

Geometry - Constructions Packet

Why we learn about constructions

The ancient Greek mathematician **Euclid** is the acknowledged inventor of geometry. He did this over 2000 years ago, and his book "Elements" is still regarded as the ultimate geometry reference. In that work, he uses these construction techniques extensively, and so they have become a part of the geometry field of study. They also provide a greater insight into geometric concepts and give us tools to draw things when direct measurement is not appropriate.



Why did Euclid do it this way?

Why didn't Euclid just measure things with a ruler and calculate lengths? For example, one of the basic constructions is **bisecting a line** (dividing it into two equal parts). Why not just measure it with a ruler and divide by two?

The answer is surprising. The Greeks could not do arithmetic. They had only whole numbers, no zero, and no negative numbers. This meant they could not for example divide 5 by 2 and get 2.5, because 2.5 is not a whole number - the only kind they had. Also, their numbers did not use a positional system like ours, with units, tens, hundreds etc, but more like the Roman numerals. In short, they could perform very little useful arithmetic.

So, faced with the problem of finding the midpoint of a line, they could not do the obvious - measure it and divide by two. They had to have other ways, and this led to the constructions using compass and straightedge or ruler. It is also why the straightedge has no markings. It is definitely not a graduated ruler, but simply a pencil guide for making straight lines. Euclid and the Greeks solved problems graphically, by drawing shapes, as a *substitute* for using arithmetic.

Introduction to constructions

Constructions: The drawing of various shapes using only a compass and straightedge or ruler. No measurement of lengths or angles is allowed.

The word construction in geometry has a very specific meaning: the drawing of geometric items such as lines and circles using only a compass and straightedge or ruler. Very importantly, you are not allowed to measure angles with a **protractor**, or measure lengths with a ruler.

Compass



The compass is a drawing instrument used for drawing circles and **arcs**. It has two legs, one with a point and the other with a pencil or lead. You can adjust the distance between the point and the pencil and that setting will remain until you change it. (This kind of compass has nothing to do with the kind used find the North direction when you are lost).

Straightedge



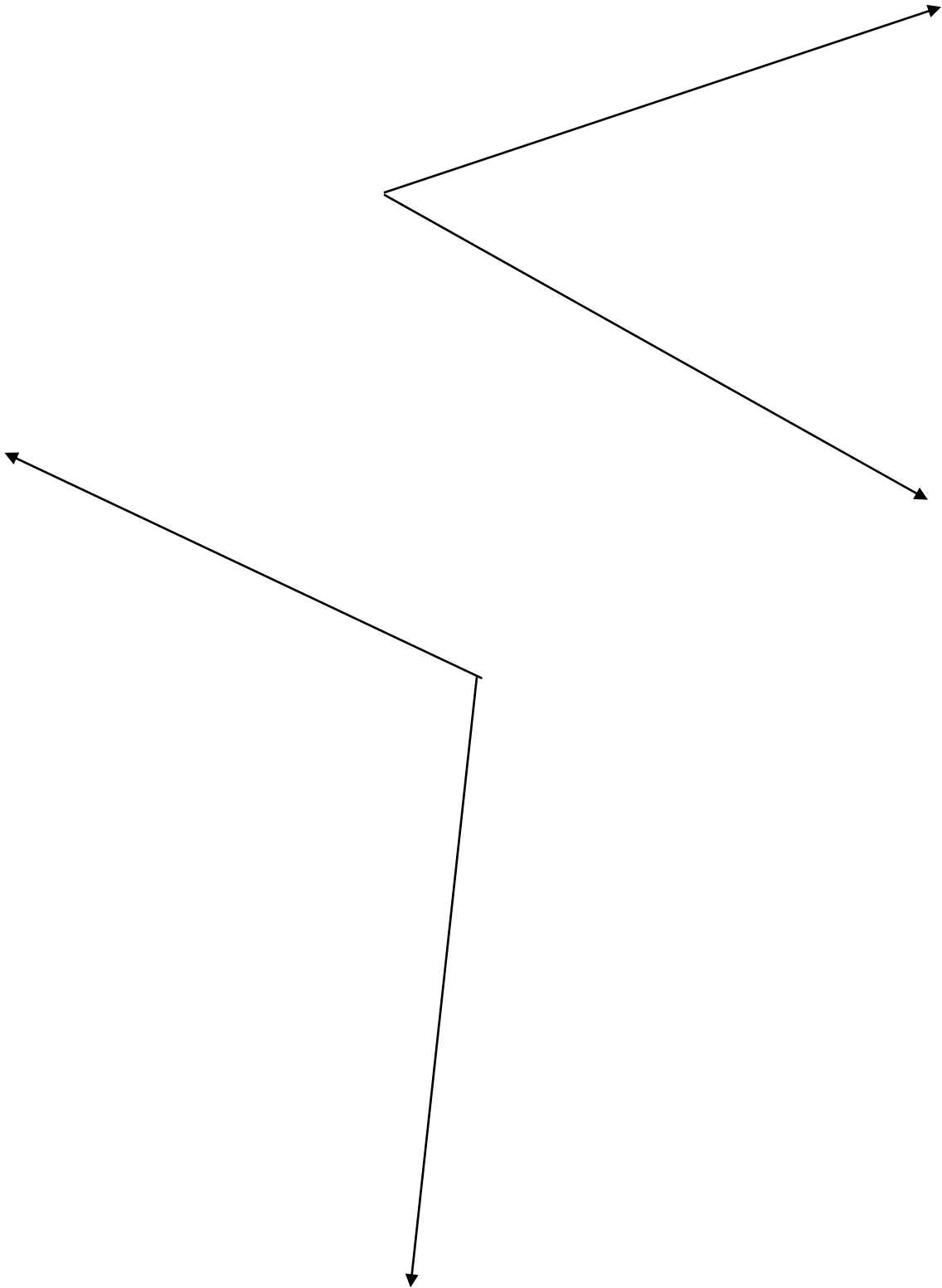
A straightedge is simply a guide for the pencil when drawing straight lines. In most cases you will use a ruler for this, since it is the most likely to be available, *but you must not use the markings on the ruler during constructions*. If possible, turn the ruler over so you cannot see them.

