowledge they learned from slide 6 (a net force causes a change in velocity, so since there is no net force here, there should not be any change in velocity). Only qualitative.

Slide 8: Translate between motion diagrams and a graphical representation. Students will see how the magnitude of the slope of the graphs relate to the length of the change in velocity vector in the motion diagram. Starting to bring in quantitative descriptions of the motion.

Slide 9: Finding a quantitative (and graphical) relationship between 2 physical quantities (mass and acceleration), and graphing the data.

Slide 10: Translating the graphical relationships found between the different physical quantities and converting them into a mathematical relationship.

Team 3 Using slides 6-10 Outline the progression from qualitative to quantitative and make a list of reasons why qualitative should precede quantitative

6. Diagrams force students to link kinematics and dynamics concepts.

7. This exercise helps construct multiple scenarios for the same diagram, emphasizing that force is proportional to acceleration and not velocity.

8. Moving beyond the idea that the sum of forces points in the direction of change in velocity towards a precise mathematical relationship. Graphs form a bridge from graphical to symbolic representations.

9. Deriving proportional relationships by graphing acceleration versus number of carts (mass) and linearizing the data.

10. Looking at limiting cases in the algebraic expression.

Representations are constructed based on students’ prior knowledge

Team 4 Using slides 6-10 Outline the progression from qualitative to quantitative and make a list of reasons why qualitative should precede quantitative

- Students can clearly see from where the equations come from
- Students won’t automatically use equations (just plug and play), they’ll see meaning in them
- Physical experience of the phenomena (for example pushing the ball) makes a deeper impact on the learning
- Multiple representations before introducing math
- Cause-effect equations naturally comes from experience
- Extreme cases add to our understanding
- The operational definition of acceleration may make more sense from ex 3.1 where the motion diagrams will help the learners’ understanding.

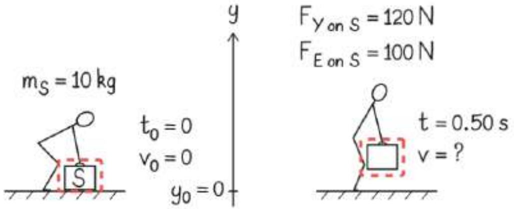
Problem solving transition

Examine the worked example below. How are the same ideas implemented in the problem solving procedure?

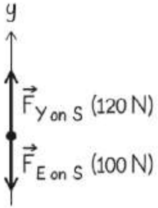
EXAMPLE 3.3 Lifting a suitcase

Earth exerts an approximately 100-N force on a 10-kg suitcase. Suppose you exert an upward 120-N force on the suitcase. If the suitcase starts at rest, how fast is it traveling after lifting for 0.50 s?

Sketch and translate First, we sketch the initial and final states of the process, choosing the suitcase as the system. The sketch helps us visualize the process and also brings together all the known information, letting our brains focus on other aspects of solving the problem. One common aspect of problems like this is the use of a two-step strategy. Here, we use Newton's second law to determine the acceleration of the suitcase and then use kinematics to determine the suitcase's speed after lifting for 0.50 s.



Simplify and diagram Next, we construct a force diagram for the suitcase while being lifted. The y-components of the forces exerted on the suitcase are your upward pull on the suitcase $F_{\text{Y on S}\,y} = +F_{\text{Y on S}} = +120\text{ N}$ and Earth's downward pull on the suitcase $F_{\text{E on S}\,y} = -F_{\text{E on S}} = -100\text{ N}$. Because the upward force is larger, the suitcase will have an upward acceleration $\vec{a}$.



Represent mathematically Since all the forces are along the y-axis, we apply the y-component form of Newton's second law to determine the suitcase's acceleration (notice how the subscripts in the equation below change from step to step):

$$a_{\text{S}y} = \frac{\Sigma F_{\text{on S}\,y}}{m_\text{S}} = \frac{F_{\text{Y on S}\,y} + F_{\text{E on S}\,y}}{m_\text{S}}$$

$$= \frac{(+F_{\text{Y on S}}) + (-F_{\text{E on S}})}{m_\text{S}} = \frac{F_{\text{Y on S}} - F_{\text{E on S}}}{m_\text{S}}$$

After using Newton's second law to determine the acceleration of the suitcase, we then use kinematics to determine the suitcase's speed after traveling upward for 0.50 s:

$$v_y = v_{0y} + a_y t$$

The initial velocity is $v_{0y} = 0$.

Solve and evaluate Now substitute the known information in the Newton's second law y-component equation above to find the acceleration of the suitcase:

$$a_{\text{S}y} = \frac{F_{\text{Y on S}} - F_{\text{E on S}}}{m_\text{S}} = \frac{120\text{ N} - 100\text{ N}}{10\text{ kg}} = +2.0\text{ m/s}^2$$

Insert this and other known information into the kinematics equation to find the vertical velocity of the suitcase after lifting for 0.50 s:

$$v_y = v_{0y} + a_y t = 0 + (+2.0\text{ m/s}^2)(0.50\text{ s}) = +1.0\text{ m/s}$$

The unit for velocity is correct and the magnitude is reasonable.

Try it yourself How far up did you pull the suitcase during this 0.50 s?

Answer

The average speed while lifting it was $(0 + 1.0\text{ m/s})/2 = 0.50\text{ m/s}$. Thus you lifted the suitcase $y - y_0 = (0.50\text{ m/s})(0.50\text{ s}) = 0.25\text{ m}$.

Team 1 Examine the worked example on slide 16. How are the same ideas implemented in the problem solving procedure?

