

Vibrational motion

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Please rename yourself: First name and the country you are from.

Links for today's workshop

Meeting folder

<https://drive.google.com/drive/folders/1var4JpRSzYlwU8ZpE7kHImAQDDXI3X9->

OALG Chapter 10 File

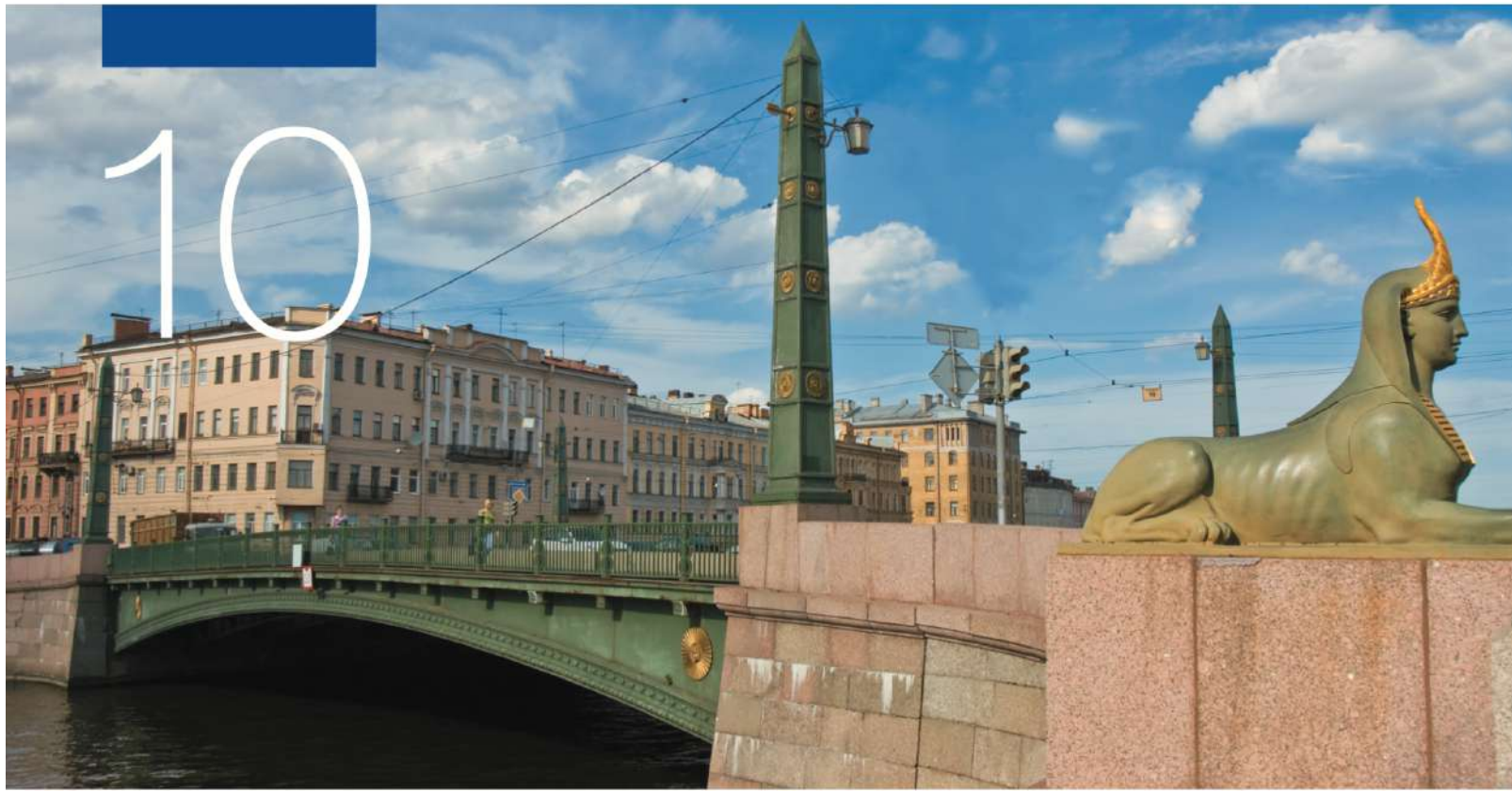
https://docs.google.com/document/d/1-z9Aji_e-5NM9uOghTpViaZHhsjNZcW3/edit#heading=h.gjdgxs KEEP THIS ONE OPEN
DURING THE WORKSHOP

ALG Chapter 10 File

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Does everyone know how to use zoom whiteboard in a breakout room?