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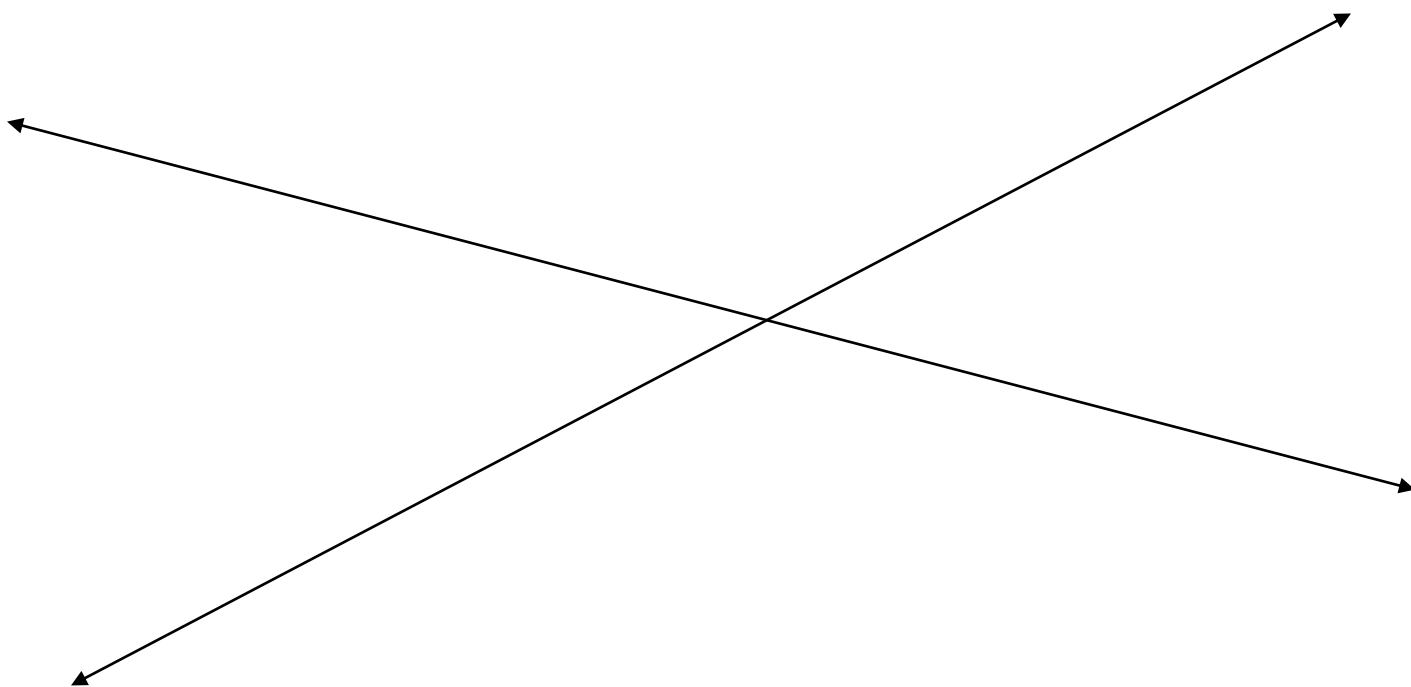
Chapter 1 Constructions