Team 2 Examine the worked example on slide 16. How are the same ideas implemented in the problem solving procedure?

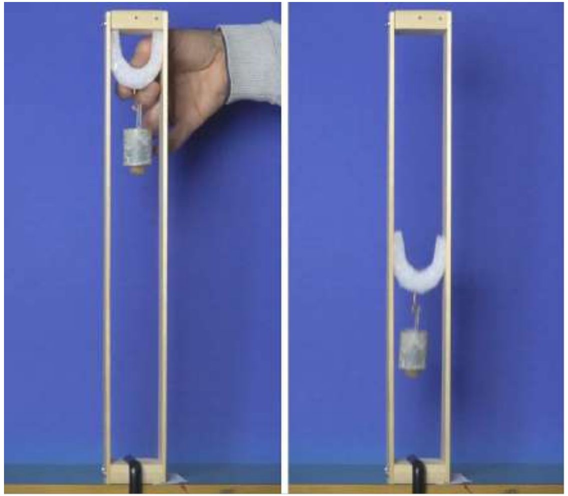
Team 3 Examine the worked example on slide 16. How are the same ideas implemented in the problem solving procedure?

Team 4 Examine the worked example on slide 16. How are the same ideas implemented in the problem solving procedure?

Read the problem below and work with your team to provide a solution. How does this problem implement conceptual to quantitative transition?

You have a V-shaped piece of foam with a metal cylinder attached to it.

You squeeze the foam and insert it between two parallel vertical wooden planks as shown in figure 2b. When you release the foam, the foam-cylinder object moves down with approximately constant acceleration of (see the video https://youtu.be/gVghm99kQXA).



Let the initial state be when the object is at rest at the top of the wooden planks (figure 2a) and the final state before the object hits the table.

a. Draw a force diagram for the object when it is somewhere between the initial and the final state.

b. Use the force diagram to write Newton’s second law in component form.

c. Find the expression the speed of the object in the final state. Evaluate the expression that you derived (check units and analyze limiting cases).

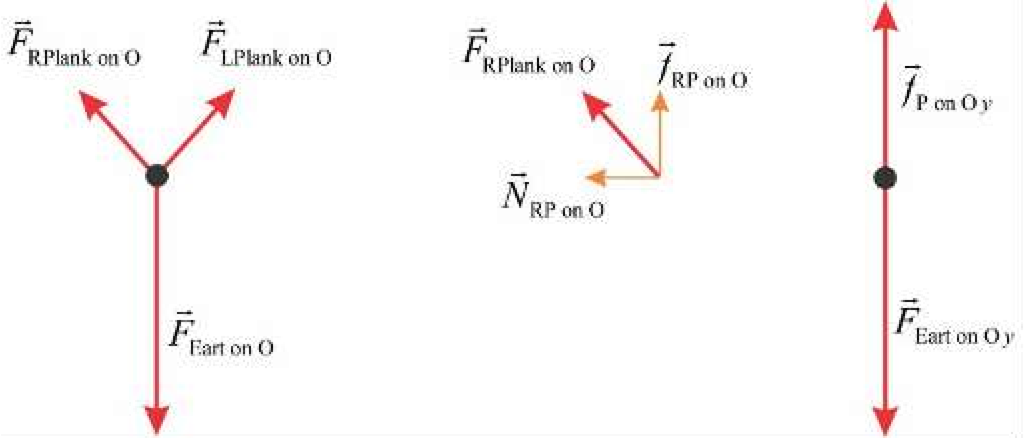
Team 1

Read the problem on slide 21 and work with your team to provide a solution. How does this problem implement conceptual to quantitative transition?

Team 2 Read the problem on slide 21 and work with your team to provide a solution. How does this problem implement conceptual to quantitative transition?

Team 3 Read the problem on slide 21 and work with your team to provide a solution. How does this problem implement conceptual to quantitative transition?

Team 4 Read the problem on slide 21 and work with your team to provide a solution. How does this problem implement conceptual to quantitative transition?



$\vec{F}_{\text{RPlank on O}}$
$\vec{F}_{\text{LPlank on O}}$
$\vec{F}_{\text{Eart on O}}$
$\vec{F}_{\text{RPlank on O}}$
$\vec{f}_{\text{RP on O}}$
$\vec{N}_{\text{RP on O}}$
$\vec{f}_{\text{P on O}\,y}$
$\vec{F}_{\text{Eart on O}\,y}$

$$a_y = \frac{\Sigma F_{\text{on O } y}}{m} = \frac{mg - f_{\text{P on O}}}{m}$$

https://docs.google.com/presentation/d/1Eu2GVstC6fW4ZCCGX88UDvFtT19APpzUJX57mgzTO9Y/edit#slide=id.g12b36bb8491_0_6

Speed of the object after moving distance *h* with a_y

$$v_{\text{f}y}{}^2 = v_{\text{i}y}{}^2 + 2a_y h$$

$$v_{\text{f}y} = \sqrt{2a_y h} = \sqrt{\frac{2h(mg - f_{\text{P on O}})}{m}} = \sqrt{2h\left(g - \frac{f_{\text{P on O}}}{m}\right)}$$

Evaluation:

Larger *h* => larger *v*, OK

Larger friction force (larger coeff. of friction, stiffer foam) => smaller *v*, OK

Larger *g* => larger *v*, OK

Larger *m* => larger *v*, OK

For certain *m* (small enough), the speed is zero, OK

Summary

What did you learn about the progression from conceptual to quantitative and how will this meeting affect your teaching?