Need to know



All together OALG 10.1.1 https://docs.google.com/document/d/1-z9Aji_e-5NM9uOghTpViaZHsjNZcW3/edit#heading=h.gjdgxs

OALG 10.1.1 Observe and find a pattern

Equipment: Two small ~200-g objects, a soft spring (if available), piece of string.

For this activity, you will need a soft spring. If you do not have one, a rubber band can be used too. In addition, find some string (dental floss will work) and a couple of small objects to attach to the string and to the spring. Conduct the following two experiments and describe common patterns concerning the behavior of the hanging object.

a. Perform the experiments and record your observations.

Experiment 1: Tie a string to a small object and let the object hang freely. Then, pull the object to the side and release it. Record your observations.

Experiment 2: Hang another object from the spring (or the rubber band), pull the object down, and release it. Record your observations.

b. Identify patterns common to both experiments.

c. Compare and contrast the motions of the objects in these experiments with the motion of objects at constant speed or at constant acceleration.

b. Put the patterns common to both experiments here.

Motion repeats

Object changes direction of motion, and has a position of equilibrium.

Passes equilibrium from 2 different sides

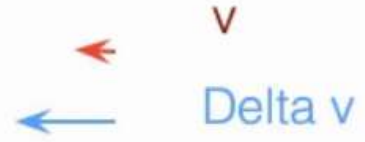
c. Compare and contrast the motions of the objects in these experiments with the motion of objects at constant speed or at constant acceleration.

Circular motion also repeats,

same direction, no eq point on the path taken

Team 1 OALG 10.1.2 https://docs.google.com/document/d/1-z9Aji_e-5NM9uOghTpViaZHsjNZcW3/edit#heading=h.gjdgxs

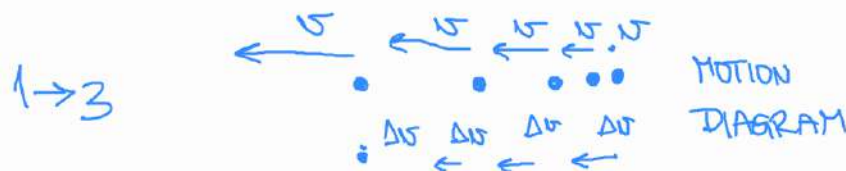
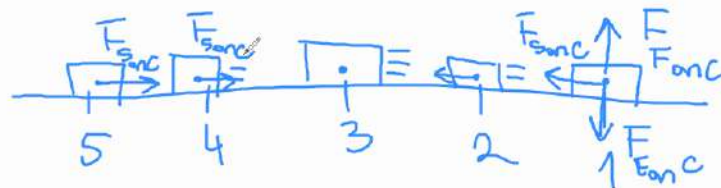
F spring on cart



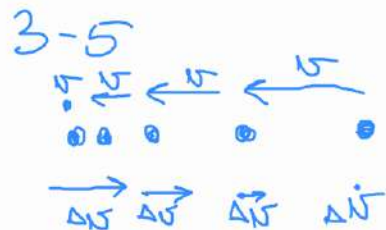
Team 2 OALG 10.1.2 https://docs.google.com/document/d/1-z9Aji_e-5NM9uOghTpViaZHhsjNZcW3/edit#heading=h.gjdgxs

a)

