

COSA Common Core State Standards Regional Series

“Mathematics in Action”

A Statewide Regional Series for District and School Leaders of CCSS

Elementary (K-2)

Mathematics Session



Locations:

April 14, 2014 – Eagle Crest Resort, Redmond, OR
April 17, 2014 – Winston Community Center , Winston, OR
April 28, 2014 – Linn County Expo Center, Albany, OR
April 30, 2014 – Medford, OR
May 6, 2014 - Convention Center, Pendleton , OR

Mathematics Presenter:

Sarah Schuhl, SMC Curriculum, sarahschuhl@yahoo.com


CCSSM in Action

K – 2

STUDENT WORK

ASSESSMENT

INSTRUCTION



Sarah Schuhl
SMc Curriculum



What Do We Expect Students To Learn?

The CCSS Requires Three Shifts in Mathematics

- 1. Focus:** Focus strongly where the standards focus.
- 2. Coherence:** *Think across grades, and link to major topics*
- 3. Rigor:** In major topics, pursue *conceptual understanding*, procedural skill and *fluency*, and *application*

Domains K – 5

Domain	K	1	2	3	4	5
Counting and Cardinality (CC)	✓					
Operations and Algebraic Thinking (OA)	✓	✓	✓	✓	✓	✓
Number and Operations in Base Ten (NBT)	✓	✓	✓	✓	✓	✓
Measurement and Data (MD)	✓	✓	✓	✓	✓	✓
Geometry (G)	✓	✓	✓	✓	✓	✓
Numbers and Operations-Fractions (NF)				✓	✓	✓

Why Do We Need the *Mathematical Practices*?


Solve.

$$\begin{array}{r} 13 \\ - 7 \\ \hline \end{array}$$

$$\frac{5}{10} + \frac{6}{12} = ?$$

$$\frac{7}{7} \times \frac{1}{4} \times \frac{3}{3} \times \frac{5}{5} = ?$$

How did you think your students will approach each problem?



How did you approach each problem?

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.

4. Model with mathematics.

5. Use appropriate tools strategically.

7. Look for and make use of structure.

8. Look for and express regularity in repeated reasoning.

1. Make sense of problems and persevere in solving them.

6. Attend to precision.



Essential Question

If the traditional algorithms are not being taught at my grade level....

What am I supposed to teach?

Big Idea: Addition & Subtraction

- Does this make sense?

$$\begin{array}{r} 1 \\ 26 \\ + 47 \\ \hline 73 \end{array}$$

- As a table group, show as many **strategies** as you can to find $26 + 47$. How do you know your strategies makes sense?

	?
26	47

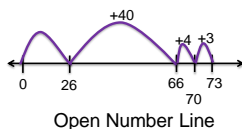
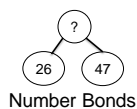
Tape Diagram



Base Ten Pieces

Place Value

$$\begin{array}{r} 26 + 47 \\ 60 + 13 \\ \hline 73 \end{array}$$



- Counting on
- Ten-Frames
- Unifix Cubes
- Number Line
- Others?**

Using High Cognitive Tasks



Instructional Tasks Matter!

"Not all tasks are created equal, and different tasks will provoke different levels and kinds of student thinking."

Stein, Smith, Henningsen, & Silver, 2000

"The level and kind of thinking in which students engage determines what they will learn."

Hiebert, Carpenter, Fennema, Fuson, Wearne, Murray, Oliver, & Human, 1997

Lower Level Demand Tasks



- Algorithmic.
- Require limited cognitive demand for successful completion. Little ambiguity in problem.
- No connection to concepts/procedures being taught.
- Focused on producing a correct answer instead of developing mathematical understanding.
- Reproduces previously learned facts, rules, formulas, or definitions or requires memorization.

--Smith, M. & Stein, M, *5 Practices for Orchestrating Productive Mathematics Discussions*, 2011 (p. 16)

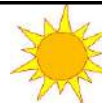
Higher Level Demand Tasks



- Focus on using procedures that develop conceptual understanding.
- Often represented in multiple ways.
- Require some cognitive effort. General procedures used cannot be followed mindlessly.
- Require complex and non-algorithmic thinking.
- Require students to explore and understand the nature of math concepts.

--Smith, M. & Stein, M, *5 Practices for Orchestrating Productive Mathematics Discussions*, 2011 (p. 16)

1.OA.1 – Field Day



It's field day! The sun is shining and the students are having fun playing games with their friends. Your teacher gives you \$7 to spend at the school store. Here are the options of what you can buy.

Store	Dollars (\$)
Water Bottle	2
Snack	4
Ball	5

- Do you have enough money to buy one of everything? How do you know?
- What can you buy using only \$7? Show how you know your answer is correct.

---www.illustrativemathematics.org

2.NBT.1 – Counting Stamps



The post office packages stamps like this:

- 10 stamps in each strip.
- 10 strips of 10 in each sheet.

- Yesterday Mike saw 4 full sheets, 7 strips, and 2 extra stamps in the drawer. He counted all the stamps and found out that there were 472 stamps in all. He said, The number 472 matches the 4 sheets, 7 strips, and 2 stamps. Cool!

Why did Mike's number match up with the numbers of sheets, strips, and extra stamps? Draw a picture to help explain your answer.

- Today Mike found 3 extra stamps, 1 sheet, and 5 strips. He said, Because of how things matched up yesterday, I guess there are 315 stamps total.

Explain why Mike's guess is incorrect. What could he have done to guess correctly?

---www.illustrativemathematics.org

Five Practices when Implementing High Cognitive Tasks

- Anticipating** likely student responses to challenging mathematical tasks.
- Monitoring** students' actual responses to the tasks (while students work on the task in pairs or small groups).
- Selecting** particular students to present their mathematical work during the whole-class discussion.
- Sequencing** the student responses that will be displayed in a specific order.
- Connecting** different students' responses and connecting the responses to key mathematical ideas.

