Helping students learn uncertainty and significant figures

Matt Blackman, Ilya Yashin

The nature of a number in physics

Dragster record on the desert In 1977, Kitty O'Neil drove a hydrogen peroxide–powered rocket dragster for a record time interval (3.22 s) and

final speed (663 km/h) on a 402-m-long Mojave Desert track. Determine her average acceleration during the race and the acceleration while stopping (it took about 20 s to stop). What assumptions did you make?

What do those numbers mean?

How should we write them to communicate the important message about how it was measured hidden in each number?

Team 1 Write the numbers in the problem as intervals using the meaning of the significant figures.

3.22 s (3.22 ± 0.005 s)   (3.22 ± 0.01 s)

663 km/h (663 ± 0.5 km/h) (663 ± 1 km/h)

a 402-m-long (402 ± 0.5 m) (402 ± 1 m)

about 20 s   What does this mean?

Team 2 Write the numbers in the problem as intervals using the meaning of the significant figures.

3.22 s (3.21 s - 3.23 s)

663 km/h (662 km/h - 664 km/h)

402 m (401 m - 403 m)

Team 3 Write the numbers in the problem as intervals using the meaning of the significant figures.

3.22 s → 3.215 to 3.224 s

663 km/h → 662.5 to 663.4 km/h

a 402-m-long → 401.5 to 402.4 m

about 20 s → 15 to 24 s (depends on what “about” means)

Team 4 Write the numbers in the problem as intervals using the meaning of the significant figures.

They all have 3 significant figures (SF), except for 20 s. That has 1 SF, or is it 2? Hmmm… Ambiguous.

3.22 +/- 0.005 s

20 +/- 5 s or 20 +/- 0.5 s ?

Anne would say 3.22 +- 0.05 or whatever estimate so decimal places match

Random uncertainty - let’s read and interrogate

[Reference materials]

Most important!

Use experimental uncertainty when the measurements are the same

Use random uncertainty for all other measures.

Multiple trials do not decrease random uncertainty, they allow us to estimate it more precisely.

Weakest link rule tells you which uncertainty is the most important.

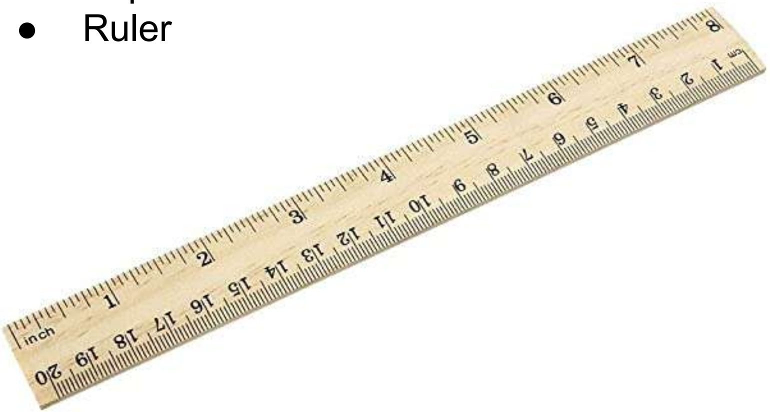
Issues with instrumental uncertainty and decreasing uncertainty

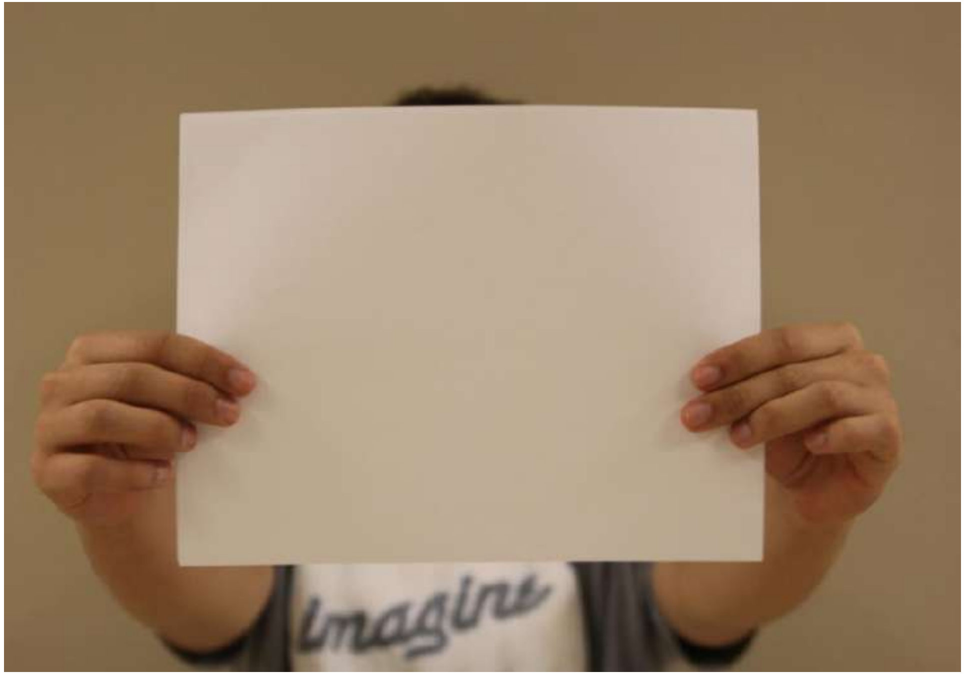
Student Prompt:

Determine the thickness of a single sheet of paper.

Tools:

- Paper
- Ruler





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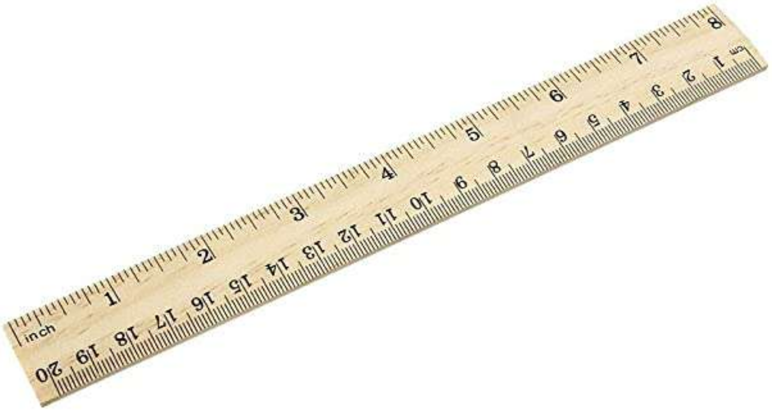
[Universe & More]

$x = 0.5\text{mm} \pm 0.5\text{mm}$

$100x = 8.0\text{mm} \pm 0.5\text{mm}$

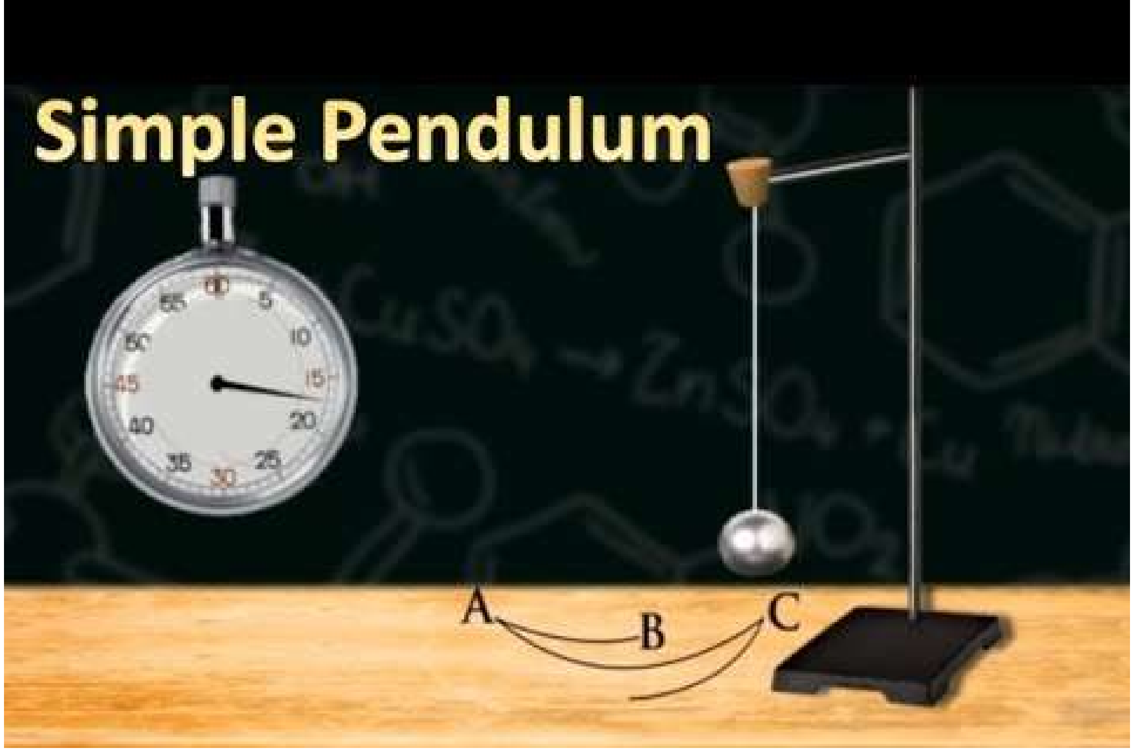
$x = (8.0\text{mm} \pm 0.5\text{mm})/100$