- 1) 1-4 Construct the bisector of an angle. Mark the congruent angles.

<http://www.mathopenref.com/constbisectangle.html>

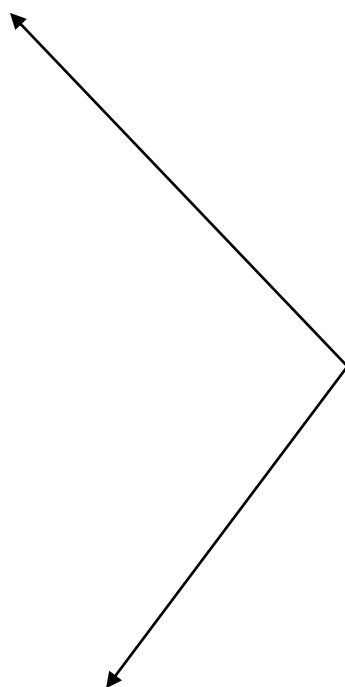
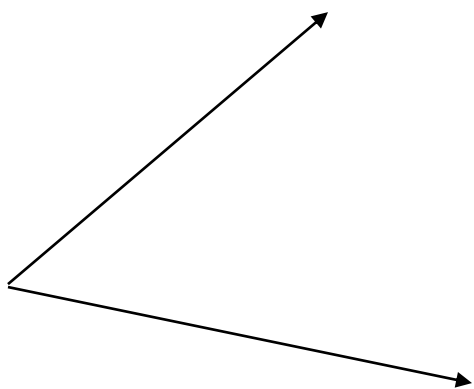


2) 1-4 Challenge Problem: Bisect all 4 angles, using the fewest arcs and lines. (record: 4 arcs 2 lines)



3) 1-4 Construct an angle congruent to a given angle. Include the markings for congruent angles.

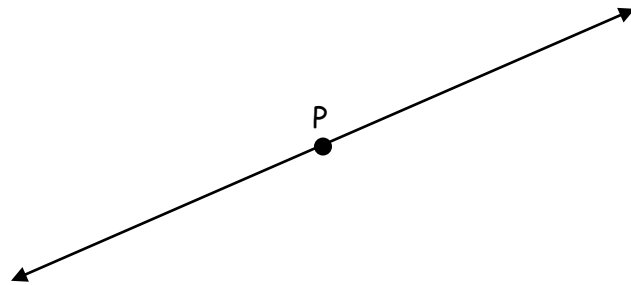
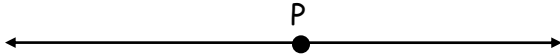
<http://www.mathopenref.com/constcopyangle.html>



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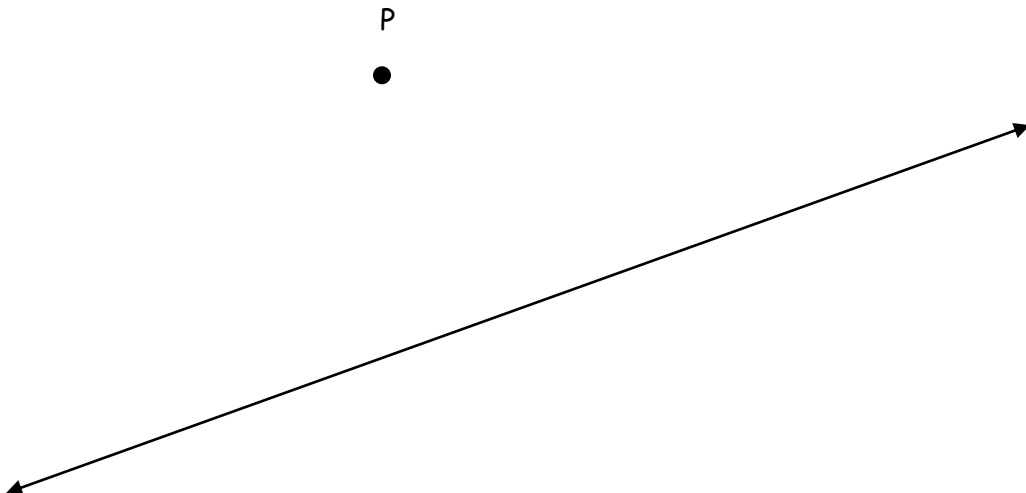
Chapter 3 Constructions