--Smith, M. & Stein, M, *5 Practices for Orchestrating Productive Mathematics Discussions*, 2011 (p. 8)

Where can I find tasks?



- www.illustrativemathematics.org – click on “Illustrations”
- www.k-5mathteachingresources.com
- www.insidemathematics.org
- www.ccsmath.org
- www.commoncoreconversation.com
- www.smarterbalanced.org
- <https://grade2commoncoremath.wikispaces.hcps.org/Grade+2+Home>

Putting it All Together



- Design a lesson using the Lesson Planning Tool that you will teach next week.
 - How will you emphasize a mathematical practice?
 - What are your assessing and advancing questions?
 - How will the lesson begin and end?
 - What are students doing during the lesson?
- Find/Create a high cognitive task to use in the next week with students.
 - How will you also teach a mathematical practice?



How will we know students have learned the CCSSM?

Formative vs. Summative Assessments

Formative

- A process during learning
- Descriptive feedback, use of rubrics, student self-assessment
- Used to support ongoing growth, improvement

Summative

- An event after learning
- Chapter tests, state assessment, end-of-year placement tests
- Used to measure achievement



SBAC Member States



SMARTER: Summative Multi-State Assessment Resources for Teachers and Educational Researchers
www.smarterbalanced.org

Four Claims Used in DRAFT SBAC Test Specifications

Claim #1
Concepts & Procedures

40%

Students can explain and apply mathematical concepts and interpret and carry out mathematical procedures with precision and fluency.

Claim #2
Problem Solving

Students can solve a range of complex well-posed problems in pure and applied mathematics, making productive use of knowledge and problem solving strategies.

Claim #4
Modeling & Data Analysis

Students can analyze complex, real-world scenarios and can construct and use mathematical models to interpret and solve problems.

60%

Claim #3
Communicating Reasoning

Students can clearly and precisely construct viable arguments to support their own reasoning and to critique the reasoning of others.

Assessment Item Types

- **Selected Response (SR)**
 - Variety of multiple choice and true/false
- **Extended Response (ER)**
 - Non-computer graded constructed response item
- **Technology Enhanced (TE)**
 - Technology embedded into items
- **Performance Tasks (PT)**
 - Rich, real-world scenarios where multiple math topics are addressed
- **Constructed Response (CR)**
 - Free response questions in the Adaptive portion of the test

Cognitive Rigor and Depth of Knowledge (DOK)




- **Level 1: Recall and Reproduction**
 Requires eliciting information such as a fact, definition, term, or a simple procedure, as well as performing a simple algorithm or applying a formula.
- **Level 2: Basic Skills and Concepts**
 Requires the engagement of some mental processing beyond a recall of information.
- **Level 3: Strategic Thinking and Reasoning**
 Requires reasoning, planning, using evidence, and explanations of thinking.
- **Level 4: Extended Thinking**
 Requires complex reasoning, planning, developing, and thinking most likely over an extended period of time.

OAKS Question – Grade 4

8 feet

4 feet



9 What is the perimeter of this figure?

(A) 12 feet
(B) 20 feet
(C) 24 feet
(D) 32 feet

SBAC – Grade 4

43023

A rectangle is 6 feet long and has a perimeter of $20\frac{1}{3}$ feet.

What is width of this rectangle? Explain how you solved this problem.



How do you create higher level DOK tasks?

Ask students to:

- Write a word problem for a given expression.
- Write a word problem with a given answer or range of answers.
- Solve a problem using more than one strategy.
- Find the error in a student solution and correct.
- Make sense of a provided solution strategy by writing the original problem or justifying the work shown.
- Solve multi-step problems.
- Solve open-ended tasks with multiple possible responses.

Does the Assessment Evaluate Student Understanding of Learning Targets?

- Are learning targets clear?
- Do proficient scores indicate student learning?
- Do low scores indicate that students need intervention?



Is There a Proportional Value Between Scores and Learning Targets on the Assessment?

- Is one learning target weighted more than others?
- Is one assessment method weighted more than another?
- If yes, is that acceptable?



What Is Proficiency?

- Rubric: Passing in all categories?
- Scoring criteria overall score or each section?

- PLC team determines.
- Look at student work.

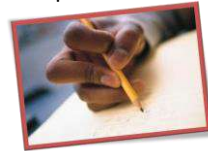


Analyze Assessments

- Which standards or learning targets are assessed?
- How are the mathematical practices assessed?
- Use the Evaluation of Assessment Tool to determine balance of DOK Levels, variety of assessment types, quality of questions and final product.
- How will the items be scored?
- What is proficiency?

Analyze an Assessment

- Look at the assessment.
- How does it measure against the rubric?
- How can it be improved?

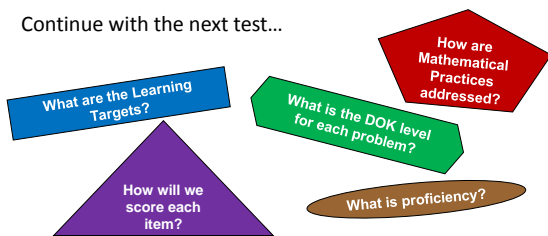


What needs to be modified on your math assessments?

Time to create/analyze our tests...

- Choose a current or next unit test
- Analyze or create it using the Evaluation of Assessment Tool
- Discuss any changes that are needed...Continue...

Continue with the next test...



Analyze Student Work



- Read the task: Apple Farm Field Trip
- What content standards and/or mathematical practices are being assessed in this task?
- What can you learn from student work?
- What can students learn from one another's work?
- How can *all* students be re-engaged in the learning of this content?

Next Steps...

- How can you make sure students are learning multiple strategies for conceptual understanding?
- How can you include the standards for mathematical practice in lessons?
- How can you use high cognitive tasks in class?
- What do you need to consider in assessments?