$x = 0.080\text{mm} \pm 0.005\text{mm}$



Important ideas:

- ***Instrumental uncertainty*** is a property of the measuring instrument.
- ***Relative uncertainty*** is related to the size of the instrumental uncertainty compared to the quantity being measured.
- To ***decrease*** instrumental uncertainty you need to make the measurement with the available instrument as large as possible.



$10T = 32.0s \pm 0.1s$

$T = (32.0s \pm 0.1s)/10$

$T = 3.20s \pm 0.01s$

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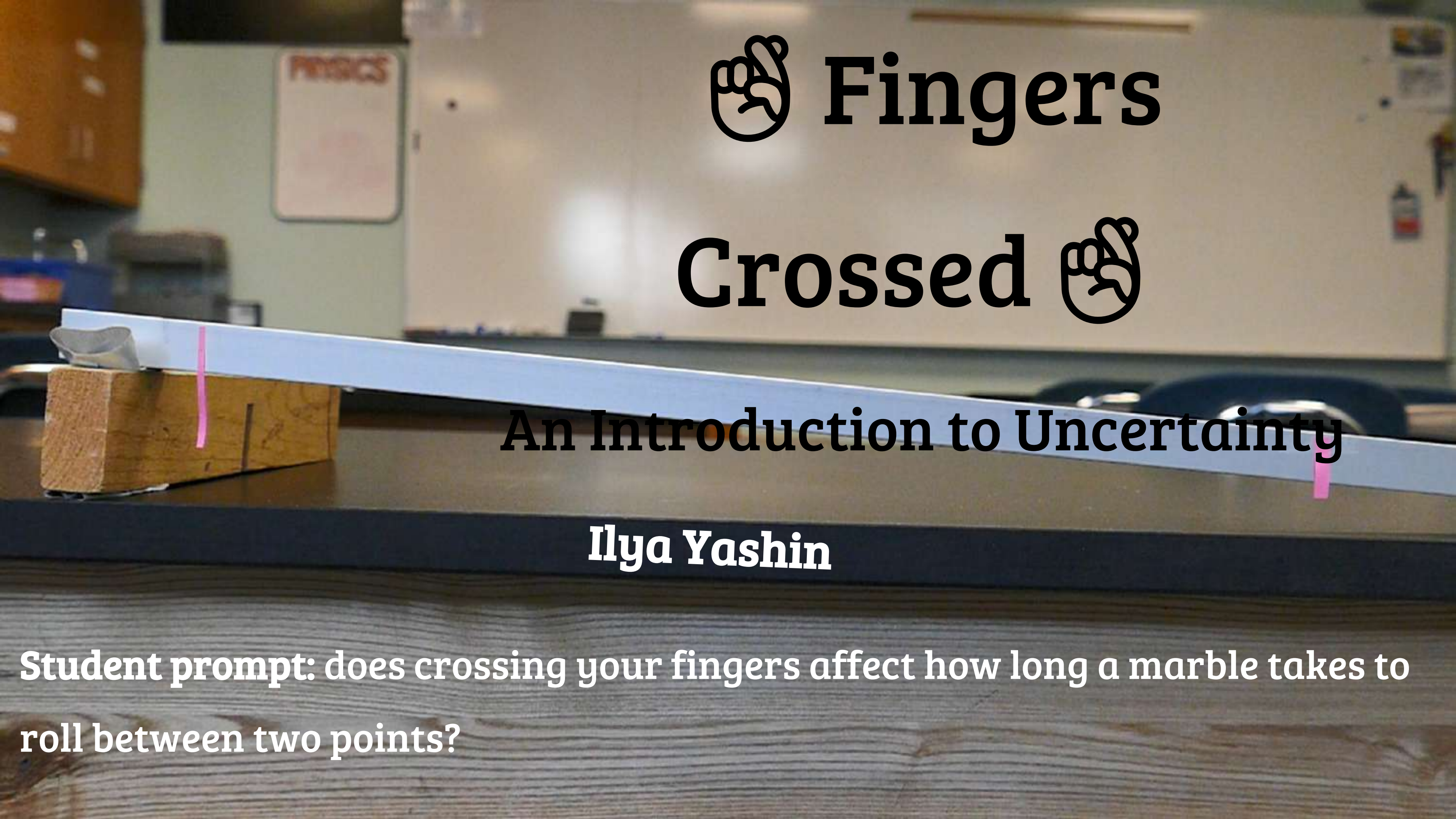


