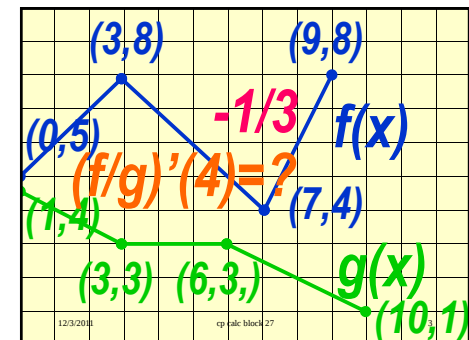
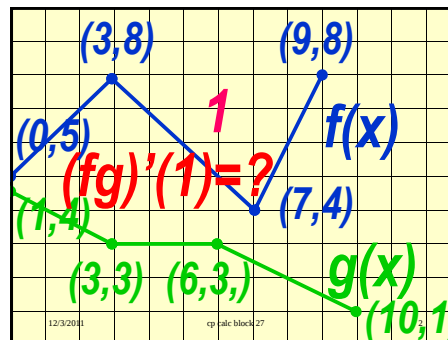


hw presentation
147/19;33;49;61;63

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cp calculus
block 27
hw p148, odds,
67-91;97-121

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Emmy DeMarco's Lilly

Ch 3.3(II)
Product/Quotient Rules and
Higher-Order Derivatives

Example 1) When a rock falls from rest near the surface of the earth, the distance it covers during the first few seconds is given by the

Example 1 equation

$s = 4.9t^2$

Find and graph $\frac{ds}{dt}$ and $\frac{d^2s}{dt^2}$

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$$a = \frac{dv}{dt} \quad v = \frac{ds}{dt} = -9.8t$$

$$= \frac{d^2s}{dt^2} = -9.8$$

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Objective

Rules of finding the velocity and acceleration of a moving particle given the position function

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Rules of finding the velocity and acceleration
 $s(t)$ = positive function
 of the moving particle

$$v = \frac{ds}{dt}; a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

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Example 2: Given the position $s=f(t)$ of a body moving along a coordinate line, find the velocity and acceleration at time

$$s = \sin t + \cos t \quad t = \frac{\pi}{4}$$

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$$\begin{aligned} s &= \sin t + \cos t \\ \text{velocity : } s' &= \cos t - \sin t \\ &= \cos\left(\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{4}\right) = 0 \end{aligned}$$

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$$\begin{aligned} s' &= \cos t - \sin t \\ \text{acceleration : } s'' &= -\sin t - \cos t \\ &= -\sin\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{4}\right) = -\sqrt{2} \end{aligned}$$

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Example 3) Find the tangent to the "Witch of Agnesi" $8x + 2y = 4$
 $y = \frac{4}{4 + x^2}$
 at the point $(2, 1)$

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Example 4a) Find the tangents to the curve $y = x^3 + x$
 $y = 4x \pm 2$
 at the points where the slope is 4.

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Example 4b) What is the smallest slope on the curve? $y = x^3 + x$
 $m = 1$ at $x = 0$

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Example 5 Find f' , f''

$$y = -\frac{1}{x}$$

$$y' = \frac{1}{x^2}, y'' = -\frac{2}{x^3}$$

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Objective
Higher-ordered Derivatives of functions

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Example 6 Find f' , f''

$$y = \frac{12}{x} - \frac{4}{x^3} + \frac{1}{x^4}$$

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$$y' = -\frac{12}{x^2} + \frac{12}{x^4} - \frac{4}{x^5}$$

$$y'' = \frac{24}{x^3} - \frac{48}{x^5} + \frac{20}{x^6}$$

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Example 7 Find f''

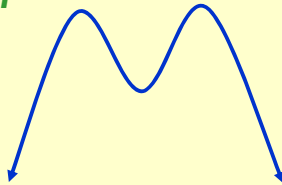
$$y = e^t \sin t$$

$$f'' = 2e^t \cos t$$

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Example 8 Sketch f' and f'' 

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Example 9 Develop a general rule for $f^{(n)}$.

$$y = \frac{1}{x}$$

$$f^{(n)} = (-1)^n n! x^{-(n+1)}$$

Example 10 Verify

$$y = 2x^3 - 6x + 10$$

$$-y''' - xy'' - 2y' = -24x^2$$

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Lesson quiz 1: Given the position $s=f(t)$ of a body moving along a coordinate line, find the **velocity** and **acceleration** at time $t = \frac{\pi}{4}$

$$s = 2 - 2 \sin t$$

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$$s = 2 - 2 \sin t$$

$$\text{velocity : } s' = -2 \cos t$$

$$= -2 \cos\left(\frac{\pi}{4}\right) = -\sqrt{2}$$

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$$s' = -2 \cos t$$

$$\text{acceleration : } s'' = 2 \sin t$$

$$= 2 \sin\left(\frac{\pi}{4}\right) = \sqrt{2}$$

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Lesson quiz 2: Find f' ,

$$f'' = -\frac{3}{x^7}$$

$$y' = \frac{21}{x^8}, y'' = -\frac{168}{x^9}$$

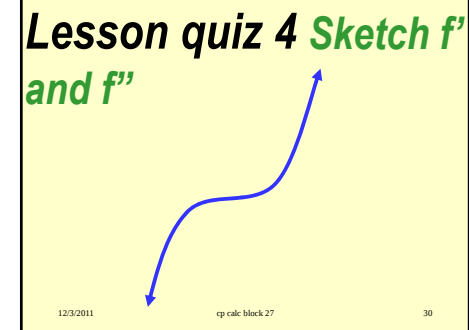
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Lesson quiz 3 Find f''

$$y = \sec x$$

$$f'' = \sec x (\sec^2 x + \tan^2 x)$$

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Lesson quiz 5 Develop a general rule for

$$xf^n + nf^{n-1}$$

$$\{xf(x)\}^n$$

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Lesson quiz 6: Find the equations of the horizontal tangents to the graph of $f(x)$

$$y = \tan x + 3 \cot x - 3$$

for $0 \leq x \leq \frac{\pi}{2}$

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$$y = \tan x + 3 \cot x - 3$$

$$y' = \sec^2 x - 3 \csc^2 x = 0$$

$$\tan x = \pm \sqrt{3}$$

$$\Rightarrow x = \frac{\pi}{3}$$

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Lesson quiz 7: Does the graph of $y=f(x)$ have any horizontal tangents in the interval $0 \leq x \leq 2\pi$ for $y = x + \sin x$

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$$y' = 1 + \cos x$$

$$y'(0) = 2, y'(2\pi) = 2$$

yes, at $x = \pi$

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Lesson quiz 8) Find the x- and y-intercepts of the line that is tangent to the curve of $y = x^3 - \frac{4}{3}, 16$ at the point $(-2, -8)$

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Lesson quiz 9: Does the graph of $y=f(x)$ have any horizontal tangents in $0 \leq x \leq 2\pi$ the interval for $y = x + \cos x$

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$y' = 1 - \sin x$
 $y'(0) = 1, y'(2\pi) = 1$
yes, horizontal
tangent at $x = \frac{\pi}{2}$

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$$y\left(\frac{\pi}{3}\right) = \tan\left(\frac{\pi}{3}\right) + 3\cot\left(\frac{\pi}{3}\right) - 3$$

$$= \sqrt{3} + 3\frac{\sqrt{3}}{3} - 3 = 2\sqrt{3} - 3$$

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