- 1) 3-1 Given a point on a line, construct a line through P, perpendicular to the line. Mark all right angles.
<http://www.mathopenref.com/constperplinepoint.html>



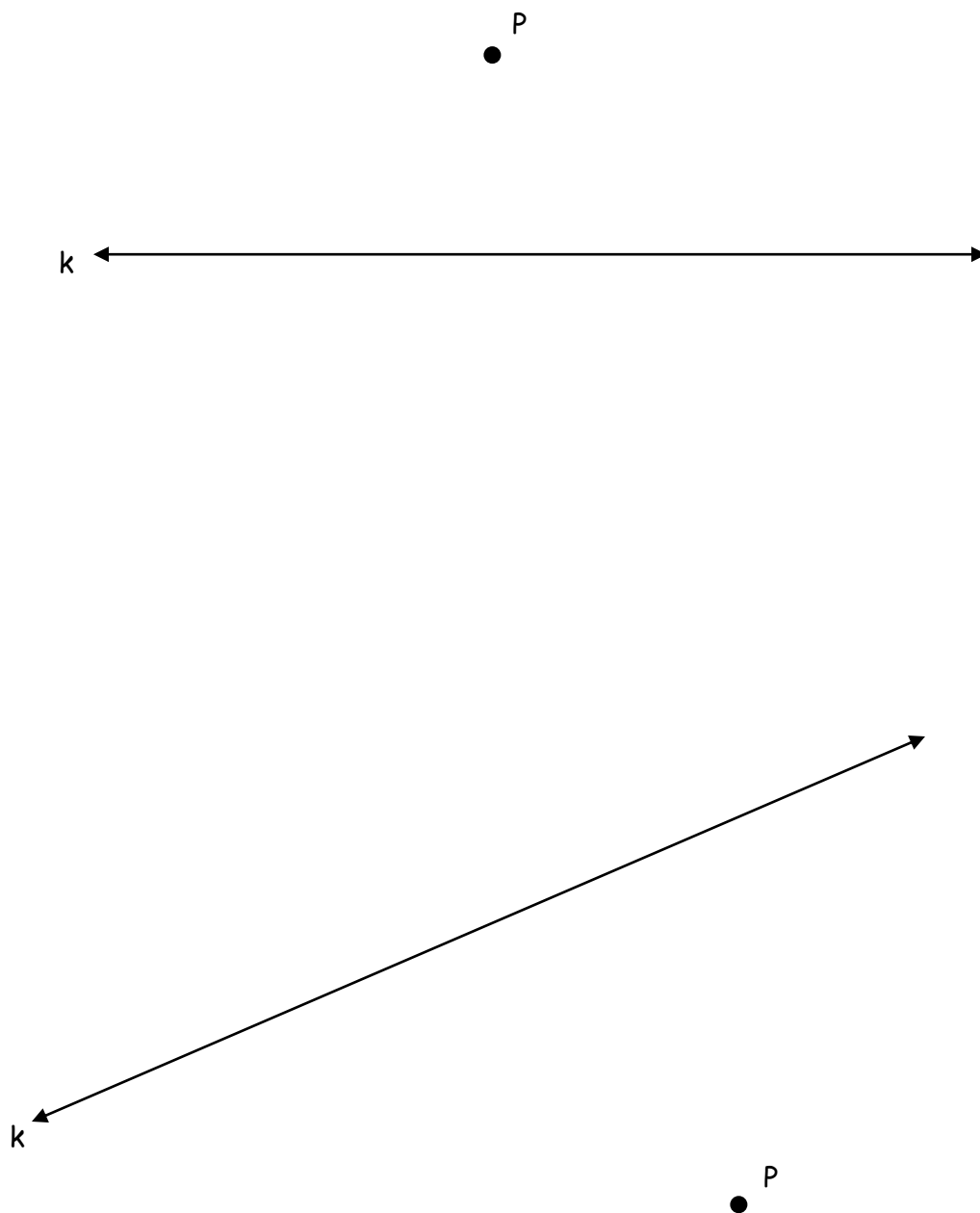
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- 2) 3-1 Given a point not on a line, construct a line through P, perpendicular to the line. Mark the right angle.