b) Yes, the points in the diagram as the velocity

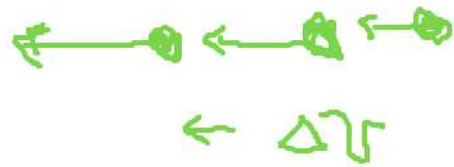


5-3 (moving right) is mirrored 1-3

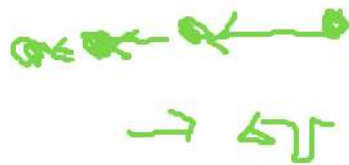


3-1 (moving right) is mirrored 3-5 moving left

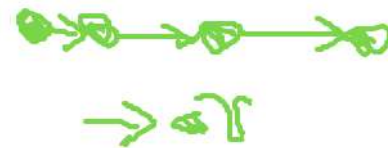
I-III



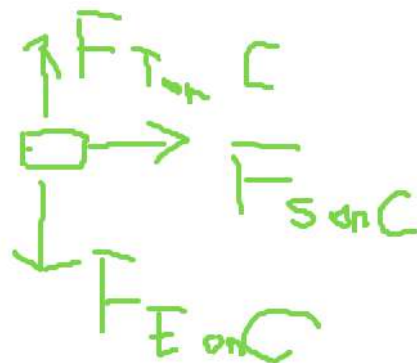
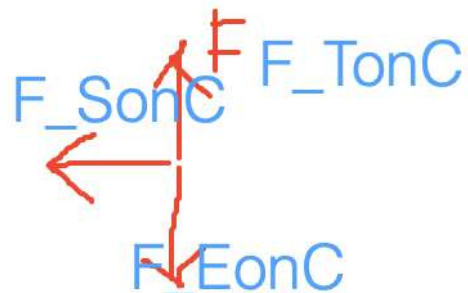
III-V



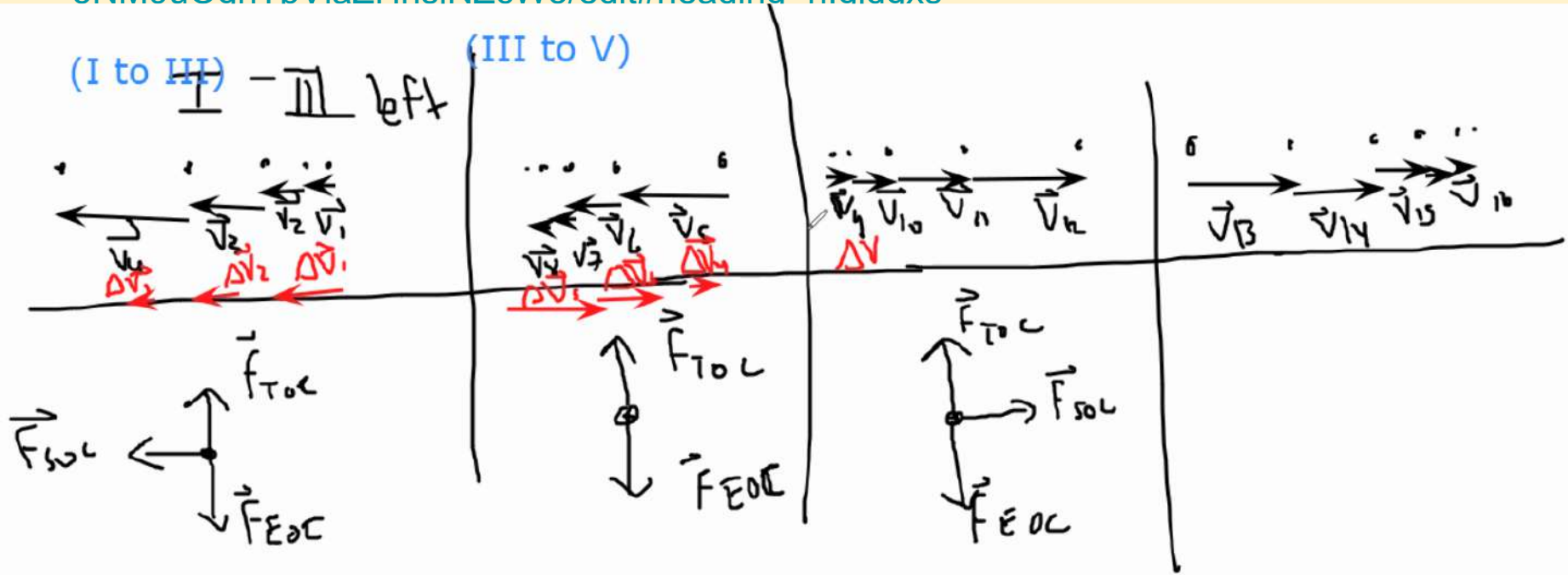
V-IV



I



Team 4 OALG 10.1.2 https://docs.google.com/document/d/1-z9Aji_e-5NM9uOahTpViaZHhsiNZcW3/edit#heading=h.aidaxs



