

Chapter 3

Laws of Algebra

- Laws of Exponents
- The Distributive Law
- The product $(a + b)(c + d)$
- Difference of two squares
- Perfect squares expressions
- Further expansion

$$a^m \times a^n = a^{m+n}$$

To **multiply** numbers with the **same base**, keep the base and **add** the exponents.

$$\frac{a^m}{a^n} = \frac{a^{m-n}}{1}, \quad a \neq 0$$

To **divide** numbers with the **same base**, keep the base and **subtract** the exponents.

$$(a^m)^n = a^{m \times n}$$

When **raising a power** to a **power**, keep the base and **multiply** the exponents.

$$(ab)^n = a^n b^n$$

The power of a product is the product of the powers.

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, \quad b \neq 0$$

The power of a quotient is the quotient of the powers.

$$a^0 = 1, \quad a \neq 0$$

Any non-zero number raised to the exponent zero is 1.

$$a^{-n} = \frac{1}{a^n} \quad \text{and} \quad \frac{1}{a^{-n}} = a^n \quad \text{and in particular} \quad a^{-1} = \frac{1}{a}, \quad a \neq 0.$$

$$9^{-3} = \frac{1}{9^3}$$

Simplify: $p^3 \times p^8 = p^{11}$

$3^5 \times 3^4 = 3^9$

Simplify: $\frac{x^7}{x^3} = x^{7-3} = x^4$

$\frac{\cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} \cancel{x} x}{\cancel{x} \cancel{x} \cancel{x}}$

$\frac{b^2}{b^9} = b^{2-9} = b^{-7} = \frac{1}{b^7}$

Simplify: $(c^3)^5$
 c^{15}

$(3d^4)^3$
 $3^3 d^{12}$
 $27d^{12}$

Simplify using $a^m \times a^n = a^{m+n}$

a $k^4 \times k^2$
 k^6

b $5^2 \times 5^6$
 5^8

c $d^3 \times d^7$
 d^{10}

d $11^4 \times 11^a$
 11^{4+a}

Simplify using $\frac{a^m}{a^n} = a^{m-n}$

e $\frac{k^{12}}{k^a}$
 k^{12-a}

f $\frac{y^6}{y}$
 y^5

g $\frac{t^m}{t^4}$
 t^{m-4}

h $\frac{x^{3a}}{x^2}$
 x^{3a-2}

Simplify using $(a^m)^n = a^{m \times n}$

a $(5^3)^2$
 5^6

b $(c^4)^3$
 c^{12}

c $(3^8)^4$
 3^{32}

d $(v^5)^5$
 v^{25}

Simplify

a $b^5 \times b^7$

b^{12}

b $\frac{t^9}{t^2}$

t^7

c $(p^6)^3$

p^{18}

d $\frac{7^6}{7^n}$

7^{6-n}

e $(x^{2s})^3$

x^{6s}

f $d^k \div d^3$

d^{k-3}

g $3^2 \times 3^7 \times 3^4$

3^{13}

h $(j^4)^{3x}$

j^{12x}

i $11^6 \times 11^1$

11^7

j $\frac{z^7}{z^{4t}}$

z^{7-4t}

k $(13^c)^{5d}$

13^{5cd}

l $w^{7p} \div w^1$

w^{7p-1}

Write as powers of 2:

$\frac{16}{1} = 2^4$

$\frac{1}{16} = \frac{1}{2^4}$

$4 \times 2^n = 2^2 \times 2^n = 2^{2+n}$

$\frac{2^m}{8} = \frac{2^m}{2^3} = 2^{m-3}$

Write as powers of 2:

a $2^1 \times 2^a$

2^{1+a}

b 4×2^b

$2^2 \cdot 2^b = 2^{2+b}$

c 8×2^t

$2^3 \cdot 2^t = 2^{3+t}$

f $\frac{2^c}{4} = \frac{2^c}{2^2}$

$= 2^{c-2}$

g $\frac{2^m}{2^{-m}}$

$2^{m-(-m)} = 2^{m+m} = 2^{2m}$

h $\frac{4}{2^{1-n}}$

$= \frac{2^2}{2^{1-n}} = 2^{2-(1-n)} = 2^{2-1+n} = 2^{1+n}$

Write as powers of 3:

a 9×3^p $3^2 \cdot 3^p$ 3^{2+p}
 b 27^a $(3^3)^a$ 3^{3a}
 c $3 \times 9^n = 3 \cdot (3^2)^n$ $3^1 \cdot 3^{2n} = 3^{1+2n}$
 f $\frac{3^y}{3}$ 3^{y-1}
 g $\frac{3}{3^y}$ 3^{1-y}
 h $\frac{9}{27^t}$ $\frac{3^2}{(3^3)^t} = \frac{3^2}{3^{3t}} = 3^{2-3t}$

Write without brackets

$(4x)^3$ $4^3 x^3$ $64x^3$
 $(\frac{x^3 y^4}{z^2})^2$ $\frac{x^6 y^8}{z^4}$

a $(2a)^2$ $4a^2$
 b $(3n)^2$ $9n^2$
 c $(5m)^3$ $125m^3$
 e $(\frac{a}{2})^3 = \frac{a^3}{2^3} = \frac{a^3}{8}$ f $(\frac{3}{m})^2 = \frac{3^2}{m^2} = \frac{9}{m^2}$ g $(\frac{p}{q})^4 = \frac{p^4}{q^4}$

Write without negative exponents

$\frac{x^{-4}}{1}$ $\frac{1}{x^4}$
 5^{-2} $\frac{1}{5^2} = \frac{1}{25}$
 $(\frac{x}{2})^{-3} = \frac{x^{-3}}{2^{-3}} = \frac{1}{x^3} \cdot \frac{8}{1} = \frac{8}{x^3}$

Simplify

$$3x^{-2}y^5 \cdot 4^{-2}x^6y^{-8}$$

$$\frac{3y^5x^6}{x^24^2y^8}$$

$$\frac{3x^6y^5}{16x^2y^8}$$

$$3 \cdot 4^{-2} \cdot x^{-2+6} \cdot y^{5+(-8)}$$

$$3 \cdot 4^{-2} x^4 y^{-3}$$

$$\frac{3x^4}{16y^3}$$

$$\frac{3x^6y^5}{16x^2y^8} = \frac{3x^{6-2}y^{5-8}}{16} = \frac{3x^4y^{-3}}{16} = \left(\frac{3x^4}{16y^3} \right)$$

$$\frac{m^{-3}n^5p^2}{m^3n^8p^{-4}} = \frac{n^5p^6}{m^6}$$

$$\left(\frac{4x^2y^{-3}}{3x^5y^{-6}} \right)^2$$

$$\frac{4^2x^4y^{-6}}{3^2x^{10}y^{-12}} = \frac{16y^6}{9x^6}$$

$\frac{y^{12}}{\sqrt{6}}$

$$\left(\frac{2x^5y^4}{5x^{-3}y^{-2}} \right)^{-3} = \left(\frac{5x^{-3}y^{-2}}{2x^5y^4} \right)^3 = \frac{5^3x^{-9}y^{-6}}{2^3x^{15}y^{12}}$$

$$\frac{2^{-3}x^{-15}y^{-12}}{5^3x^9y^6}$$

$$= \frac{125}{8x^{24}y^{18}}$$