<http://www.mathopenref.com/constperpextpoint.html>



- 3) 3-1 Given a line and a point, construct a line through the point P, parallel to the given line k.
Include markings for parallel lines.

<http://www.mathopenref.com/constparallel.html>



4) 3-1 Include all markings for right angles and congruent segments and angles where applicable.

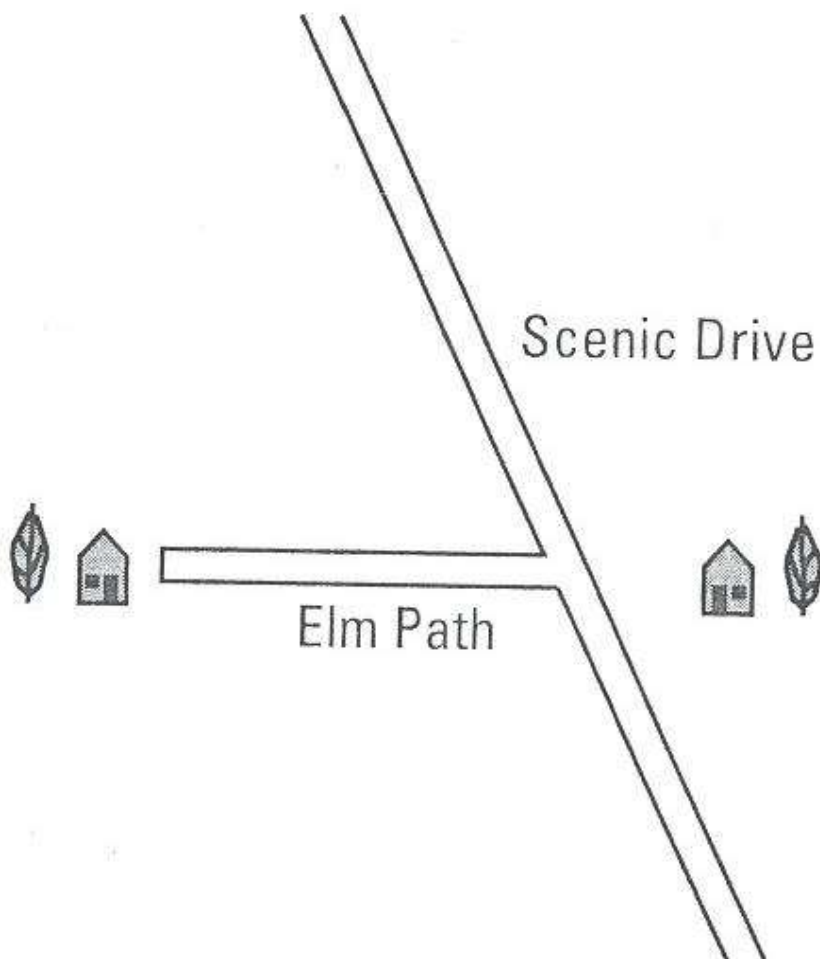
a) A new road, Shadow Lane, is to be built perpendicular to Elm Path at its midpoint.

Construct and label Shadow Lane.

<http://www.mathopenref.com/constbisectline.html>

b) Another road, Park Crossway, is to be built parallel to Elm Path through the intersection of Shadow Lane and Scenic Drive. On the drawing above, construct and label Park Crossing.

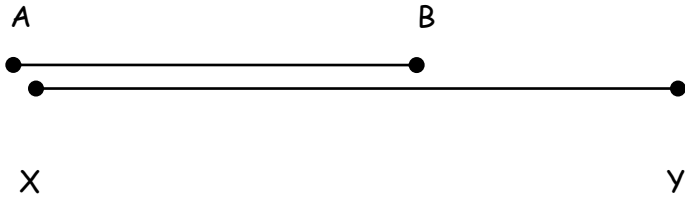
<http://www.mathopenref.com/constparallel.html>



Chapter 4-1 Constructions

- 1) 4-1 Construct a segment that is congruent to each segment below. Mark the congruent segments.

