Derivatives Worksheet
AP Physics C

For problems 1 – 8, find the derivative of *y* with respect to *x*.

1. $y = 5 \quad \frac{dy}{dx} = 0$

2. $y = x^4 \quad \frac{dy}{dx} = 4x^3$

3. $y = 7x^4 + 9x^3 + 3x + 2 \quad \frac{dy}{dx} = 28x^3 + 27x^2 + 3$

4. $y = (2x^3 - 4x^2)(3x^5 + x^2) \quad \frac{dy}{dx} = 48x^7 - 84x^6 + 10x^4 - 16x^3$

5. $y = \frac{3-2x}{3+2x} \quad \frac{dy}{dx} = \frac{-12}{3+2x\ \ ^2}$

6. $y = \frac{3}{x^5} \qquad \frac{dy}{dx} = -15x^{-6}$

7. $y = 3\sin x - 2\cos x \qquad \frac{dy}{dx} = 3\cos x + 2\sin x$

8. $y = 5\cos(3x+2) \qquad \frac{dy}{dx} = -15\sin\ \ 3x+2$

Find *v*(*t*) and *a*(*t*) for the following position vectors:

9. $\vec{r}(t) = (5t^2 + 3t)\hat{i} + 5\hat{j} \qquad v\ \ t\ \ =\ \ 10t+3\ \ \hat{i}\ ;\ a\ \ t\ \ = 10\hat{i}$

10. $\vec{r}(t) = 5t\hat{i} + (3t^3 - 5t^2)\hat{j} + (4t^2 - 6t + 4)\hat{k} \qquad v\ \ t\ \ = 5\hat{i} +\ \ 9t^2 - 10t\ \ \hat{j} +\ \ 8t - 6\ \ \hat{k}\ ;$
    $a\ \ t\ \ =\ \ 18t - 10\ \ \hat{j} + 8\hat{k}$

11. $\vec{r}(t) = (A\cos\omega t)\hat{i} + (A\sin\omega t)\hat{j} \qquad v\ \ t\ \ =\ \ -\omega A\sin\omega t\ \ \hat{i} +\ \ \omega A\cos\omega t\ \ \hat{j}\ ;$
    $a\ \ t\ \ =\ \ -\omega^2 A\cos\omega t\ \ \hat{i} +\ \ -\omega^2 A\sin\omega t\ \ \hat{j}$

Solve the following problems:

12. For the equation $y = x^2 - 4x + 3$, find

    a. The equation for the slope of its tangent line at any point $\quad \frac{dy}{dx} = 2x - 4$

    b. The equation of the tangent line at the point (4,3) using the point slope form. (*y-y′) = m(x - x′)* At $x = 4$, $m = 4$; $y - 3 = 4\ \ x - 4$ so $y = 4x - 13$

13. A point moves along the curve $y = x^3 - 3x + 5$ so that $x = \frac{t^2}{2} + 3$, where *t* is the elapsed time. At what rate is *y* changing (the *y* component of velocity) when *t* = 4 s? (hint: use the chain rule.)

$\frac{dy}{dx} = 3x^2 - 3$ and $\frac{dx}{dt} = t$ From the chain rule: $\frac{dy}{dt} = \left(\frac{dy}{dx}\right)\left(\frac{dx}{dt}\right)$ So

$\frac{dy}{dt} = \; 3x^2 - 3 \;\; t$ and since $x = \frac{t^2}{2} + 3$ the value of x at t = 4 is 11. Substituting x = 11 and t = 4 into the derivative function and simplifying, we get: $\frac{dy}{dt} = 1440\, {}^{m}\!/\!{}_{s}$

14. A particle undergoes straight-line motion with its displacement at any time given by the following equation: $x = 2t^3 - 4t^2 + 2t + 5$. Find:

    a. The times at which the particle is motionless. t = ⅓s, t = 1s

    b. The time at which it is moving to the right. when t > 1 and when t < ⅓

    c. The times at which it is moving to the left. when ⅓ < t < 1

    The particle will be motionless when the velocity = 0; $\frac{dx}{dt} = 6t^2 - 8t + 2$ Set equal to 0 and solve for t. Alternatively, plot the velocity function to find the values for t when v = 0

    The particle will move right when v is positive and left when v is negative.