Definite Integrals as Accumulated Change
AP Calculus

Name: Answers

Remember this important concept:

Accumulated (or total or net) change is given by the definite integral whose integrand is the rate of change. More specifically, if f is the rate of change of F, then:

$$\int_a^b f(t)\,dt = \text{Change in } F \text{ from } t=a \text{ to } t=b = F(b)-F(a).$$

Using this knowledge, write a sentence to answer each of the following questions.

1. If $h(t)$ is the rate of change of the height of a conical pile of sand measured in feet per hour, what does $\int_0^5 h(t)\,dt$ represent? Answer in correct units. $\int_0^5 h(t)\,dt$ represents the amount of sand, i.e its height in feet, accumulated in the pile from 0 to 5 hours.

2. If $v(t)$ is the velocity of a particle moving along the x-axis, measured in feet per second, what does $\int_3^{10} v(t)\,dt$ represent? Answer in correct units. $\int_3^{10} v(t)\,dt$ represents the displacement or change in position from 3 to 10 seconds, in feet

3. If $b(t)$ is the rate of growth of the number of bacteria in a dish, measured in number of bacteria per hour, what does $\int_2^6 b(t)\,dt$ represent? Answer in correct units.

$\int_2^6 b(t)\,dt$ represents the net change in the number of bacteria from 2 to 6 hours.

4. If $v(t)$ is the velocity of a particle moving along the x-axis at time t, and the position $x(t)$ is 5 at time $t=2$, (a) write an integral expression that represents the position of the particle at time $t=10$, and (b) write an integral expression that gives the total distance traveled by the particle from time $t=2$ to time $t=10$.

a) $p(t) = 5 + \int_2^{10} v(t)\,dt$ b) total dist from 2 to 10 $= \int_2^{10} |v(t)|\,dt$

5. If $p(t)$ is the rate of growth of a rabbit population, measured in rabbits per year, and there were 100 rabbits in the year 2005 ($t=0$), write an integral expression that represents the rabbit population in 2007.

$$\text{population} = 100 + \int_0^2 p(t)\,dt$$