Circular motion and Gravitation

Eugenia Etkina

Summary of my posts about circular motion

Need to know

How to get the students to invent and test the idea that the sum of the forces exerted on an object moving in a circle at constant speed points towards the center of the circle.

How to explain WHY the sum of the forces should point towards the center.

Need to know

https://arstechnica.com/.../nascar-driver-stuns.../...

https://www.youtube.com/watch?v=dSDb9oKMCRc&t=19s

How to get the students to invent and test the idea that the sum of the forces exerted on an object moving in a circle at constant speed points towards the center of the circle.

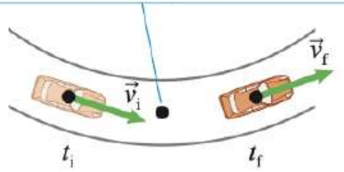
https://mediaplayer.pearsoncmg.com/assets/_frames.true/secs-egv2e-forces-exerted-on-an-object-moving-in-a-circle-at-constant-speed

https://mediaplayer.pearsoncmg.com/assets/_frames.true/secs-egv2e-does-the-sum-of-the-forces-exerted-on-an-object-moving-at-constant-speed-in-a-circle-point-toward-the-center-of-the-circle-

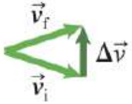
How to explain WHY the sum of the forces should point towards the center.

PHYSICS TOOL BOX 5.1 **Estimating the direction of acceleration during two-dimensional motion**

1. Choose the point at which you want to determine the direction of acceleration and draw velocity vectors at equal distances before and after the point.



2. Place the $\vec{v}_i$ and $\vec{v}_f$ arrows tail to tail. Draw a $\Delta\vec{v}$ arrow from the head of $\vec{v}_i$ to the head of $\vec{v}_f$.



$$\vec{v}_i + \Delta\vec{v} = \vec{v}_f$$

or

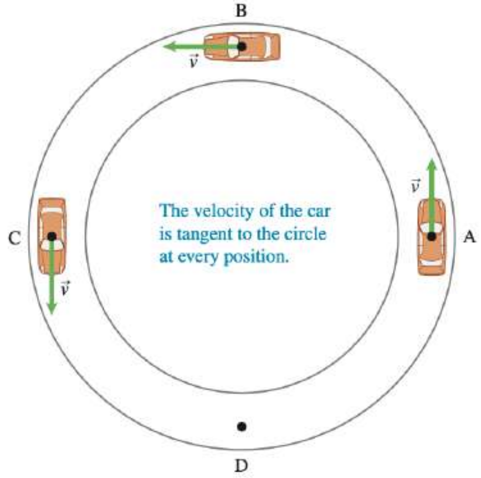
$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

3. The acceleration arrow $\vec{a}$ is in the direction of $\Delta\vec{v}$.

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

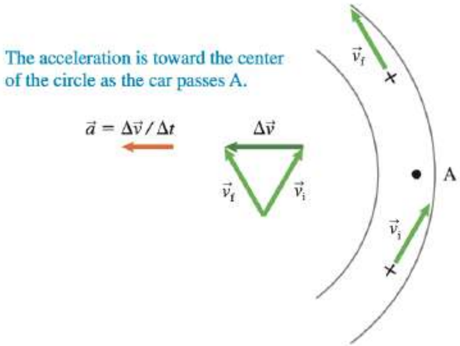
CONCEPTUAL EXERCISE 5.1 Direction of a racecar's acceleration

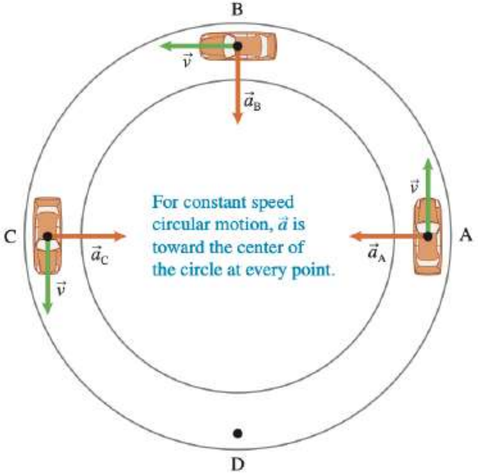
Determine the direction of the racecar's acceleration at points A, B, and C in the figure below as the car travels at constant speed on the circular path.



Sketch and translate A top view of the car's path is shown above. We are interested in the car's acceleration as it passes points A, B, and C.

Simplify and diagram To find the direction of the car's acceleration at each point, we use the velocity change method (shown for point A above, right). When done for all three points, notice that a pattern emerges: the acceleration at different points along the car's path has a different direction, but in every case it points toward the center of the circular path.





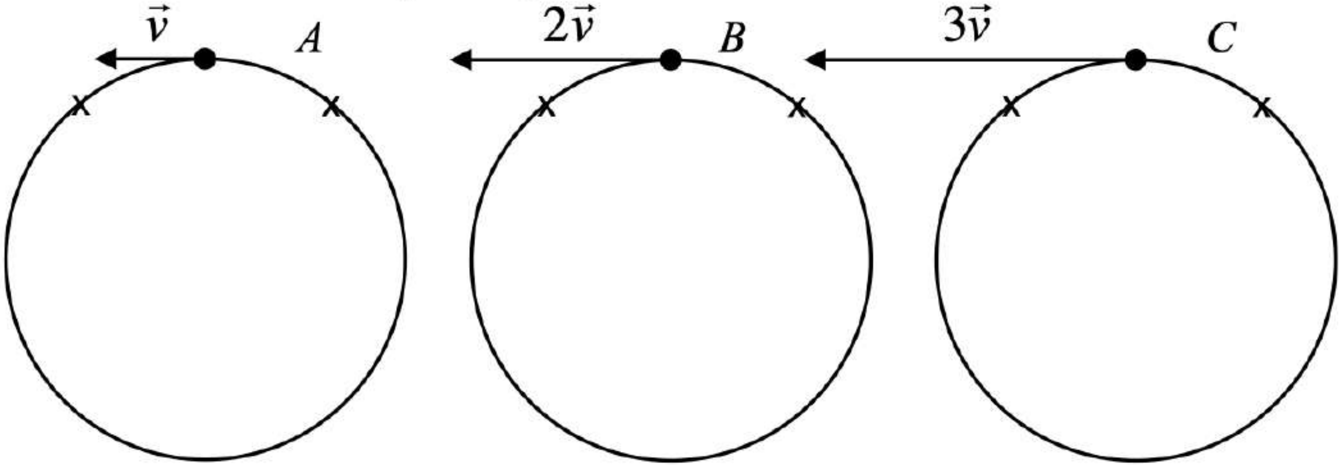
Next step: what is the value of the radial acceleration?