Contact Information

Sarah Schuhl , SMc Curriculum
sarahschuhl@yahoo.com

Shannon McCaw, SMc Curriculum
mccaws@smccurriculum.com

800-708-5259

Temperature Check



1. Which mathematical practices have you been teaching students this year?

2. Which content standards have you taught this year?

3. What are three “big ideas” you want students to come to you knowing next year?

4. How are you feeling about implementing the Common Core State Standards in Mathematics?

Track Your Progress: Common Core State Standards for Mathematics in Action

Shade each rectangle to show your current understanding of each learning target.

- I can describe strategies for teaching the priority content standards with the mathematical practices.

Starting ...	Getting There ...	Got It!
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- I can create assessments aligned to SBAC claims and DOK levels.

Starting ...	Getting There ...	Got It!
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- I can analyze student work to increase student achievement.

Starting ...	Getting There ...	Got It!
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Next Steps...



CCSSM (SBAC) Priority Clusters K – 2

Kindergarten	Grade 1	Grade 2
<p><u>Counting and Cardinality</u> Know number names and the count sequence.</p> <p>Count to tell the number of objects.</p> <p>Compare numbers.</p> <p><u>Operations and Algebraic Thinking</u> Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.</p> <p><u>Number and Operations in Base Ten</u> Work with numbers 11-19 to gain foundations for place value.</p>	<p><u>Operations and Algebraic Thinking</u> Represent and solve problems involving addition and subtraction.</p> <p>Understand and apply properties of operations and the relationship between addition and subtraction.</p> <p>Add and subtract within 20.</p> <p>Work with addition and subtraction equations.</p> <p><u>Number and Operations in Base Ten</u> Extending the counting sequence.</p> <p>Understand place value.</p> <p>Use place value understanding and properties of operations to add and subtract.</p> <p><u>Measurement and Data</u> Measure lengths indirectly and by iterating length units.</p>	<p><u>Operations and Algebraic Thinking</u> Represent and solve problems involving addition and subtraction.</p> <p>Add and subtract within 20.</p> <p><u>Number and Operations in Base Ten</u> Understand place value.</p> <p>Use place value understanding and properties of operations to add and subtract.</p> <p><u>Measurement and Data</u> Measure and estimate lengths in standard units.</p> <p>Relate addition and subtraction to length.</p>

CCSSM (SBAC) Supporting Clusters K – 2


Kindergarten	Grade 1	Grade 2
<p><u>Measurement and Data</u> Classify objects and count the number of objects in categories.</p> <p>Describe and compare measurable attributes.</p> <p><u>Geometry</u> Identify and describe shapes.</p> <p>Analyze, compare, create, and compose shapes.</p>	<p><u>Measurement and Data</u> Represent and interpret data.</p> <p>Tell and write time.</p> <p><u>Geometry</u> Reason with shapes and their attributes.</p>	<p><u>Operations and Algebraic Thinking</u> Work with equal groups of objects to gain foundations for multiplication.</p> <p><u>Measurement and Data</u> Work with time and money.</p> <p>Represent and interpret data.</p> <p><u>Geometry</u> Reason with shapes and their attributes.</p>

Essential Skills – CCSSM Content Standards

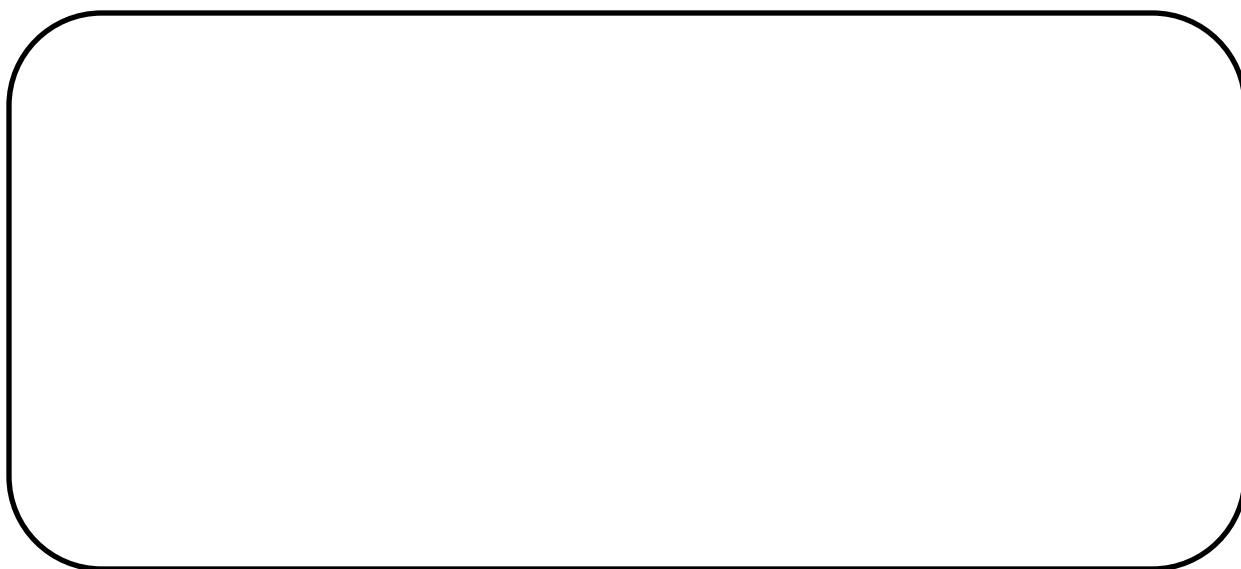
Review the Priority and Supporting Clusters. Read the accompanying content standards.

My Grade Level: _____

1. What are 7 – 10 Essential Skills students in my grade must learn?



2. What are 7 – 10 Essential Skills students should come to my grade having learned?



Mathematical Practices K – 2

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Write the number for the mathematical practice best evidenced by each student description.