Team 1 OALG 10.1.3 https://docs.google.com/document/d/1-z9Aji_e-5NM9uOghTpViaZHhsjNZcW3/edit#heading=h.gjdgxs

Team 2 OALG 10.1.3 https://docs.google.com/document/d/1-z9Aji_e-5NM9uOghTpViaZHhsjNZcW3/edit#heading=h.gjdgxs

Team 3 OALG 10.1.3 https://docs.google.com/document/d/1-z9Aji_e-5NM9uOghTpViaZHhsjNZcW3/edit#heading=h.gjdgxs

Team 3 OALG 10.1.3 https://docs.google.com/document/d/1-z9Aji_e-5NM9uOghTpViaZHhsjNZcW3/edit#heading=h.gjdgxs

All together - to illustrate the progress of student work, time for telling and practice

OALG 10.1.4 Reason and explain

Summarize the results of Activities 10.1.3–10.1.4 to describe and explain the motion of the cart. The description should include your observations, and the explanations should include your reasoning based on force and energy analyses for the observed phenomena.

OALG 10.1.5 Read and interrogate

Read and interrogate the sub-section “Period and frequency” in Section 10.1 in the textbook and answer Review Question 10.1.

OALG 10.1.6 Practice

Answer Questions 1-3 on page 309 in the textbook and solve Problems 2 and 4 on page 310.

Period and frequency

Another important quantity is the time interval for one complete vibration. Consider the cart attached to a spring in Figure 10.2. The time interval for one complete vibration from $x = +A$ to $-A$ and back to $+A$ is called the **period** T of the vibration.

We are familiar with this quantity from our study of circular motion. For example, one day is the time interval (the period) for Earth to make one complete rotation on its axis—the time interval for the appearance of the Sun above the horizon on two consecutive mornings.

Period The period T of a vibrating object is the time interval needed for the object to make one complete vibration—from the clock reading when it passes through a position while moving in a certain direction until the next clock reading when it passes through that *same* position moving in the *same* direction. The SI unit of period is the second.

The number of vibrations each second is called the **frequency** f of the vibration. If an object makes 4 vibrations each second, the frequency is 4 vib/s and the period is 0.25 s. The most commonly used frequency unit is the hertz (Hz), where 1 hertz equals 1 vibration per second: $1 \text{ Hz} = 1 \text{ vib/s}$ (or 1 cycle/s). The words vibration and cycle indicate what is being counted but are not themselves units. These words (cycle and vibration) can be removed from the units in equations involving frequency. Thus, an appropriate unit for frequency is $1/\text{s}$ or s^{-1} .

1. What are the features that make vibrational motion *different* from circular motion? Choose all that apply.
 - (a) Vibrational motion is periodic.
 - (b) Vibrational motion repeats itself.
 - (c) During vibrational motion, there is a periodic change in the form of system energy.
 - (d) Vibrational motion has a specific equilibrium point through which the system passes from different directions.
2. What does it mean if the amplitude of an object's vibration is constant and equal to 0.10 m?
 - (a) The object's maximum distance from the starting point is 0.10 m.
 - (b) The object's displacement from the equilibrium position is ± 0.10 m.
 - (c) The magnitude of the object's maximum displacement from the equilibrium position is 0.10 m.
3. What does it mean if the period of an object's vibration equals 0.60 s?
 - (a) The motion lasts for 0.60 s.
 - (b) The time interval needed for the object to move between the two farthest points from the equilibrium position is 0.60 s.
 - (c) The time interval needed for the object to move through the same point in the same direction is 0.60 s.

2. * You have a ball bearing and a bowl. You let the ball roll down from the top of the bowl; it moves up and down the bowl's wall for some time.
(a) Is this vibrational motion? Explain why or why not. (b) Draw a force diagram for the ball for four different positions and indicate what force or force component is responsible for the acceleration toward the equilibrium position. (c) Draw energy bar charts for these four ball-Earth system positions.
3. Draw a sketch of a pendulum. Indicate the equilibrium position. Show this object vibrating and indicate the amplitude of vibration on the sketch.
4. * (a) Draw a sketch of an object attached to a vertical spring. Indicate the equilibrium position. (b) Show this object vibrating up and down and indicate the amplitude of vibration on the sketch. (c) Describe an experiment you would perform to determine the period of vibration of the object. (d) Perform the experiment. Include experimental uncertainty in your result.