We will be working with the document at

OALG Chapter 5 Final.docx

OALG 5.3.1

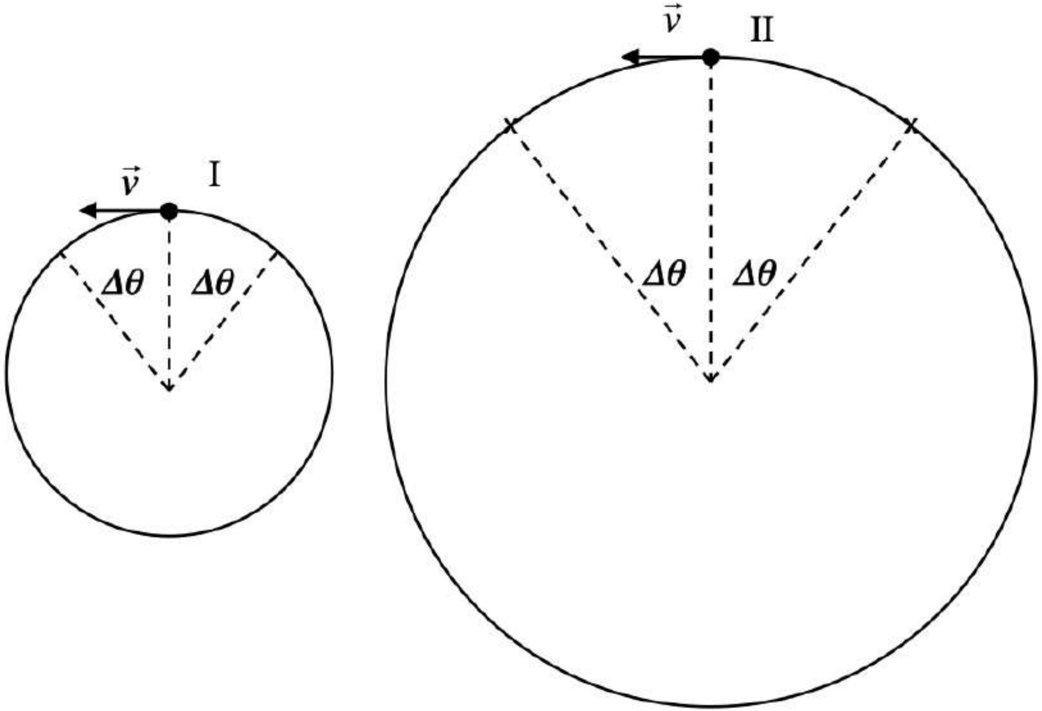
Imagine three small toy cars travel at constant speed in identical-radii horizontal circular paths (a top view is shown below). Car *A* moves at speed v, car *B* at speed $2v$, and car *C* at speed $3v$. Use the velocity technique (Physics Toolbox 5.1 in the textbook) to determine how the magnitude of the acceleration of the cars depends on their speeds. Remember that acceleration is $\Delta\vec{v}/\Delta t$ and that you need to compare the velocity change $\Delta\vec{v}$ vectors for the three speeds and also the time interval Δt needed for the velocity changes in each of the three cases.



OALG 5.3.2

Two small toy cars travel at the same constant speed in horizontal circular paths (a top view is shown below). Car I moves in a circle of radius r and car II in a circle of radius $2r$.

a. Use the velocity technique (Physics Toolbox 5.1 in the textbook) to determine how the magnitudes of the accelerations of the cars depend on the radii of the circles. Do not forget to consider the time intervals needed for the velocity changes.



b. Combine the results from Activities 5.3.1 and 5.3.2a to write a general expression for the magnitude of the acceleration during constant-speed circular motion.

