

### Differential Equation with tangent line 2<sup>nd</sup> derivative

Do this free response question on a separate piece of paper and show clear work. Clearly label your answers. Remember: Don't ever use the word "it" and use notation whenever possible, such as  $W(t)$  or  $dW/dt$ , rather than unclear words like "the function" or "the rate". And remember that the notation in part

(b) is just Leibniz's notation for the 2<sup>nd</sup> derivative....  $\frac{d^2W}{dt^2}$  is just another way of writing  $W''(t)$

At the beginning of 2010, a landfill contained 1400 tons of solid waste. The increasing function  $W$  models the total amount of solid waste stored at the landfill. Planners estimate that  $W$  will satisfy the differential equation  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$  for the next 20 years.  $W$  is measured in tons, and  $t$  is measured in years from the start of 2010.

- (a) Use the line tangent to the graph of  $W$  at  $t = 0$  to approximate the amount of solid waste that the landfill contains at the end of the first 3 months of 2010 (time  $t = \frac{1}{4}$ ).
- (b) Find  $\frac{d^2W}{dt^2}$  in terms of  $W$ . Use  $\frac{d^2W}{dt^2}$  to determine whether your answer in part (a) is an underestimate or an overestimate of the amount of solid waste that the landfill contains at time  $t = \frac{1}{4}$ .
- (c) Find the particular solution  $W = W(t)$  to the differential equation  $\frac{dW}{dt} = \frac{1}{25}(W - 300)$  with initial condition  $W(0) = 1400$ .