<http://www.mathopenref.com/constcopysegment.html>



- 2) 4-1 Given a line segment as one side, construct an equilateral triangle.

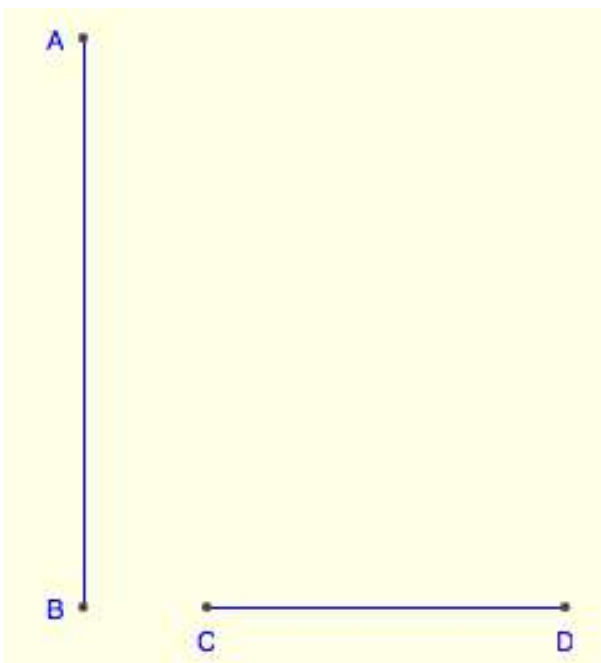
Show markings for congruent sides.

<http://www.mathopenref.com/constequilateral.html>



- 3) 4-1

<http://www.mathopenref.com/constisosceles.html>

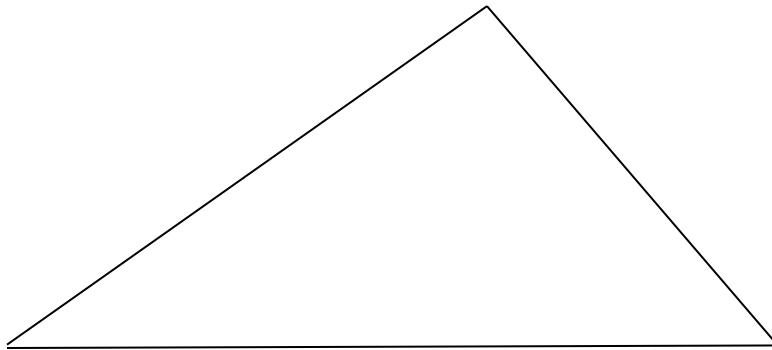


Name _____ Hour _____

Chapter 4-2 Constructions

Construct a triangle congruent to the given triangle. Include markings for congruent corresponding sides and angles.

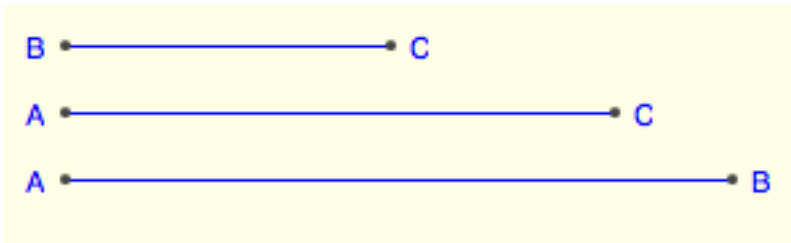
<http://www.mathopenref.com/constcopytriangle.html>



Chapter 4-3-4-5 Constructions

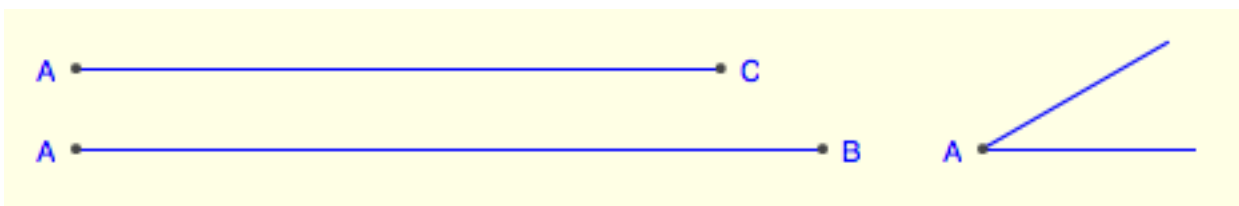
- 1) Construct a triangle given all three sides. (SSS) Include markings for congruent corresponding sides.