Team 1 OALG 5.3.1 and 5.3.2 OALG Chapter 5 Final.docx

a_1

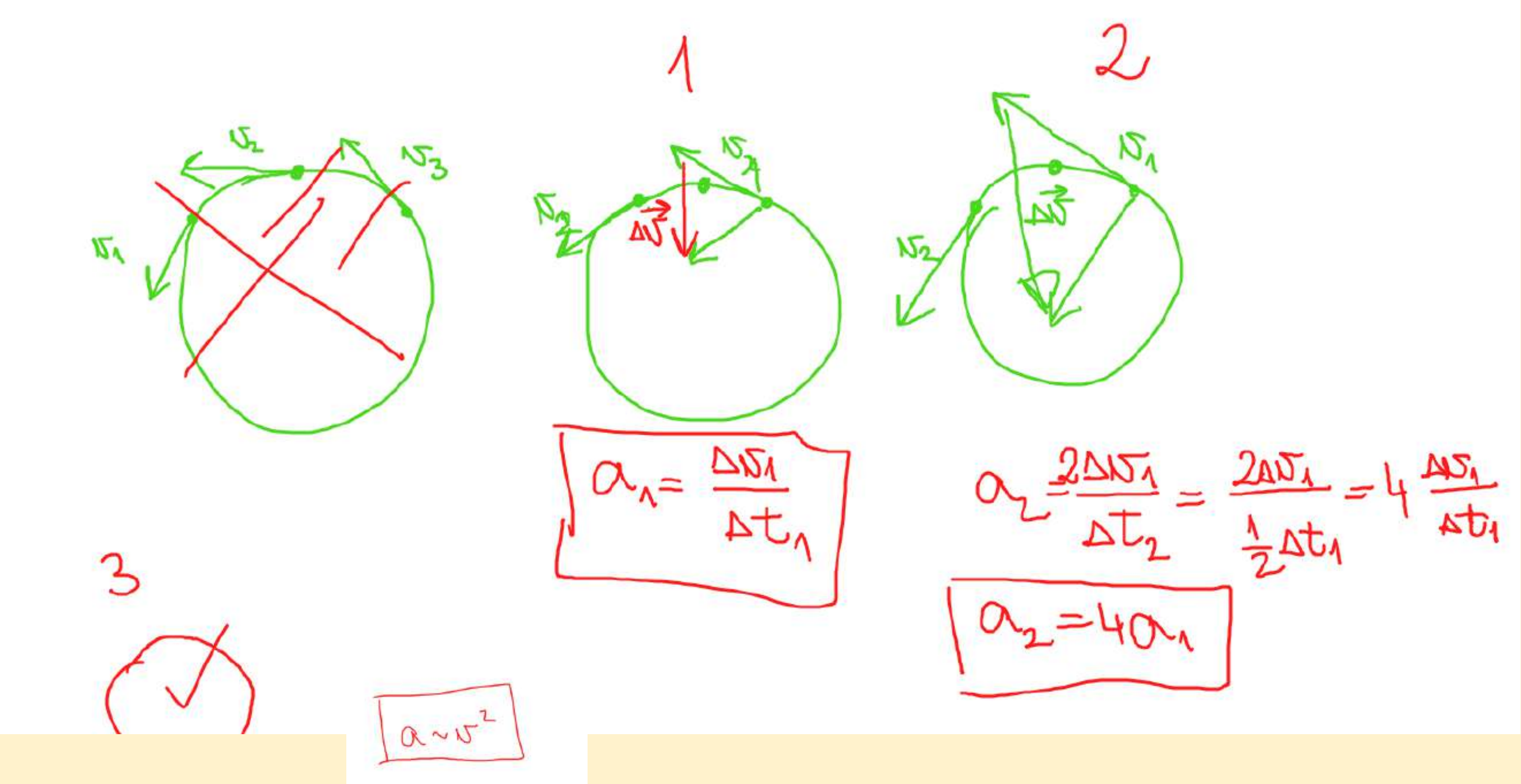
$a_2=4a_1$

$a_3=9a_1$

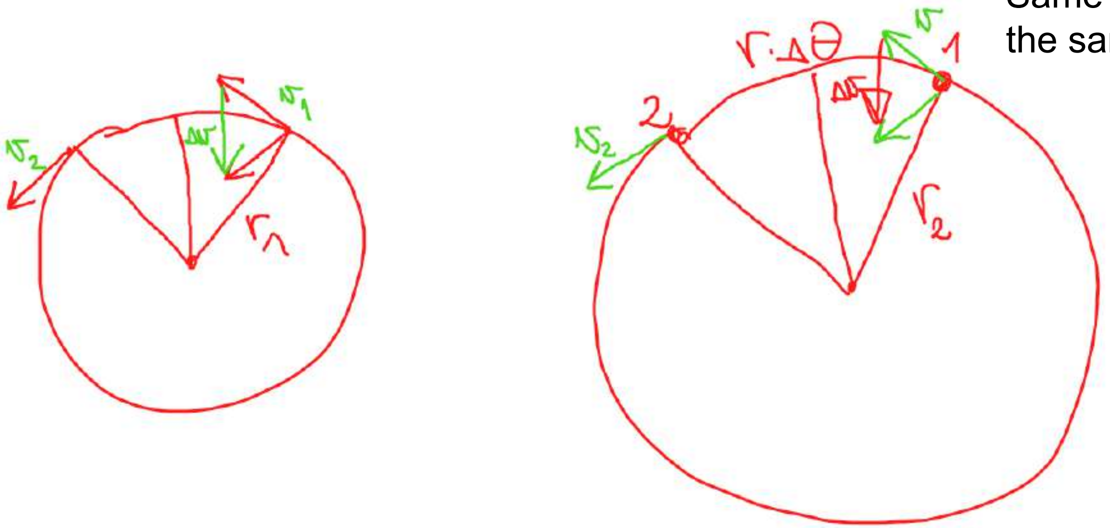
Inverse relationship between change in velocity and radius

Team 2 OALG 5.3.1 and 5.3.2 [OALG Chapter 5 Final.docx]