🤞 Fingers Crossed 🤞

An Introduction to Uncertainty

Ilya Yashin

Student prompt: does crossing your fingers affect how long a marble takes to roll between two points?

1. Get student consensus for experimental protocol.

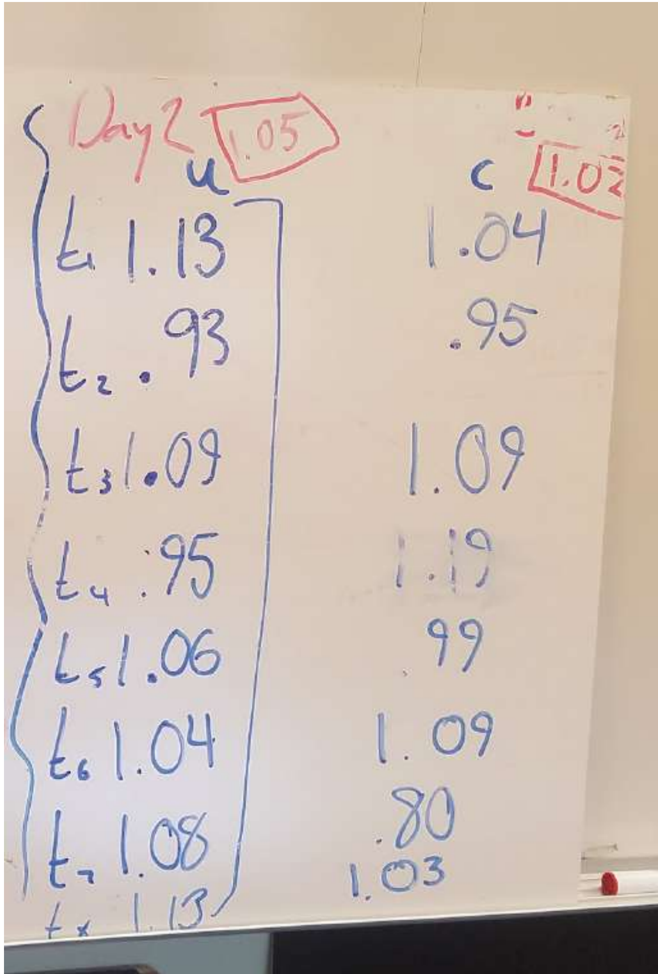
2. Students test with stopwatches (maybe slo-mo recording?), put data & conclusion on boards.

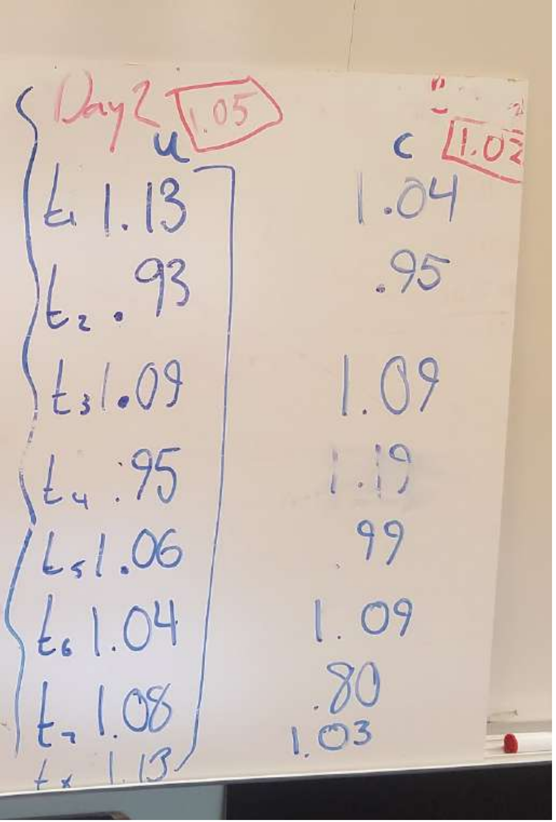
3. Discussion.

"We're just bad at timing!"