	Student Description	MP
A	A student adds $9 + 3$ by finding $(9 + 1) + 2 = 10 + 2 = 12$.	
B	A student finding $23 + 67$ begins by drawing base ten pieces and then uses that thinking to instead find the total by summing the tens and the ones.	
C	A student shows the counterexample $8 + 0 = 8$ to show how a sum is not always greater than its two addends.	
D	A student gives an example showing how a quantity represented by 10 could be less than one represented by 5.	
E	A student uses subtraction to determine how many more students brought lunch than will buy lunch in her class.	
F	When subtracting $15 - 9$, one student decides to use a number line, while another decides to use ten frames.	
G	Students use correct mathematical language during discussions and calculate accurately. They ask... " <i>Does my answer make sense?</i> "	
H	A student skip-counting by 5s recognizes that the digit in the ones place alternates between 0 and 5 for each consecutive number in the sequence.	

Problem Solving Graphic Organizer

<p style="text-align: center;"><u>Words</u></p> <p>Kyle has 8 blocks. He finds 3 more. Kyle needs 20 blocks to build his tower.</p> <p>How many more blocks does Kyle need?</p>	<p style="text-align: center;"><u>Objects</u></p>
<p style="text-align: center;"><u>Picture</u></p>	<p style="text-align: center;"><u>Equation</u></p>
<p><u>Answer</u></p>	

Addition & Subtraction K – 2

1. Show as many strategies as you can to find $26 + 47$.
2. Use a strategy that is not the standard algorithm to find $32 + 59$.
3. Use two different strategies to find $78 - 49$.

Big Idea: Place Value

- I have 12 ones and 3 tens. Who am I?
- I have 40 tens and 2 ones. Who am I?
- I have 3 hundreds, 15 ones and 2 tens. Who am I?
- How many ways can you use tens and ones to describe the number 74?
Explain how you know you have found all the ways to use tens and ones to describe 74.

TABLE 1. Common addition and subtraction situations.⁶

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown ¹
Put Together/ Take Apart²	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$, $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare³	("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?	(Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?	(Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?
	("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$, $5 - 2 = ?$	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$, $3 + 2 = ?$	(Version with "fewer"): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$, $? + 3 = 5$

¹These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean is the same number as.

²Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.

³For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.

⁶Adapted from Box 2-4 of Mathematics Learning in Early Childhood, National Research Council (2009, pp. 32, 33).

TABLE 2. Common multiplication and division situations.⁷

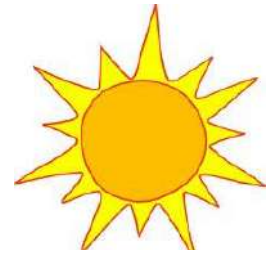
	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$, and $18 \div 3 = ?$	$? \times 6 = 18$, and $18 \div 6 = ?$
Equal Groups	There are 3 bags with 6 plums in each bag. How many plums are there in all? <i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?	If 18 plums are shared equally into 3 bags, then how many plums will be in each bag? <i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?	If 18 plums are to be packed 6 to a bag, then how many bags are needed? <i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?
Arrays,⁴ Area⁵	There are 3 rows of apples with 6 apples in each row. How many apples are there? <i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?	If 18 apples are arranged into 3 equal rows, how many apples will be in each row? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?	If 18 apples are arranged into equal rows of 6 apples, how many rows will there be? <i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?
Compare	A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost? <i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?	A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost? <i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?	A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat? <i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?
General	$a \times b = ?$	$a \times ? = p$, and $p \div a = ?$	$? \times b = p$, and $p \div b = ?$

⁴The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

⁵Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

⁷The first examples in each cell are examples of discrete things. These are easier for students and should be given before the measurement examples.

Grade 1
1.OA.1 – Field Day



It's field day! The sun is shining and the students are having fun playing games with their friends. Your teacher gives you \$7 to spend at the school store. Here are the options of what you can buy.

Store	Dollars (\$)
Water Bottle	2
Snack	4
Ball	5

1. Do you have enough money to buy one of everything? How do you know?
2. What can you buy using only \$7? Show how you know your answer is correct.

Grade 2

2.NBT.1 – Counting Stamps



The post office packages stamps like this:

- 10 stamps in each strip.
- 10 strips of 10 in each sheet.

1. Yesterday Mike saw 4 full sheets, 7 strips, and 2 extra stamps in the drawer. He counted all the stamps and found out that there were 472 stamps in all. He said,

The number 472 matches the 4 sheets, 7 strips, and 2 stamps. Cool!

Why did Mike's number match up with the numbers of sheets, strips, and extra stamps? Draw a picture to help explain your answer.

2. Today Mike found 3 extra stamps, 1 sheet, and 5 strips. He said,

Because of how things matched up yesterday,
I guess there are 315 stamps total.

Explain why Mike's guess is incorrect. What could he have done to guess correctly?

Figure 2.12: CCSS Mathematical Practices Lesson-Planning Tool

Unit:	Date:	Lesson:
Learning target: As a result of today's class, students will be able to _____		
Formative assessment: How will students be expected to demonstrate mastery of the learning target during in-class checks for understanding?		
Probing Questions for Differentiation on Mathematical Tasks		
Assessing Questions (Create questions to scaffold instruction for students who are "stuck" during the lesson or the lesson tasks.)	Advancing Questions (Create questions to further learning for students who are ready to advance beyond the learning target.)	
Targeted Standard for Mathematical Practice: Which Mathematical Practice will be targeted for proficiency development during this lesson?		
Tasks (Tasks can vary from lesson to lesson.)	What Will the Teacher Be Doing? (How will the teacher present and then monitor student response to the task?)	What Will the Students Be Doing? (How will students be actively engaged in each part of the lesson?)
Beginning-of-Class Routines How does the warm-up activity connect to students' prior knowledge, or how is it based on analysis of homework?		