5



Team 2 OALG 5.3.1 and 5.3.2 OALG Chapter 5 Final.docx



Same speed, the velocity change is the same

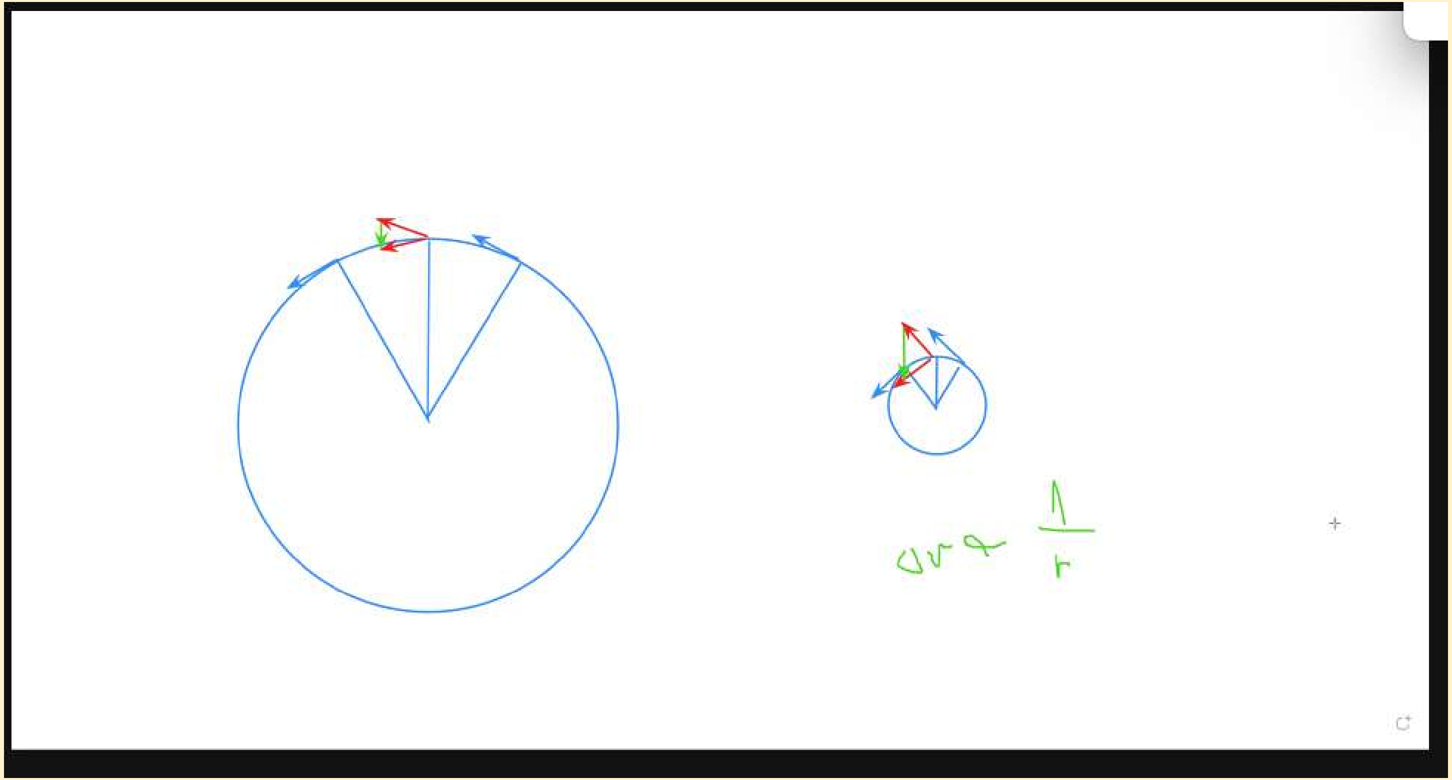
$$a_1 = \frac{\Delta v_1}{\Delta t_1}$$

$$a_2 = \frac{\Delta v_2}{\Delta t_2} = \frac{\Delta v_1}{2\Delta t_1}$$

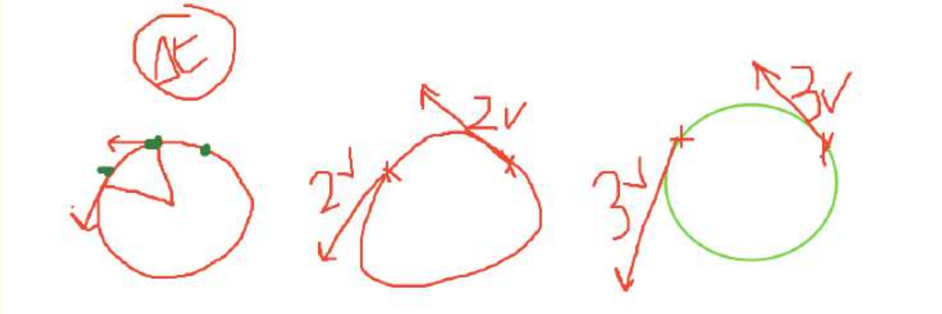
$$\boxed{a_2 = \frac{1}{2} a_1}$$

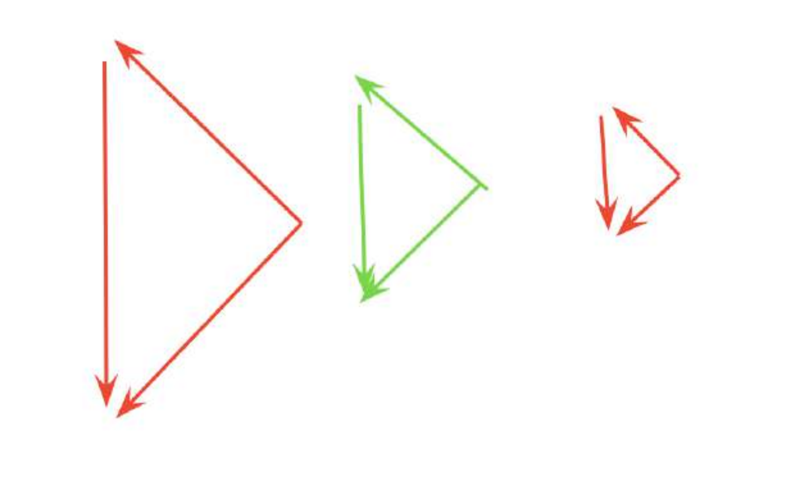
$$a \sim \frac{1}{r}$$

Team 3 OALG 5.3.1 and 5.3.2 [OALG Chapter 5 Final.docx]



Team 4 OALG 5.3.1 and 5.3.2 OALG Chapter 5 Final.docx





$$a_1 = \frac{\Delta v}{\Delta t}$$

$$a_2 = 4\frac{\Delta v}{\Delta t}$$

$$a_3 = 9\frac{\Delta v}{\Delta t}$$

OALG 5.3.3 Test the relation

Equipment: a ruler, a protractor.

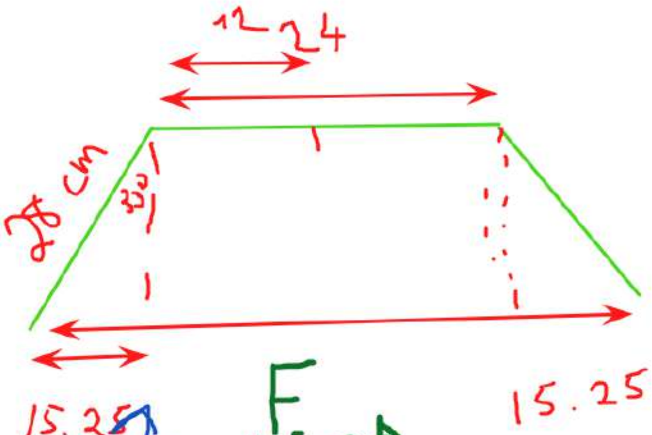
Use the video https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-OALG-5-3-3 to test whether Newton's second law works for circular motion, namely. For this experiment, you will need a ruler and a protractor.

