CP Calculus
Block 19
HW-Review p88,
84;90;96;108;114;
120;126;

11/2/2012 CP Calculus Block 19 1

CP Calculus
Block 19
HW p98,x3's
3-48

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Ch 2.4
Continuity and
One-sided Limit

Do Now:

$$f(x)=\begin{cases}\dfrac{x-2}{x-1}, & x\le 0\\[2mm] \dfrac{1}{x^2}, & x>0\end{cases}$$

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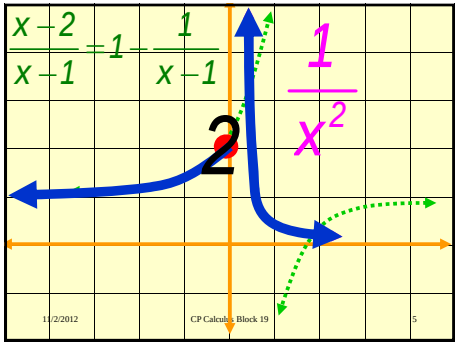

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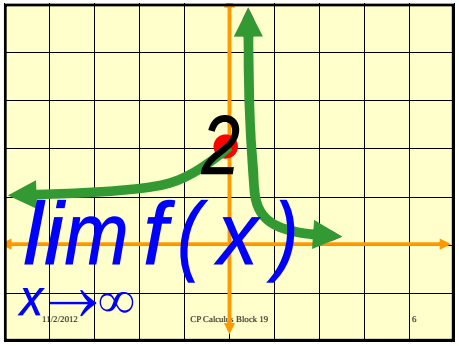

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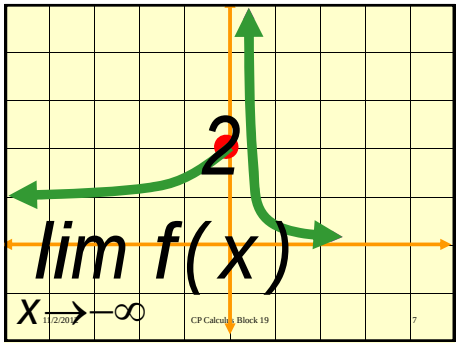

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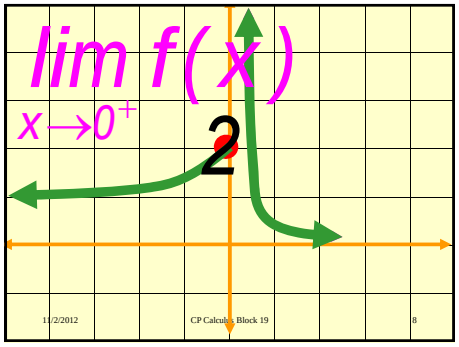

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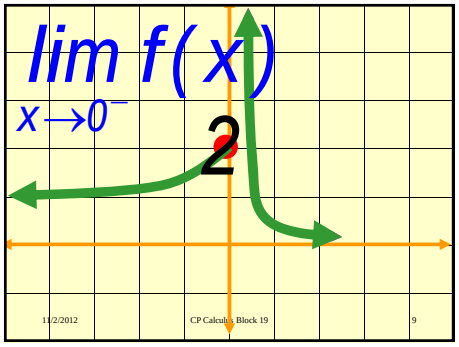

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Objective 1:

Determine the continuity of the functions

Definition A:

Continuity at an interior point

$$\lim_{x \to c} f(x) = f(c)$$

Definition B:

Continuity on closed interval [a,b], left endpoint a or right endpoint b

$$\lim_{x \to a^+} f(x) = f(a)$$

$$\lim_{x \to b^-} f(x) = f(b)$$

Definition C: A function is continuous if it is continuous at each point of its domain

The Continuity Test

The function $y=f(x)$ is continuous at $x=c$ if and only if all of the following are true:

i) $f(c)$ exitsts

ii) $\lim_{x \to c} f(x)$ exists

iii) $\lim_{x \to c} f(x) = f(c)$

Existence of Limit

$$\lim_{x \to c^+} f(x) = \lim_{x \to c^-} f(x) = L$$

Properties of Continuity I

Scalar Multiple: $bf(x)$

Properties of Continuity ii

Sum or difference

$f + g$, or $f - g$

Properties of Continuity iii

Product

$f(x)g(x)$

Properties of Continuity iv

Quotient

$f(x)/g(x)$ $g(x) \neq 0$

Continuity of Composite Functions

$f(g(x))$ or $g(f(x))$

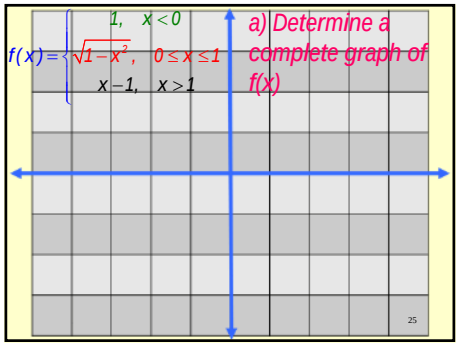
Theorem: If f is continuous at c and g is continuous at $f(c)$, then the composite $g \circ f$ is continuous at c.