Team 1 OALG 10.2.1

- a) They are consistent with the motion diagram we drew, e.g., they started the maximum position (maximum a), and zero velocity.
- b) They are consistent, e.g., when the slope of the x vs t graph is zero, the v is zero in v vs t graph.

We want students ...

Team 2 OALG 10.2.1

- a) Yes. Even our choice of positive being to the right is consistent with the graphs.
- b) It is. The slope of the $x(t)$ graph is changing (from 0 to negative, back to zero, to positive, ...)
- c) Yes, those two always point in the same direction. If we define positive towards the right and put the card on the far right point, the restoring force is maximum and negative.
- d) The force and acceleration point in the same direction, so if force has a - ($F(\text{SoC}) = -kx$), the acceleration graph will also always mirror the displacement graph.
- e) sine/cosine
- f) it's the same for all 3

Team 3 OALG 10.2.1

- (a) When $x(t)$ is maximum, $v(t)$ is zero. When $x(t)$ is zero, $v(t)$ is at a positive or negative maximum (depending on which way the cart is moving through the equilibrium position). When $x(t)$ is minimum, $v(t)$ is zero. The slope of the $x(t)$ graph is equal to the $v(t)$ graph, which makes the graphs consistent with each other and the motion diagrams.
- (b) Answered in part (a).
- (c) Yes, the force and acceleration are consistent with each other because they have the same sign (or direction) at all times.
- (d) When the cart is displaced in the positive direction, the force that the spring exerts on the cart is pointed in the negative direction. These two quantities are always pointed in opposite directions and this is consistent with Hooke's Law.
- (e) A sinusoidal function would work because the graphs resemble what a sinusoidal function would look like.
- (f) The period is the same for all 3 graphs.

Team 4 OALG 10.2.1

- a. They are consistent because at the furthest most position the acceleration is the greatest and velocity is zero, and at the equilibrium position the acceleration is zero and the velocity is the greatest
- b. Yes at the maximum and minimum position the slope is zero and the velocity at those points is zero. Slope of the position-time plot is the steepest when velocity is the greatest.
- c. The acceleration is the greatest when we are at the farthest points and the direction is changing.
- d. The force is proportional to the distance from the equilibrium point and the negative sign so that the acceleration (acceleration is in same direction as force, $a = \text{sum of } F/m$) will be opposite in direction to the displacement and will mirror it..
- e. The function of the position-time graph is proportional to a cosine function
- f. Two lines (when the position starts at most positive position and gets back to the most positive position with velocity in same direction as before)

All together OALG 10.2.2 and 10.2.3

Talking: Eugenia USA

$$x = A \cos \theta = A \cos \left(\frac{2\pi}{T} t \right)$$

$$T; t$$

$$0; T/2$$

$$v = -\frac{2\pi A}{T} \sin \left(\frac{2\pi}{T} t \right)$$

$$T; R=A$$

$$\cancel{a} = -\frac{4\pi^2}{T^2} x$$

$$a = -\frac{kx}{m}$$

$$\frac{4\pi^2}{T^2} = \frac{k}{m} \quad \boxed{T = 2\pi \sqrt{\frac{m}{k}}}$$

$$a = -\frac{4\pi^2}{T^2} A \cos \left(\frac{2\pi}{T} t \right)$$

$$T = 2\pi \sqrt{\frac{m}{k}}$$

$m \rightarrow 0$
 $k \rightarrow \infty$

$$\frac{\text{kg} \cdot \text{s}^2}{\text{kg} \cdot \text{m}}$$

$a = \text{const}$ — descr.

$\Sigma F = \text{const}$ — expl.

$x = A \cos\left(\frac{2\pi}{T} t\right)$ — D. d.

$F = -kx$ — G/e

All together

OALG 10.2.4 Reason

- a.** Use your results from Activity 10.2.3 to write mathematical representations for $x(t)$, $v(t)$, and $a(t)$ as cosine or sine functions of time. (In your expressions, try to use the quantities amplitude A and period T .) How do you know whether the representations you wrote make sense?
- b.** Compare and contrast your representations with Equations 10.2; 10.3 and 10.4 on pages 290-291 in the textbook.