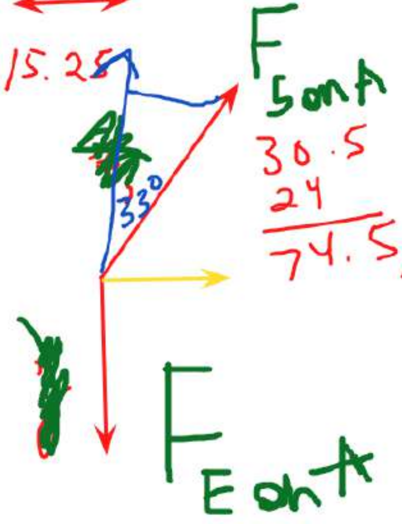
a. Watch two spheres A and B of different masses move in a circle (mA=29 g and mB=89 g). What do you notice about their motion? Write all your observations below (do not measure anything). Draw force diagrams to explain the tilt of the strings qualitatively.

b. Use the data in the video for sphere A to determine whether the angle is consistent with the equation under test. What other quantities do you need to determine to make this judgment? Describe how you will model the objects, interactions, and processes that you will use in your mathematical model. Construct force diagrams as appropriate.

c. Consider the uncertainties in your data. How do they affect your judgment?

d. How can you explain that the tilt of the string for sphere B is the same as for sphere A although sphere B has about three times larger mass?





30.5
24
74.5/2 = 27.25

Y :

$F_y = \cos 33° \cdot F_S$

$F_{E on A}$

$m \cdot g = \cos \alpha \cdot F_S$

$m = 2 kg$

$\sin 33° = \frac{x}{28}$

$x = 28 \cdot \sin 33 =$

$x = 15.25 cm$

$R = x + 12 = 27,25$

$a = \frac{v^2}{R}$

$F_S = \frac{m \cdot g}{\cos \alpha}$

$F_g =$

x :

$F_{Sx} = \sin 33° F_S$

$S on A +$

$\frac{\sin \alpha \, m g}{\cos \alpha} = \frac{m v^2}{R}$

$\tan \alpha \cdot g = \frac{v^2}{R}$

$\tan \alpha = \frac{v^2}{R g}$

$\alpha = \text{arcos} \frac{v^2}{R g}$

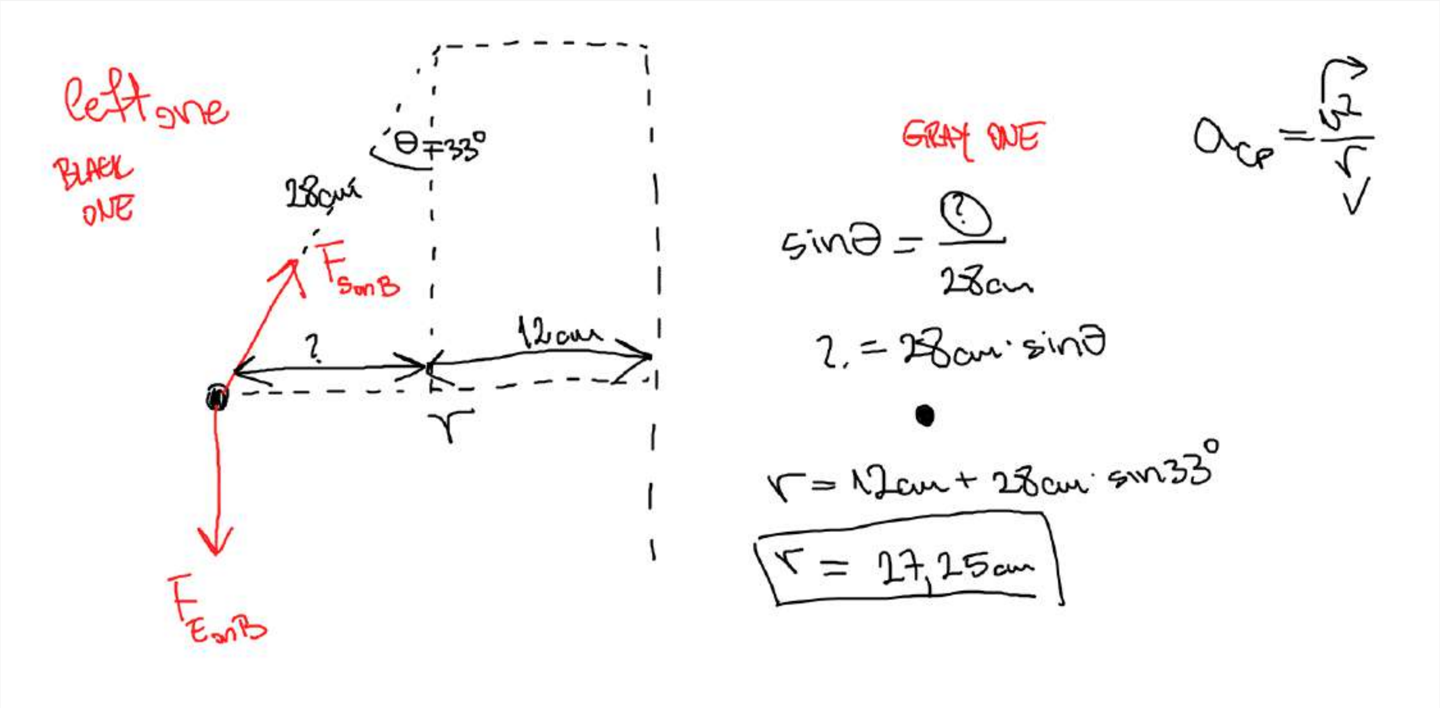
$F_{Sx} = m \cdot a$

$F_{Sx} = m \cdot \frac{v^2}{r}$

$\sin \alpha \cdot F_S = m \cdot \frac{v^2}{r}$

Team 2 OALG 5.3.3 [OALG Chapter 5 Final.docx]

(a) Both balls swing out at the same angle, even though they have different masses. Both balls are the same distance from the center of rotation.

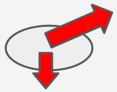


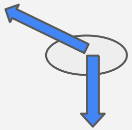
Team 3 OALG 5.3.3 OALG Chapter 5 Final.docx

1. **a.** They're both moving in the same circular path at the same speed. And the tilt of the string to keep them in those circular paths is exactly the same.

