Differential Equations and tangent lines 2 Name: Answers

AP Calculus

1) Consider the differential equation $\dfrac{dy}{dx} = \dfrac{3-x}{y}$

a) Let y = f(x) be the particular solution to the given differential equation for 1<x<5 such that the line y = -2 is tangent to the graph of f. Find the x-coordinate of the point of tangency.

b) Let y = g(x) be the particular solution the given differential equation for -2<x<8, with the initial condition g(6) = -4. Find g(x).

c) Show that $\frac{dy}{dx} = \frac{3-x}{y}$ given g(x) is what you got in part b.

a) slope of f(x) at $y=-2$ is zero.

$$0 = \frac{3-x}{-2} \qquad \text{At } (3,-2) \quad f'(x) = \frac{dy}{dx} = 0$$

$$0 = 3 - x$$

$$\boxed{x = 3}$$

b) $\int y\,dy = \int (3-x)\,dx$

$$\frac{1}{2}y^2 = 3x - \frac{1}{2}x^2 + C_1$$

$$y^2 = 6x - x^2 + C_2$$

$$y = \pm\sqrt{6x - x^2 + C_2}$$

$$(6,-4):\ -4 = -\sqrt{6(6) - (6)^2 + C_2}$$

$$16 = C$$

$$\boxed{g(x) = -\sqrt{6x - x^2 + 16}}$$

c) $g(x) = -(6x - x^2 + 16)^{\frac{1}{2}}$

$$g'(x) = -\frac{1}{2}(6x - x^2 + 16)^{-1/2}(6 - 2x)$$

$$= -\frac{3-x}{\sqrt{6x - x^2 + 16}} = \frac{3-x}{y}$$

since $y = -\sqrt{6x - x^2 + 16}$

2) Consider the differential equation $\frac{dy}{dx} = e^y\left(3x^2 - 6x\right)$. Let $y = f(x)$ be the particular solution to the differential equation that passes through $(1, 0)$.

(a) Write an equation for the line tangent to the graph of f at the point $(1, 0)$. Use the tangent line to approximate $f(1.2)$.

(b) Find $y = f(x)$, the particular solution to the differential equation that passes through $(1, 0)$.

a) slope of tangent line at $(1,0)$ $= e^0(3(1)^2 - 6(1))$

$= -3$

equation of line: $y = -3(x-1)$ or $y = -3x+3$

approx. of $f(1.2) = -3(1.2)+3 = -.6$

b) $\frac{dy}{e^y} = (3x^2 - 6x)\,dx$

$\int e^{-y}\,dy = \int (3x^2 - 6x)\,dx$

$-1e^{-y} = x^3 - 3x^2 + C_1$

$e^{-y} = -x^3 + 3x^2 + C_2$

$-y = \ln(-x^3 + 3x^2 + C_2)$

$y = -\ln(-x^3 + 3x^2 + C_2)$

given $(1,0)$

$0 = -\ln(-1+3+C)$

$0 = \ln_e(2+C)$

$e^0 = 2+C$

$-1 = C$

so

$f(x) = -\ln(-x^3+3x^2-1)$

$\boxed{f(x) = -\ln(-x^3+3x^2-1)}$

actual value of $f(1.2) = -.465$