"Come on, the difference is SO tiny!!"

"HUMAN ERROR!!!"





4. Introducing uncertainty

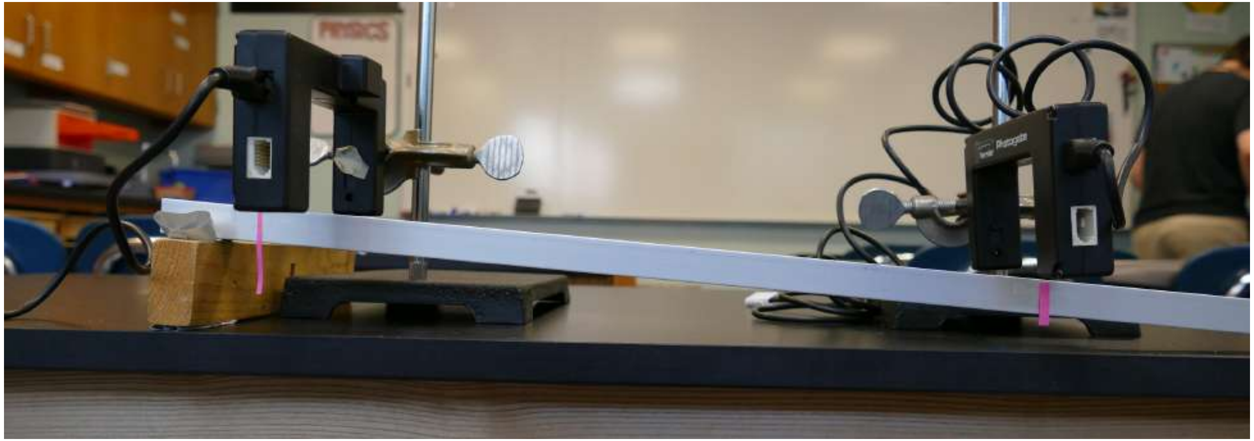
Factoring in

instrumental uncertainty

OVERLAP

	Time between (s)	
	Uncrossed	**Crossed**
	1.1	1.0
	0.9	1.0
	1.1	1.1
	1.0	1.2
	1.1	1.0
	1.0	1.1
	1.1	0.8
	1.1	1.0
Average:	1.1	1.0
Range:	0.2	0.4

5. Photogates!



	Time between gates (s)	
	Uncrossed	Crossed
	0.4284	0.4271
	0.4271	0.4259
	0.4288	0.4237
	0.4302	0.4236
	0.4272	0.4250
	0.4282	0.4243
	0.4270	0.4235
	0.4265	0.4215
Average:	0.4279	0.4243
Range:	0.0037	0.0056

OVERLAP

What is the timing precision of photogate measurements?

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0.025 s

For Blocked-to-Blocked and Unblocked-to-Unblocked transitions

Vernier photogates have an electronic timing resolution of 1 microsecond. You will, however, typically get measurements with as much as +/-25 millisecond variation. This is due to variations in optical-response tolerances of the photodiodes and the geometry of the object relative to the path it takes through the photogate.

Key takeaways:

- Uncertainty allows us to compare two quantitative results. Averages aren’t enough!
- “Human error” can be quantified.
- Single value → fuzzy range
- Equal → overlap

Even experts make mistakes (Jiulio F. Caballero. F. Harris, There Seems To Be Uncertainty about the Use of Significant Figures in Reporting Uncertainties of Results, Journal of Chemical Education • Vol. 75 No. 8 August 1998)

Table 1. Examples of Inappropriate and Appropriate Rounding

Published Value	Ref	Appropriate Rounding	Uncer-tainty[a]
771 ± 135	*3*	$(7.7 \pm 1.4) \times 10^2$	14
65.9 ± 10.1	*3*	$(6.6 \pm 1.0) \times 10^1$	10
$e^{-(646 \pm 69)/T}$	*4*	$e^{-(650 \pm 70)/T}$	7
1.79 ± 0.78	*5*	1.8 ± 0.8	8
$e^{(8450 \pm 850)/T}$	*6*	$e^{(8400 \pm 800)/T}$ [b]	8
$(1.4488 \pm 0.0290) \times 10^{-46}$	*7*	$(1.449 \pm 0.029) \times 10^{-46}$	29
–19.9998 SE 0.100842	*8*	–20.00 SE 0.10	10

[a]This column shows that each uncertainty has a value between 3 and 30, whatever the place value.

[b]This example illustrates laboratory rounding. When rounding involves 5 exactly, round to the even number.