Show all work for each problem.

①

x	$f(x)$	$g(x)$	$f'(x)$
−4	0	−9	5
−2	4	−7	4
0	6	−4	2
2	7	−3	1
4	10	−2	3

The table above gives values of the differentiable functions f and g, and f', the derivative of f, at selected values of x. If $g(x) = f^{-1}(x)$, what is the value of $g'(4)$?

(A) $-\frac{1}{3}$ (B) $-\frac{1}{4}$ (C) $-\frac{3}{100}$ (D) $\frac{1}{4}$ (E) $\frac{1}{3}$

$$g'(4) = \frac{1}{f'(g(4))} = \frac{1}{f'(-2)} = \frac{1}{4}$$

② Let f be the function defined by $f(x) = x^3 + x$. If $g(x) = f^{-1}(x)$ and $g(2) = 1$, what is the value of $g'(2)$?
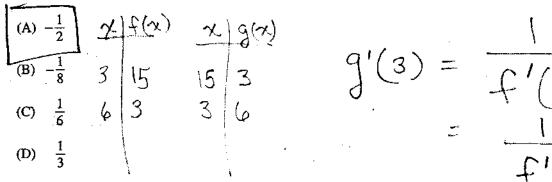
(A) $\frac{1}{13}$ (B) $\frac{1}{4}$ (C) $\frac{7}{4}$ (D) 4   (E) 13

$$g'(2) = \frac{1}{f'(g(2))}$$

$$= \frac{1}{f'(1)}$$

$$= \frac{1}{4}$$

x	$f(x)$
1	2

x	$g(x)$
2	1

$2 = x^3 + x$ ✓

$f'(x) = 3x^2 + 1$

$f'(1) = 3(1)^2 + 1 = 4$

over →

③ Let f be a differentiable function such that $f(3) = 15$, $f(6) = 3$, $f'(3) = -8$, and $f'(6) = -2$. The function g is differentiable and $g(x) = f^{-1}(x)$ for all x. What is the value of $g'(3)$?

(A) $-\dfrac{1}{2}$

(B) $-\dfrac{1}{8}$

(C) $\dfrac{1}{6}$

(D) $\dfrac{1}{3}$

(E) The value of $g'(3)$ cannot be determined from the information given.

x	$f(x)$
3	15
6	3

x	$g(x)$
15	3
3	6

$$g'(3) = \frac{1}{f'(g(3))}$$

$$= \frac{1}{f'(6)}$$

$$= -\frac{1}{2}$$

④ The functions f and g are differentiable, and $f(g(x)) = x$ for all x. If $f(3) = 8$ and $f'(3) = 9$, what are the values of $g(8)$ and $g'(8)$?

$f(x)$ and $g(x)$ are inverses.

(A) $g(8) = \dfrac{1}{3}$ and $g'(8) = -\dfrac{1}{9}$

(B) $g(8) = \dfrac{1}{3}$ and $g'(8) = \dfrac{1}{9}$

(C) $g(8) = 3$ and $g'(8) = -9$

(D) $g(8) = 3$ and $g'(8) = -\dfrac{1}{9}$

(E) $g(8) = 3$ and $g'(8) = \dfrac{1}{9}$

x	$f(x)$
3	8

x	$g(x)$
8	3

$$g(8) = 3$$

$$g'(8) = \frac{1}{f'(g(8))} \Rightarrow \frac{1}{f'(3)} = \boxed{\frac{1}{9}}$$