Tasks (Tasks can vary from lesson to lesson.)	What Will the Teacher Be Doing? (How will the teacher present and then monitor student response to the task?)	What Will the Students Be Doing? (How will students be actively engaged in each part of the lesson?)
Task 1 How will the students be engaged in understanding the learning target?		
Task 2 How will the task develop student sense making and reasoning?		
Task 3 How will the task require student conjectures and communication?		
Closure How will student questions and reflections be elicited in the summary of the lesson? How will students' understanding of the learning target be determined?		

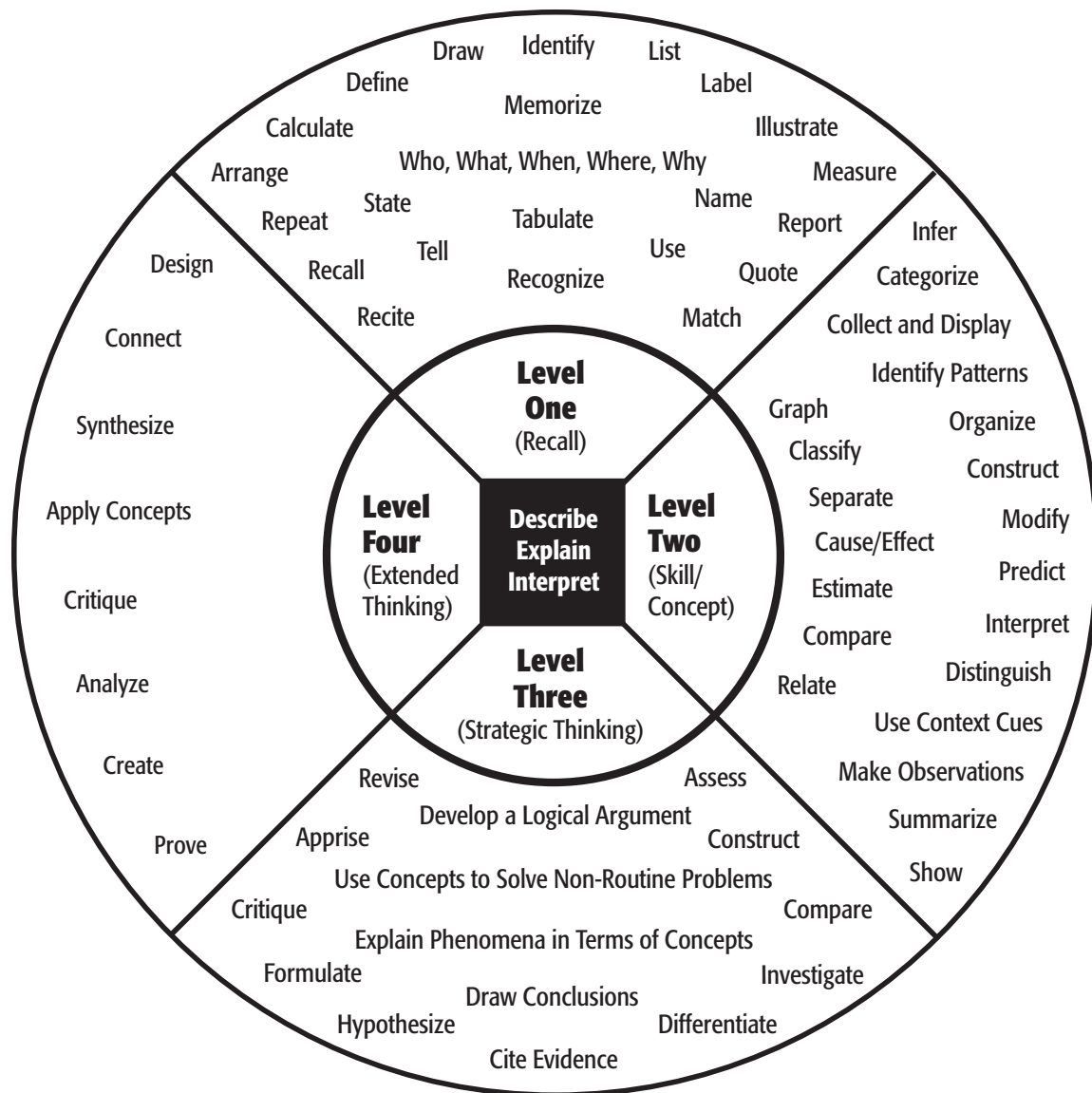
Depth of Knowledge (DOK)

Source: www.smarterbalanced.org Mathematics Content Specifications

A “Snapshot” of the Cognitive Rigor Matrix (Hess, Carlock, Jones & Walkup, 2009)