Sphere A

Sphere B





b. Use the data in the video for sphere A to determine whether the angle is consistent with the equation under test. What other quantities do you need to determine to make this judgment? Describe how you will model the objects, interactions, and processes that you will use in your mathematical model. Construct force diagrams as appropriate.

$$a = F_{net}/m = v^2/r$$

c. Consider the uncertainties in your data. How do they affect your judgment?

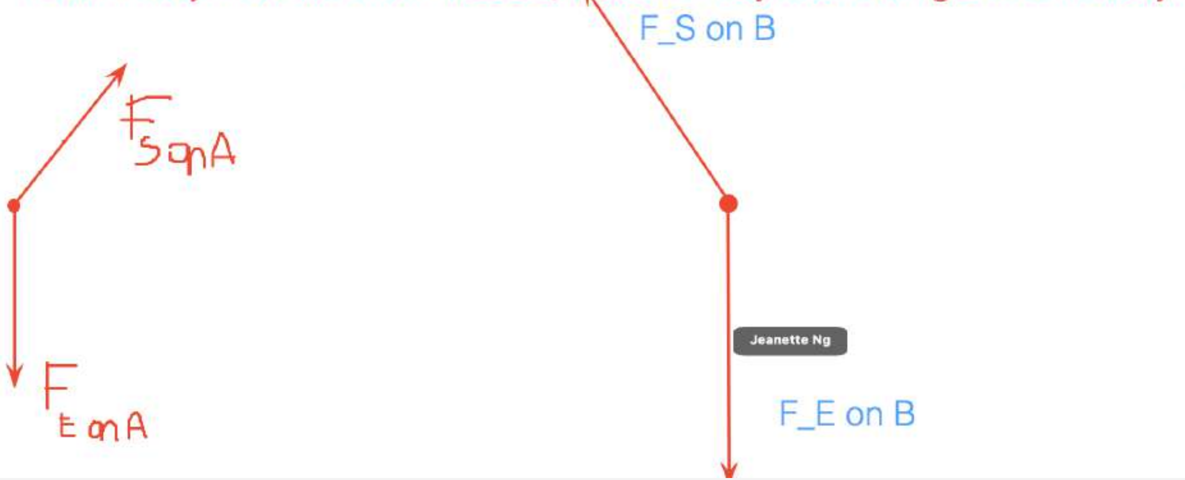
d. How can you explain that the tilt of the string for sphere B is the same as for sphere A although sphere B has about three times larger mass?

Team 4 OALG 5.3.3 [OALG Chapter 5 Final.docx]

Watch two spheres A and B of different masses move in a circle (mA=29 g and mB=89 g). What do you notice about their motion? Write all your observations below (do not measure anything). Draw force diagrams to explain the tilt of the strings qualitatively.

They move apart from the vertical about 33 degree

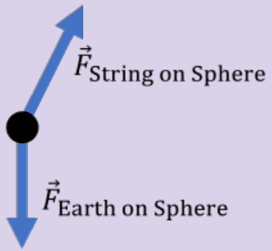
Even if they have different masses, it seems they are moving at the velocity

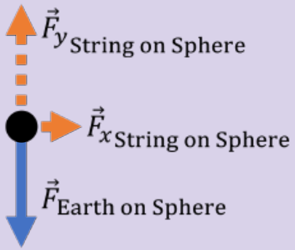


Model solution

a.Both spheres swing outwards so that their strings tilt at an angle to the vertical. Both strings seem to form the same angle with the horizontal, even though the spheres have different masses.

The string is angled so that the the vertical components of both forces cancel out, leaving the sum of the forces to point in the horizontal direction.





This is qualitatively consistent with what we’ve learned since that would leave the sum of forces pointing towards the center of the circle through which the sphere is moving.

Model solution

b. & c.

Hypothesis: Newton's 2nd Law applies to circular motion and the coefficient is 1 ($a = \frac{v^2}{r} = \frac{\Sigma F}{m}$)

Prediction: We can predict the angle the string will make with the vertical if we measure the period and radius of the sphere's swinging:

> **Period** is measured to be 40 frames (the spheres go around from 340 to 380) or 1.33s. The uncertainty in this measurement is 1 frame (1/30 s)
>
> **Radius** of the circle was measured to be 27cm by counting pixels. The uncertainty in this measurement is about 1 cm.

Model solution

b.

Apply Newton's 2nd Law in the x- and y- directions:

$$a_x = \frac{\Sigma F_x}{m} = \frac{v^2}{r}$$

$$\frac{F_{x\,\text{String on Sphere}}}{m} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$\frac{F_{\text{String on Sphere}} \sin\theta}{m} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$\sin\theta = \frac{m\left(\frac{2\pi r}{T}\right)^2}{F_{\text{String on Sphere}} \cdot r}$$

$$a_y = \frac{\Sigma F_y}{m} = 0$$

$$F_{y\,\text{String on Sphere}} - F_{\text{Earth on Sphere}} = 0$$

$$F_{\text{String on Sphere}} \cos\theta = mg$$

$$\cos\theta = \frac{mg}{F_{\text{String on Sphere}}}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{2\pi r}{T}\right)^2}{gr}$$

$$\theta = \tan^{-1}\left(\frac{\left(\frac{2\pi(.27\text{m})}{1.33\text{s}}\right)^2}{\left(9.8\frac{\text{N}}{\text{kg}}\right)(.27\text{m})}\right) = 32° \pm 2°$$

Model solution

b. & c.

Outcome & Judgment: The angle of the string, as measured in the video, is 33°. This is within the bounds of our prediction, 32°±2°, so from this experiment we cannot reject our hypothesis that Newton's 2nd Law applies to objects moving in circles.

Model solution

d.

The angle is the same for both spheres for the same reason the freefall acceleration is the same for all objects regardless of mass: All forces exerted on the spheres in this experiment depend linearly on their mass, so the mass cancels out when using Newton’s 2nd Law.