Example 0 Given $f(x)$

$$f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{1 - x^2}, & 0 \le x \le 1 \\ x - 1, & x > 1 \end{cases}$$

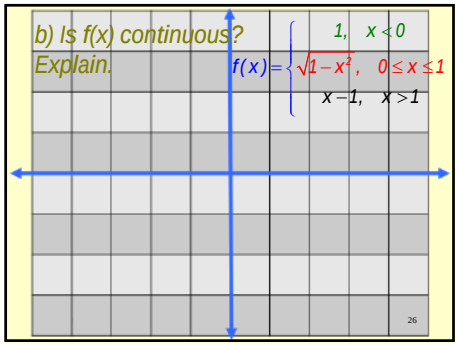
$$f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{1 - x^2}, & 0 \le x \le 1 \\ x - 1, & x > 1 \end{cases}$$

a) Determine a complete graph of $f(x)$



b) Is $f(x)$ continuous? Explain.

$$f(x) = \begin{cases} 1, & x < 0 \\ \sqrt{1 - x^2}, & 0 \le x \le 1 \\ x - 1, & x > 1 \end{cases}$$



Example 1 Find the points at which the function is not continuous

$$f(x) = \frac{1}{(x+2)^2}$$

Example 2 Find the points at which the function is not continuous

$$f(x)=\frac{x+3}{x^2-3x-10}$$

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Example 3 Find the points at which the function is not continuous

$$f(x)=\frac{1}{x^2+1}$$

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Example 4 Find the points at which the function is not continuous

$$f(x)=|2x+3|$$

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Example 5 Find the points at which the function is not continuous

$$f(x)=\frac{|x|}{x}$$

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Example 6 Find the points at which the function is not continuous

$$f(x)=\sqrt[4]{3x-1}$$

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Example 7 Find the points at which the function is not continuous

$$f(x)=\sqrt[5]{2-x}$$

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Example 8 Define g(x) so that $g(x)=(x^2-9)/(x-3)$ is continuous at $x=3$

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Example 9 Define f(1) so that $f(x)=(x^3-1)/(x^2-1)$ is continuous at $x=1$

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Example 10 Given g(x)

$$g(x)=\begin{cases} x^3, & x<\frac{1}{2} \\ bx^2, & x\geq\frac{1}{2} \end{cases}$$

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What value should be assigned to b to make g(x) continuous at x=1/2?

Example 11: Find the limit

$$\lim_{x \to 0} \tan x$$

Example 12: Find the limit

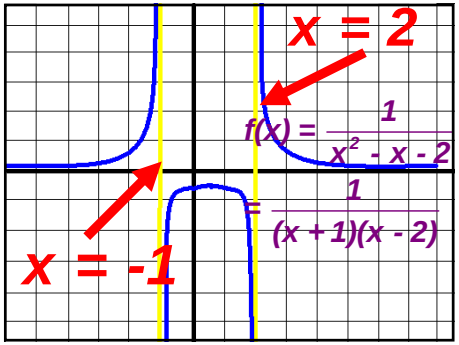
$$\lim_{x \to 0} \sin\left(\frac{\pi}{2}\cos(\tan x)\right)$$

Example 13: Find the vertical asymptote(s) of

$$f(x) = \frac{3}{(x+2)}$$

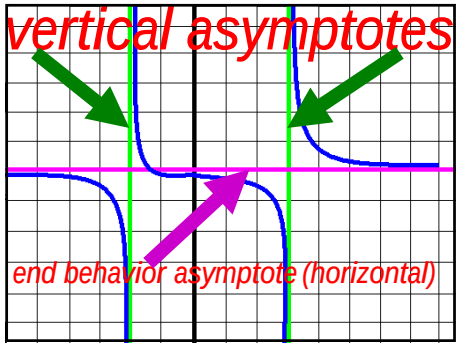
Example 14: Find the vertical asymptote(s) of

$$f(x) = \frac{1}{x^2 - x - 2}$$



Example 15: Find the end behavior asymptote(s) of

$$f(x) = \frac{x+1}{x^2 - x - 6}$$



Example 16

Find $\lim_{x \to \pm\infty} f(x)$ if

$$f(x) = \frac{2x^3 + 7}{x^3 - x^2 + x + 7}$$

Example 17

Find $\lim_{x\to\pm\infty} f(x)$ if

$$f(x) = \frac{3x+7}{x^2-2}$$

Example 18

Find $\lim_{x\to\pm\infty} f(x)$ if

$$f(x) = \frac{x^4}{x^3+1}$$

Example 19

Find $\lim_{x\to\pm\infty} f(x)$ if

$$f(x) = \frac{10x^5 + x^4 + 31}{x^6}$$

Example 20

Find $\lim_{x\to\pm\infty} f(x)$ if

$$f(x) = \frac{-x^4}{x^4 - 7x^3 + 7x^2 + 9}$$

Lesson quiz 1: Given

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 \le x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x \le 3 \end{cases}$$

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 \le x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x \le 3 \end{cases}$$

a) Does $f(1)$ exist?

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 \le x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x \le 3 \end{cases}$$

b) $\lim_{x\to 1} f(x)$ exist?

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 \le x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x \le 3 \end{cases}$$

c) Does $\lim_{x\to 1} f(x) = f(1)$?

$$f(x) = \begin{cases} x^2 - 1, & -1 \le x < 0 \\ 2x, & 0 \le x < 1 \\ 1, & x = 1 \\ -2x + 4, & 1 < x < 2 \\ 0, & 2 < x \le 3 \end{cases}$$

d) Is $f(x)$ continuous at $x = 1$?

$$f(x)=\begin{cases} x^2-1, & -1\le x<0 \\ 2x, & 0\le x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \end{cases}$$

e) At what values of x is $f(x)$ continuous?

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$$f(x)=\begin{cases} x^2-1, & -1\le x<0 \\ 2x, & 0\le x<1 \\ 1, & x=1 \\ -2x+4, & 1<x<2 \\ 0, & 2<x\le 3 \end{cases}$$

f) How should h be defined to make h a continuous extension of f to the point $x = 1$?

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