Depth of Thinking (Webb) + Type of Thinking (Revised Bloom)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
Remember	<ul style="list-style-type: none"> Recall conversations, terms, facts 			
Understand	<ul style="list-style-type: none"> Evaluate an expression Locate points on a grid or number on number line Solve a one-step problem Represent math relationships in words, pictures, or symbols 	<ul style="list-style-type: none"> Specify, explain relationships Make basic inferences or logical predictions from data/observations Use models/diagrams to explain concepts Make and explain estimates 	<ul style="list-style-type: none"> Use concepts to solve non-routine problems Use supporting evidence to justify conjectures, generalize, or connect ideas Explain reasoning when more than one response is possible Explain phenomena in terms of concepts 	<ul style="list-style-type: none"> Relate mathematical concepts to other content areas, other domains Develop generalizations of the results obtained and the strategies used and apply them to new problem situations
Apply	<ul style="list-style-type: none"> Follow simple procedures Calculate, measure, apply a rule (e.g., rounding) Apply algorithm or formula Solve linear equations Make conversions 	<ul style="list-style-type: none"> Select a procedure and perform it Solve routine problem applying multiple concepts or decision points Retrieve information to solve a problem Translate between representations 	<ul style="list-style-type: none"> Design investigation for a specific purpose or research question Use reasoning, planning, and supporting evidence Translate between problem & symbolic notation when not a direct translation 	<ul style="list-style-type: none"> Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solves the problem, and reports results
Analyze	<ul style="list-style-type: none"> Retrieve information from a table or graph to answer a question Identify a pattern/trend 	<ul style="list-style-type: none"> Categorize data, figures Organize, order data Select appropriate graph and organize & display data Interpret data from a simple graph Extend a pattern 	<ul style="list-style-type: none"> Compare information within or across data sets or texts Analyze and draw conclusions from data, citing evidence Generalize a pattern Interpret data from complex graph 	<ul style="list-style-type: none"> Analyze multiple sources of evidence or data sets
Evaluate			<ul style="list-style-type: none"> Cite evidence and develop a logical argument Compare/contrast solution methods Verify reasonableness 	<ul style="list-style-type: none"> Apply understanding in a novel way, provide argument or justification for the new application
Create	<ul style="list-style-type: none"> Brainstorm ideas, concepts, problems, or perspectives related to a topic or concept 	<ul style="list-style-type: none"> Generate conjectures or hypotheses based on observations or prior knowledge and experience 	<ul style="list-style-type: none"> Develop an alternative solution Synthesize information within one data set 	<ul style="list-style-type: none"> Synthesize information across multiple sources or data sets Design a model to inform and solve a practical or abstract situation.

Depth of Knowledge (DOK) Levels



Level One Activities	Level Two Activities	Level Three Activities	Level Four Activities
Recall elements and details of story structure, such as sequence of events, character, plot and setting.	Identify and summarize the major events in a narrative.	Support ideas with details and examples.	Conduct a project that requires specifying a problem, designing and conducting an experiment, analyzing its data, and reporting results/ solutions.
Conduct basic mathematical calculations.	Use context cues to identify the meaning of unfamiliar words.	Use voice appropriate to the purpose and audience.	Apply mathematical model to illuminate a problem or situation.
Label locations on a map.	Solve routine multiple-step problems.	Identify research questions and design investigations for a scientific problem.	Analyze and synthesize information from multiple sources.
Represent in words or diagrams a scientific concept or relationship.	Describe the cause/effect of a particular event.	Develop a scientific model for a complex situation.	Describe and illustrate how common themes are found across texts from different cultures.
Perform routine procedures like measuring length or using punctuation marks correctly.	Identify patterns in events or behavior.	Determine the author's purpose and describe how it affects the interpretation of a reading selection.	Design a mathematical model to inform and solve a practical or abstract situation.
Describe the features of a place or people.	Formulate a routine problem given data and conditions.	Apply a concept in other contexts.	
	Organize, represent and interpret data.		

Figure 4.4:
Evaluation Tool for Assessment Instrument Quality

Assessment indicators	Description of Level 1	Requirements of the Indicator Are Not Present	Limited Requirements of This Indicator Are Present	Substantially Meets the Requirements of the Indicator	Fully Achieves the Requirements of the Indicator	Description of Level 4
Identification and emphasis on learning targets	Learning targets are unclear or absent from the assessment instrument. Too much attention is given to one target.	1	2	3	4	Clearly stated learning targets are on the assessment and connected to the assessment questions.
Visual presentation	Assessment is sloppy, disorganized, and difficult to read. There is no room for teacher feedback.	1	2	3	4	Assessment is neat, organized, easy to read, and well spaced. There is room for teacher feedback.
Time allotment	Few students can complete the assessment in the time allowed.	1	2	3	4	Test can be successfully completed in time allowed.
Clarity of directions	Directions are missing or unclear.	1	2	3	4	Directions are appropriate and clear.
Clear and appropriate scoring rubrics	Scoring rubric is either not in evidence or not appropriate for the assessment task.	1	2	3	4	Scoring rubric is clearly stated and appropriate for each problem.
Variety of assessment task formats	Assessment contains only one type of questioning strategy and no multiple choice. Calculator usage is not clear.	1	2	3	4	Test includes a variety of question types, assesses different formats, and includes calculator usage.
Question phrasing (precision)	Wording is vague or misleading. Vocabulary and precision of language is problematic for student understanding.	1	2	3	4	Vocabulary is direct, fair, and clearly understood. Students are expected to attend to precision in responses.
Balance of procedural fluency and demonstration of understanding	Test is not balanced for rigor. Emphasis is on procedural knowledge. Minimal cognitive demand for demonstration of understanding is present.	1	2	3	4	Test is balanced with product- and process-level questions. Higher-cognitive-demand and understanding tasks are present.

What does a Common Core Assessment look like?

Depth of Knowledge Levels

Level 1: Recall and Reproduce
(25% of seat time on assessment)

Level 2: Basic Skills and Concepts
(50% of seat time on assessment)

Level 3: Strategic Thinking and Reasoning
(25% of seat time on assessment)

Level 4: Extended Thinking
(Separate assessment – performance task)

Claims

1. **Concepts and Procedures**
(40% of overall score on SBAC)
2. **Problem-Solving** (40% of overall score on SBAC)
3. **Communicating Reasoning**
(20% of overall score on SBAC)

Styles of Items

1. **Selected Response**
 - multiple choice
 - select all that apply
 - true/false or yes/no
 - drag and drop
2. **Constructed Response**
 - fill in the blank
 - numerical answer
3. **Extended Response**
 - explain your reasoning
 - show how you know your answer is correct
 - writing a note to convince someone
4. **Performance Task**

Assessment Evaluation Tool

Item Number	DOK Level	Claim	Item Type

Grade 2 Unit 3: Multiplication Foundations

Name _____

Date _____

2.NBT.2 Learning Target: *I can count and skip-count within 1,000.*

1. Count by **100's** to 1,000. Write the numbers in the grid below.

5 points _____

2. Count by **5's** to 100. Write the numbers in the grid below.

5 points _____

3. Count by **10's** to 100. Write the numbers in the grid below.

--	--	--	--	--	--	--	--	--	--

5 points _____

2.OA.3 Learning Target: I can explain if the number of items in a group are even or odd.