Team 1 OALG 10.2.5

Team 2 OALG 10.2.5

Team 3 OALG 10.2.5

Team 3 OALG 10.2.5

Possible project or a lab

OALG 10.2.7

Team 1 OALG 10.3.1

Team 2 OALG 10.3.1

Team 3 OALG 10.3.1

Team 4 OALG 10.3.1

All together OALG 10.3.2

The goal of this activity is to help you derive an expression for the period of the cart's vibration at the end of the spring shown in Activity 10.1.3. Begin the derivation by answering two questions (part **a.** and part **b.**).

- a.** What physical quantities might affect the period?
- b.** Describe experiments that you could perform to decide whether these quantities do in fact affect the period.

Now we move to the actual step-by-step derivation (parts **c.**–**e.**).

- c.** Write down Newton's second law for the cart ($a = \dots$)
- d.** Substitute your expressions for $x(t)$ and $a(t)$ into the Newton's second law equation you came up with in Activity 10.2.4.
- e.** Come up with an expression for T that is necessary for the mathematical representation in part **d.** to be true.

All together ALG 10.3.3 - discuss -

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10.3.3 Test an idea

PIVOTAL Lab: *Equipment per group:* whiteboard, markers, ring stand, C-clamp, spring, small objects of different masses (to hang from the spring), hanger, meter stick, digital scale, motion sensor, stopwatch, tape, force sensor.

Design an experiment to test the validity of the mathematical expression for T that you just created. Work with your group to address the following points:

- a. Devise and write a brief outline of the experimental procedure you will use to determine the spring constant of the spring. Include a labeled sketch.
- b. Include the mathematical procedure you will use to calculate the spring constant from the data you record. What assumptions did you make in the mathematical procedure? Which of them is the most questionable? Decide whether this assumption is valid. This may require performing a small side experiment. Yes, you should write this up in your report.
- c. Brainstorm: Now that you know the spring constant of your spring, how will you test the validity of the mathematical relationship you just derived? The values of which measurable quantities could you predict using the equation you're testing?
- d. What are the sources of experimental uncertainty? How will you minimize these uncertainties?
- e. Perform your experiment.
- f. Report the results of your experiment in appropriate formats (numbers, tables, graphs, etc...) Did the outcome match your prediction within experimental uncertainties?
- g. What conclusion can you draw about the validity of the mathematical relationship you're testing?
- h. Your friend is confused – the expression for the period that you derived does not have the amplitude in it. Shouldn't the period depend on the amplitude? How would you test his idea? Design and experiment, make the prediction based on your friend's idea, then conduct the experiment and compare the result to the prediction. What is your judgment concerning your friend's idea?

Team 1 OALG 10.4.1

Team 2 OALG 10.4.1

Team 3 OALG 10.4.1

Team 4 OALG 10.4.1

Team 1 OALG 10.5.1

Team 2 OALG 10.5.1

Team 3 OALG 10.5.1

Team 4 OALG 10.5.1

All together OALG 10.8.2 and connect it to the need to know

I learned how to derive the x , v , and a equations without using calculus!

What are the most important ideas that I learned today?

I work with these two topics, I learned conceptual questions, I use some of them, but here I got a new ones, every workshop makes me feel it opens my head, or makes it bigger: new ideas, new representations, new connections... to give one example this time it was new to me to test the expression of period.

Drawing the motion diagram for activity 10.1.2 vertically was helpful

The inclusion of energy analysis is the thing I usually skip, but it should not be

The depth of comparing circular motion with vibrational motion (x -components of position/velocity/acc).

Having students observe periodic motion and then think about the relationships between position, velocity, acceleration can be very helpful to build those connections and relationships in their minds.

It's nice to see that the multiple representations (motion diagram, force diagram, graphs, energy..) come together for this unit.

I liked waiting to use any numbers and spending the time to qualitatively observe



Eugenia USA



Cecilia Argentina



Ting-Hui Lee (Kentucky, USA)



Andrew Yolleck (he/him/his)



Stephanie USA



Dorota SF Bay Area USA



Žan, Slovenia



Michelle USA



Anne Caraley (she/her) SUNY Oswego



Redentor, Philippines



Jose Salcido, USA



Frank (USA)



Yuehai (John) Yang (US)