Let’s go back to the “need to know”

Team 1 OALG 5.5.1 OALG Chapter 5 Final.docx

$$a = \frac{v^2}{r}$$

$$2.69 \times 10^{-3}\ m/s^2$$

$$a =$$

$$v = 1010\ \frac{m}{s}$$

$$\Delta t = \frac{27.3\ days}{} \cdot \frac{24\ hr}{1\ day} \cdot \frac{3600\ s}{1\ hr}$$

$$= 2360000\ s$$

$$r = 3.8 \times 10^{8}\ m$$

$$C = 2\pi r = 2.39 \cdot 10^{9}\ m$$

$$\frac{\left(\frac{C}{\Delta t}\right)^2}{r} = 2.69 \cdot 10^{-3}\ \frac{m}{s^2}$$

Team 2 OALG 5.5.1 [OALG Chapter 5 Final.docx]

$$a_M = 2{,}69 \cdot 10^{-3} m/s^2$$

$$r = 3{,}8 \cdot 10^8 m$$

$$T = 27{,}3 d = 27{,}3 \cdot 3600 s \cdot 24$$

$$a_M = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = 2{,}7 \cdot 10^{-3} m/s^2 \approx \frac{1}{3600} g = \frac{F_{E on M}}{m_M}$$

Team 3 OALG 5.5.1 OALG Chapter 5 Final.docx

$A = v^2/r$

$A = (2* \quad *r/T)^2/r = 4* \quad ^2*r/T^2$

$= 4*3.14^2*3.8x10^8/(27.3*24*60*60)^2$

$=0.002696\ m/s^2=2.7 \times 10^{-3} m/s^2$

Team 4 OALG 5.5.1 OALG Chapter 5 Final.docx

Stai visualizzando lo schermo di Eugenia Etkina REC Visualizza opzioni

obs → $g = 9.8\ m/s^2$

$F_{E\ on\ O} = m_O g$ N2

$F_{E\ on\ M} \sim m_M$

$F_{E\ on\ M} \sim \frac{1}{r^2}$ ← obs

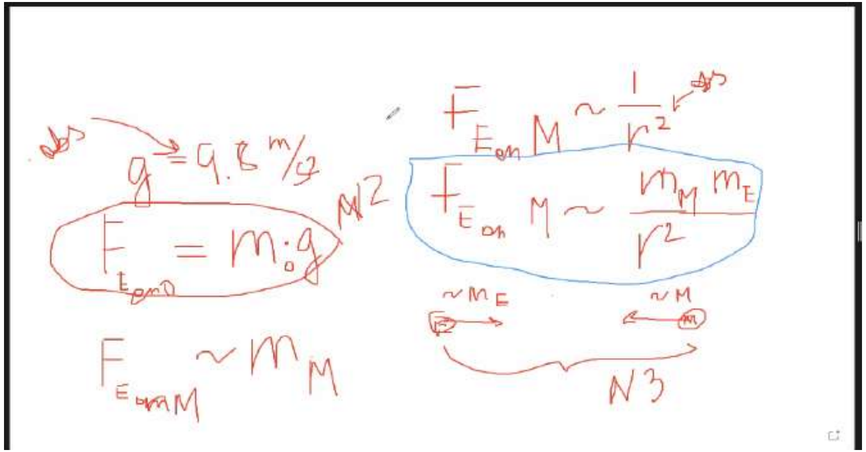
$F_{E\ on\ M} \sim \frac{m_M m_E}{r^2}$

$\sim m_E$ $\sim M$

E → ← M

N3

Solution for 5.5.1



second law to determine the gravitational force. He knew that the Moon was separated from Earth by about $r_M = 3.8 \times 10^8$ m, roughly 60 Earth radii ($60R_E$). With this distance and the Moon's orbital period of 27.3 days, Newton was able to use Eq. (5.5) to determine the Moon's radial acceleration:

$$a_{r \text{ at } R=60R_E} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(3.8 \times 10^8 \text{ m})}{[(27.3 \text{ days})(86{,}400 \text{ s/day})]^2}$$

$$= \frac{4\pi^2(3.8 \times 10^8 \text{ m})}{(2.36 \times 10^6 \text{ s})^2} = 2.69 \times 10^{-3} \text{ m/s}^2$$

He then did a similar analysis for a second situation. He performed a thought experiment in which he imagined what would happen if the Moon were condensed to a small point-like object (while keeping its mass the same) located near Earth's surface. This would place the "Moon particle" at a distance of one Earth radius ($1\,R_E$) from Earth's center. If the Moon only interacted with Earth, it would have the same free-fall acceleration as any object near Earth's surface (9.8 m/s^2), since this acceleration is independent of mass. Now Newton knew the Moon's acceleration at two different distances from Earth's center.

Newton then used this information to determine how the gravitational force that one object exerts on another depends on the separation of the objects, assuming that the force causing the acceleration of the Moon is the gravitational force exerted on it by Earth and that this force changes with the separation of the objects. He took the ratio of these two accelerations using his second law:

$$\frac{a_{r \text{ at } R=60R_E}}{a_{r \text{ at } R=1R_E}} = \frac{F_{\text{E on Moon at } R=60R_E}/m_{\text{Moon}}}{F_{\text{E on Moon at } R=1R_E}/m_{\text{Moon}}} = \frac{F_{\text{E on Moon at } R=60R_E}}{F_{\text{E on Moon at } R=1R_E}}$$

Notice that the Moon's mass cancels when taking this ratio. Substituting the two accelerations he had observed, Newton made the following comparison between the forces:

$$\frac{F_{\text{E on Moon at } R=60R_E}}{F_{\text{E on Moon at } R=1R_E}} = \frac{a_{r \text{ at } R=60R_E}}{a_{r \text{ at } R=1R_E}} = \frac{2.69 \times 10^{-3} \text{ m/s}^2}{9.8 \text{ m/s}^2} = \frac{1}{3600} = \frac{1}{60^2}$$

$$\Rightarrow F_{\text{E on Moon at } R=60R_E} = \frac{1}{60^2} F_{\text{E on Moon at } R=1R_E}$$