4.

Total number of stars _____

Are there an even or odd number of stars? _____

Explain how you know your answer is correct.

2 points _____

5. There are an equal number of boys and girls. There are 7 boys. Write an equation to show how many children there are altogether.

Is the total number of children odd or even?_____

Explain how you know your answer is correct.

4 points_____

2.OA.4 Learning Target: *I can find the total number of objects in an array.*

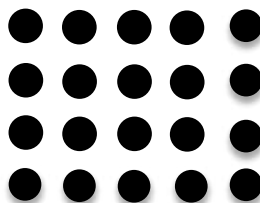
6. Which expressions show how to find the sum of the dots in the array?
Fill in the circle for each correct answer.

2 points_____

☐ $4 + 4 + 4 + 4 + 4$

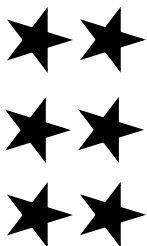
☐ $5 + 5 + 5 + 5$

☐ $5 + 5 + 5 + 5 + 5$



7. Write an addition equation to show how many stars are in the array. Be sure to include the numbers you are adding and the total.

2 points_____



8. Jill has a bag with 12 marbles. Draw an array she can make using all the marbles and write an equation to match the array.

3 points_____

Scoring

2.NBT.2 I can count and skip-count within 1,000.

Test Questions #1 – 3: 5 points if all numbers are correct
 4 points if 80% of the answers are correct
 3 points if 60% of the answers are correct
 2 points if 40% of the answers are correct
 1 point if 20% of the answers are correct

2.OA.3 I can explain if the number of items in a group are even or odd.

Test Question #4: 1 point for “even”
 1 point for explanation
 Test Question #5: 2 points for equation (addends and sum)
 1 point for “even”
 1 point for explanation

2.OA.4 I can find the total number of objects in an array.

Test Question #6 1 point per correct answer
 Test Question #7 1 point for the addends
 1 point for the sum
 Test Question #8 1 point for a correct array
 1 point for the addends
 1 point for the sum

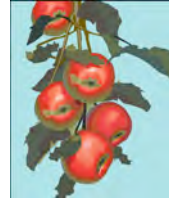
Student Reflection

Learning Target	Test Questions	Score	How did I do? (Circle one.)
2.NBT.2 I can count and skip-count within 1,000.	#1 – 3	_____ out of 15	I got it! Still learning it...
2.OA.3 I can explain if the number of items in a group are even or odd.	#4 – 5	_____ out of 6	I got it! Still learning it...
2.OA.4 I can find the total number of objects in an array.	#6 - 8	_____ out of 7	I got it! Still learning it...

Learning Targets I know and can do:

Learning Targets I still need to learn:

Apple Farm Field Trip



1. 63 second graders are going on the field trip. 19 parents will also go.
How many people are going on the field trip?

Show how you know your answer is correct.

_____people

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

_____miles

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.

_____ ¢

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

_____ students

Student A is typical of the 30% of the students who scored the maximum number of points on this task. Most students working at the maximum were able to correctly identify an operation in context, and could set up a two-digit addition and subtraction problem correctly. It was not unusual for students at this level to pick a strategy they were comfortable with and stay with it. This student used a counting up strategy. The evidence reveals that the student is counting by ones, no matter how big the difference being measured might be. When considering reengagement lessons, think about how to make connections with these students among various strategies.

Student A

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

Show how you know your answer is correct.

$$\begin{array}{r} 63 \\ + 19 \\ \hline 82 \end{array}$$

82 people

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92

34 miles

Apple Farm Fi

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.

48¢, 49¢, 50¢, 51¢, 52¢, 53¢, 54¢, 55¢, 56¢, 57¢, 58¢, 59¢, 60¢, 61¢, 62¢, 63¢, 64¢, 65¢

17¢

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63

27 students

Base ten blocks have long been used to represent the standard algorithm for multi-digit subtraction with regrouping. In this model, when the student “needs more one”, s/he must *exchange* a ten rod for ten units. **Student CC** is using base ten blocks to model the subtraction problems in Parts 3 and 4. What evidence is there that the student is using the drawings to model his/her own thinking about how the subtraction works? What would it look like if s/he were exchanging for units? Where does this student get more units when they are needed?

What value is there in the student being able to use the base ten blocks to model their own thinking, rather than to mimic the rote procedures of the algorithm? How can decomposition be used to help record their thinking, since it doesn’t match the recording notation of the standard algorithm? (See box below for a possible recording of the actions in Part 4. Spend a few minutes following the recording with the drawing. Where is each number and action represented in the model?)

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.



4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.



Student CC

Student CC – Part 4

$$36 = 30 + 3 + 3$$

$$63 - 30 = 33$$

$$33 - 3 = 30$$

$$30 = 10 + 10 + 10$$

$$10 - 3 = 7$$

$$10 + 10 + 7 = 27$$

Students D to K represent the wide range of understanding students have with the standard addition and subtraction algorithms. Students may be confused on the format, process, and underlying meaning of the algorithms, yet they aren't taking advantage of other, number-sense based strategies. Not all students forget the process for the standard algorithm. Perhaps they are making deep sense of the steps, or perhaps they have a good memory. The evidence presented here gives us good insight into the gaps of understanding that are happening here, and may also be happening under even successful algorithms, if we don't help students build their number sense.

