

Domain and Range

Domain means the values that x can take, and **Range means the values that y can take**. That's it!

Ok. Now, how do we use this?

Math uses several different ways to say the same thing in some key areas. I will give you a table that will help you keep things straight.

x	y
Domain	Range
Independent variable	Dependent variable
Input	Output
Explanatory variable	Response variable
Time	Anything else

That last one is for help deciding which is the “ x ” variable and which is the “ y ” variable in a word problem. Sometimes it doesn't matter which one is x and which one is y , but if the problem involves time, time is always x .

When you are unsure which variable is the x and which is the y , the easiest way to decide is to ask yourself a question based on the variables in the problem.

Example: Does water pressure depend on depth, or does depth depend on water pressure?

It is more common to measure the water pressure based on depth than it is to measure depth based on the water pressure. Therefore, you would make depth your x and pressure your y

Example: Does temperature depend on your altitude (on a mountain), or does your altitude depend on the temperature?

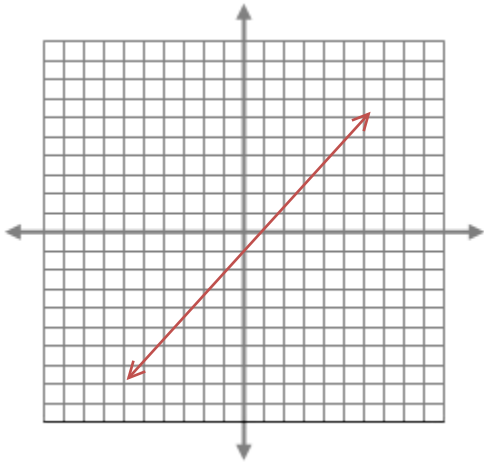
You are going to get a more accurate estimate of the temperature based on your altitude than you would if you tried to estimate your altitude based on the temperature. Weather such as rain or snow would drop the temperature, but if the storm cleared quickly, the temperature would rise. But tables and books list an average temperature for a location based on altitude. So your x -axis would be altitude and your y -axis would be temperature.

Example: Does the population of a city depend on how long the city has existed, or does the current date determine the population?

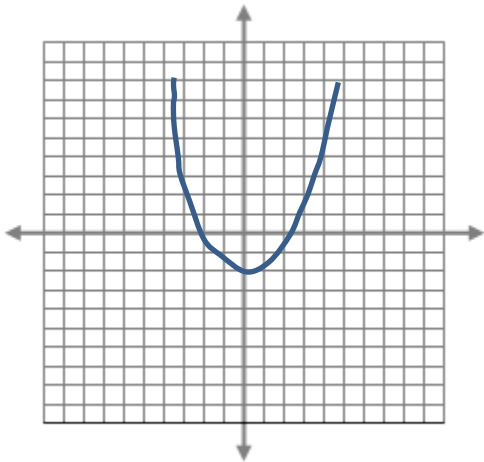
The population for most cities grows over time, so it would be more accurate to guess the population of a city based on the year rather than guessing what year it is based on the population of the city. In this case you must be given a base year (for example, the city had a population of 10,000 people in 1990) and a growth rate (for example, the city grows at a rate of 500 people per year).

NOTE: Time is almost always the x variable, so that would be another indication to put the year as the x variable.

With *regular straight lines*, the domain and range are both the set of real numbers. In other words, there are no restrictions on x or y . They can be any number in the universe. In the graph below, it is clear that both x and y go on forever in both directions. The mathematical shortcut for writing domain and range for this graph looks like this: $\{x \in \mathbb{R}, y \in \mathbb{R}\}$ which literally translates to “ x is in the set of all real numbers, y is in the set of all real numbers.” You can also say, “Both the domain and the range are the set of all real numbers.”

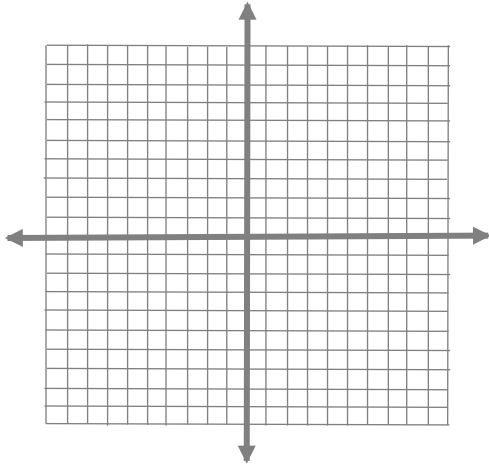


With other graphs, there may be restrictions. For example: $y = x^2 - 2$ looks like this:



While x still has no restrictions, y can never drop below -2 . So here the domain is still the set of real numbers, but the range is $y \geq -2$. This is also written $\{x \in \mathbb{R}, y \geq -2\}$. You can also say, “The domain is the set of all real numbers, and the range is the set of all real numbers greater than or equal to negative two.”

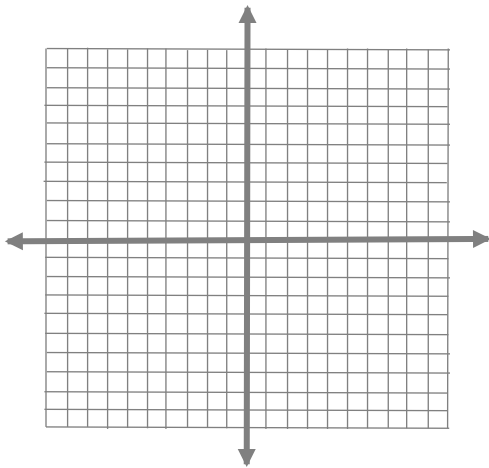
Sometimes you have restrictions because of a square root. For example: $y = \sqrt{x} + 2$.



In this case, x cannot be smaller than 0, and y cannot be less than 2, since you cannot have the square root of a negative number. So the domain and range look like this: $\{x \geq 0, y \geq 2\}$. You can also say, "The domain is all real numbers greater than or equal to zero, and the range is all real numbers greater than or equal to 2." It just takes longer. 😊

Sometimes you have a restriction because you have a variable in the denominator of a fraction.

For example: $y = \frac{4}{x}$



This time, since you cannot have 0 in the denominator of a fraction, there is an asymptote at both $x = 0$ and $y = 0$, so the domain and range look like this:

$$\{x \in \mathbb{R}, x \neq 0; y \in \mathbb{R}, y \neq 0\}$$

You can also say, "The domain is all real numbers except zero, and the range is all real numbers except zero."