The force exerted on the Moon when the Moon is at a distance of 60 times Earth's radius is $1/60^2$ times the force exerted on the Moon when near Earth's surface (about 1 times Earth's radius). It appears that as the distance from Earth's center to the Moon *increases* by a factor of 60, the force decreases by a factor of $1/60^2 = 1/3600$. This suggests that the gravitational force that Earth exerts on the Moon depends on the inverse square of its distance from the center of Earth:

$$F_{\text{E on Moon at } r} \propto \frac{1}{r^2} \tag{5.10}$$

dds

$$g = 9.8\ m/s^2$$

$$F_{E\,on\,O} = m_o \cdot g$$

N2

$$F_{E\,on\,M} \sim m_M$$

dds

$$F_{E\,on\,M} \sim \frac{1}{r^2}$$

$$F_{E\,on\,M} \sim \frac{m_M\, m_E}{r^2}$$

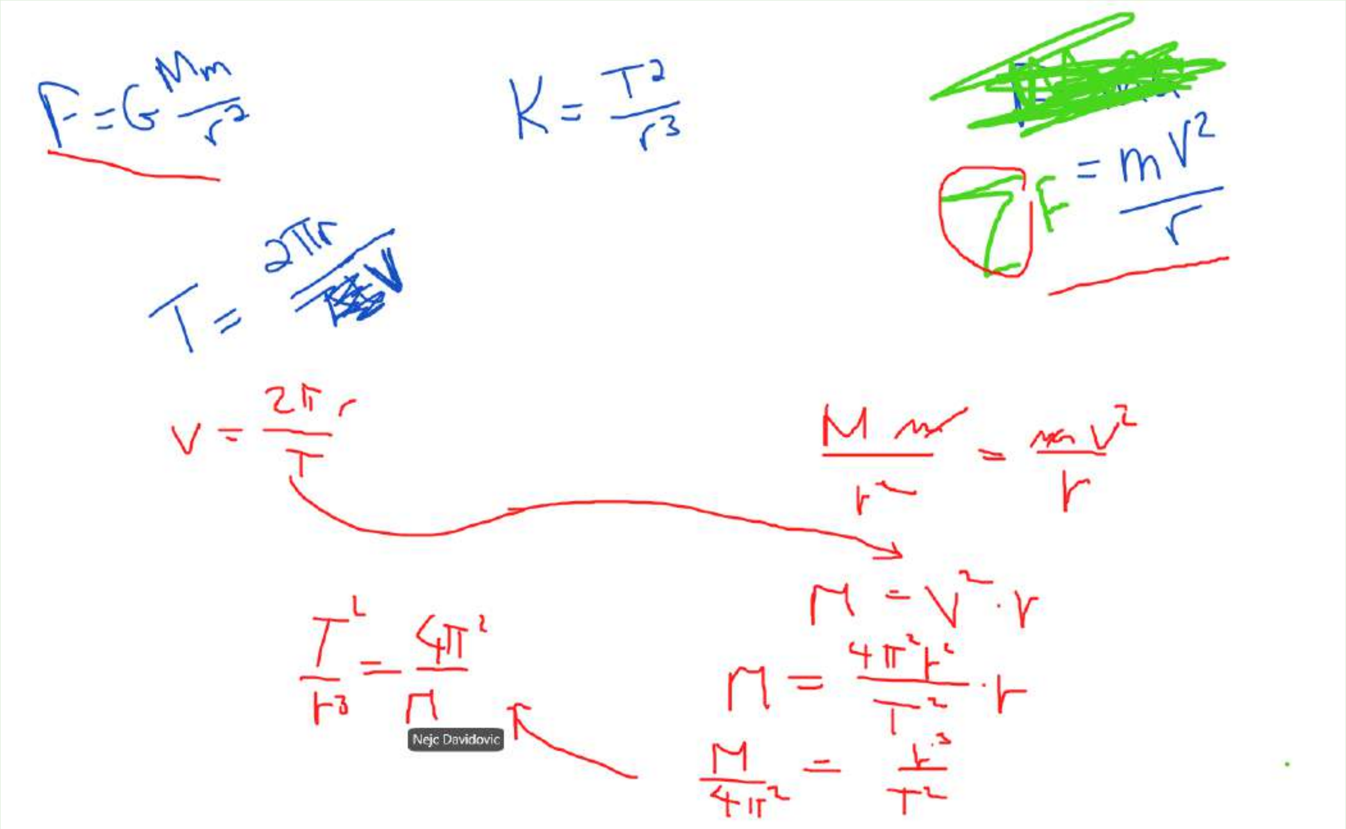
$\sim m_E$ $\sim M$

N3

All together 5.5.2 OALG Chapter 5 Final.docx

All together 1 5.5.3 OALG Chapter 5 Final.docx

Team 1 5.5.5 [OALG Chapter 5 Final.docx]



Team 2 5.5.5 [OALG Chapter 5 Final.docx]

$$a_E = \frac{\Sigma F}{m_E} = \frac{G\frac{M_s m_E}{r^2}}{m_E} = G\frac{M_s}{r^2}$$

$$a_E = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{\frac{4\pi^2 r^2}{T^2}}{\frac{r}{1}} = \frac{4\pi^2 r}{T^2}$$

$$G\frac{M_s}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$G M_s T^2 = 4\pi^2 r^3 \quad / : r^3$$

$$\frac{T^2}{r^3} G M_s = 4\pi^2$$

$$1{,}97\cdot 10^{-19}\,\frac{s^2}{m^3} = \frac{T^2}{r^3} = \frac{4\pi^2}{G M_s} = K$$

Team 3 5.5.5 OALG Chapter 5 Final.docx

Team 4 5.5.5 OALG Chapter 5 Final.docx

$F = GmM/r^2$, and sum of $F = ma = mv^2/r$

So $GmM/r^2 = mv^2/r$

$GmM/r^2 = m(2* \;\; *r/T)^2/r = 4 \;\; {}^2r/T^2$

$T^2/r^3 = 4* \;\; {}^2/(GM) =$ constant

What will I take with me from today's meeting?

Progression of how Law of Universal Gravitation led to discovery of Neptune and eventually Pluto

The fact that Halley sponsored Newton's publication of Principia

Newton's second law applies to circular motion

How to help students find the relationships of universal gravitation law and centripetal acceleration themselves

Law of universal gravitation uses Newton's 2nd and 3rd laws, as well as observational data

From the first part, we can work with centripetal acceleration, without differential geometry

Sometimes we can derive relationships via reasoning from data rather than experiments (ie, Newton and the Moon)