Students D and E evidence two common errors. **Student D** has set up the problem correctly, but in Part 2, s/he is treating each of the place value digits as a separate problem, where the larger digit (regardless of placement) subtracts the smaller digit. Therefore, **Student D** subtracts 2 ones from 8 ones and place 6 ones in the ones place. In the tens place, 5 is subtracted from 9 and a 4 is placed in the tens place. How is **Student E**'s work similar to **Student D**'s? How is it different? What do these two papers reveal about how the students are thinking about place value and quantity, versus digits?

Student D

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

Show how you know your answer is correct.

$$\begin{array}{r} 63 \\ + 19 \\ \hline 82 \end{array} \checkmark$$

82 people

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

$$\begin{array}{r} 92 \\ - 58 \\ \hline 46 \end{array} \checkmark$$

46 miles

Student E

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.

$$\begin{array}{r} 48 \\ \times 65 \\ \hline 23 \end{array} \quad \begin{array}{r} 23 \end{array} \times \times \quad 8$$

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

$$\begin{array}{r} 36 \\ \times 63 \\ \hline 33 \end{array} \quad \begin{array}{r} 33 \end{array} \times \times \quad \begin{array}{|c|} \hline 33 \\ \hline \end{array} \quad \begin{array}{|c|} \hline 10 \\ \hline \end{array}$$

Student H

3. Molly wants to buy an apple pen. She sees a red apple pen that cost 48¢. She sees a sparkle apple pen for 65¢. How much more does the sparkle apple pen cost?

Show how you know your answer is correct.

Handwritten work: $\begin{array}{r} 48 \\ +65 \\ \hline 113 \end{array}$ I added it by the right side and then the left side. $10 \text{ } 13 \text{¢}$

4. Time to leave! 36 students got on the bus. They waited until all 63 students were there. How many students were late to the bus?

Show how you know your answer is correct.

Handwritten work: $\begin{array}{r} 36 \\ +63 \\ \hline 99 \end{array}$ I crossed out the 3 and the 6. 100

How is **Student H** handling the extra group of 10 units created in Part 3? Where does s/he keep track of it with notation? Where does s/he keep track of the extra group of 10 tens created? How is it reflected in the answer? What do we know about the depth of thinking, when “nearly fifty” and “more than fifty” add up to just “13”?

Student I also needs to decide how to handle the extra group of 10 units created when 3 is added to 9. How is “carrying over” handled in these problems? In what ways does **Student J**’s decomposition reflect a deeper level of number sense and understanding than the standard algorithm as executed by **Student I**? In what ways might Student K’s work be used to help Student J understand what happens to that extra group of tens?

Student I

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

Show how you know your answer is correct.

Handwritten work: $\begin{array}{r} 63 \\ +19 \\ \hline 82 \end{array}$ ✓

$\times 72$ people $\times 6$

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

Handwritten work: $\begin{array}{r} 92 \\ -58 \\ \hline 34 \end{array}$ $\times 0$

Student J

1. 63 second graders are going on the field trip. 19 parents will also go. How many people are going on the field trip?

Show how you know your answer is correct.

Handwritten work: $63 + 19$ $\begin{array}{r} 63 \\ +19 \\ \hline 82 \end{array}$ ✓ 82 people

2. The Apple Farm is 92 miles from the school. They have traveled 58 miles so far. How many more miles do they have to go?

Show how you know your answer is correct.

Handwritten work: $10 + 10 + 10 + 10 = 40$ ✓ $58 \xrightarrow{10} 68 \xrightarrow{10} 78 \xrightarrow{10} 88 \xrightarrow{4} 92$ 34 miles

Student Task	Set up and perform two-digit addition and subtraction calculations in context. Choose an appropriate operation and number sentence to solve the problem. Use a variety of strategies for performing addition or subtraction with accuracy and flexibility. Justify an answer using words, numbers or pictures.
Core Idea 1 Number Properties	Understand numbers, ways of representing numbers, relationships among numbers, and number systems. <ul style="list-style-type: none"> Understand whole numbers and represent and use them in flexible ways, including relating, composing, and decomposing numbers. Demonstrate an understanding of the base-ten number system and place value concepts.
Core Idea 2 Number Operations	Understand the meanings of operations and how they relate to each other, make reasonable estimates, and compute fluently <ul style="list-style-type: none"> Demonstrate fluency in adding and subtracting whole numbers. Communicate reasoning using pictures, numbers, and/or words

Mathematics of the task:

- Ability to read a problem in context and choose an operation to solve the problem.
- Ability to set up a number sentence that represents the problem.
- Accurately and efficiently solves two-digit addition and subtraction equations.
- Understands how to compose and decompose numbers.
- Can prove a sum or difference is correct using pictures, words, and/or symbols.

Based on teacher observation, this is what second graders knew and were able to do:

- Recognize combining situations in context, and add two-digit numbers with regrouping.
- Use the standard algorithm for addition or subtraction.
- Most students were able to set up subtraction correctly, whether or not they could find an accurate answer.
- Most students were able to identify whether the context was subtraction or addition.
- Some students had access to a number of strategies to solve comparison subtraction, including pictures, models, counting up, and taking away.

Areas of difficulty for second graders:

- Adding, instead of comparing, the two numbers in Parts 2, 3, and 4.
- Used inefficient counting strategies, such as counting by units instead of groupings of 5, 10, or more.
- If students didn't have access to the standard algorithm for two-digit subtraction, they were limited in other strategies.
- Many students mixed up the subtrahend and minuend when setting up the standard algorithm.

Strategies used by successful students:

- Extract relevant information, then use given numbers appropriately when setting up the addition or subtraction problem.
- Understand comparison could be solved by subtracting or by missing addend addition and counting up.
- Use pictures, models, and tools for counting, including standard addition and subtraction algorithm.
- Composed and decomposed fluently to find differences between two numbers.