<http://www.mathopenref.com/consttrianglesss.html>



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- 2) Construct a triangle given all three sides. (SAS)

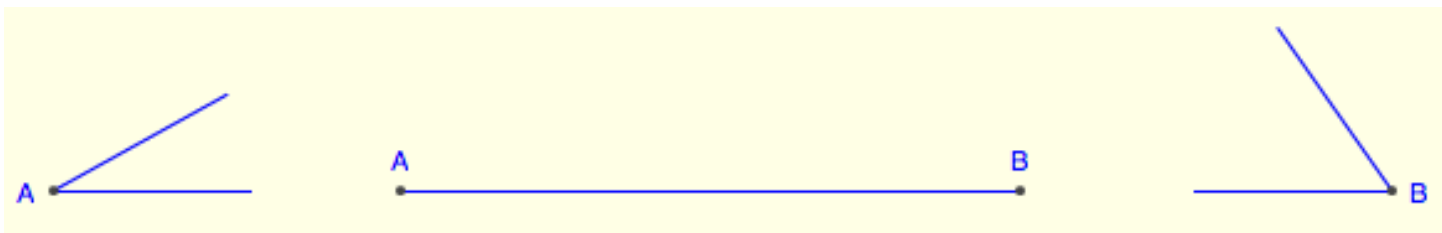
<http://www.mathopenref.com/consttrianglesas.html>



4) 4-5 Construct a triangle given one side and adjacent angles. (ASA)

Include markings for congruent corresponding sides and angles.

<http://www.mathopenref.com/consttriangleasa.html>



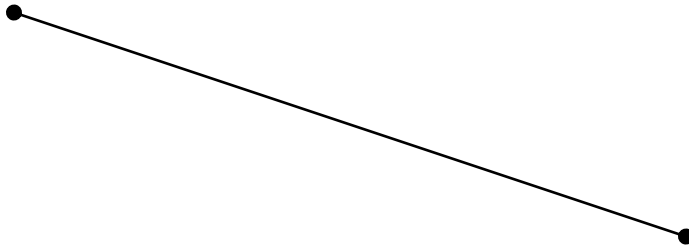
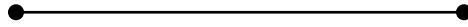
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Name _____ Hour _____

Chapter 5-2 Constructions

- 1) 5-2 Construct the midpoint/perpendicular bisector of a line segment. Mark congruent segments and right angles. Label the midpoint "M".

<http://www.mathopenref.com/constbisectline.html>



2) 5-2 Include all markings for right angles and congruent segments and angles where applicable.

A) Determine the **circumcenter** of the three triangles below. **Label it "C"**.

<http://www.mathopenref.com/constcircumcenter.html>

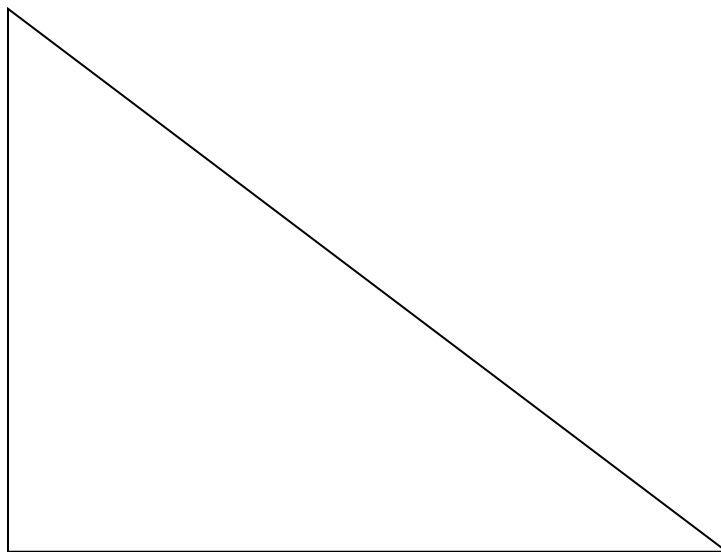
B) Then **circumscribe** a circle about each triangle.

<http://www.mathopenref.com/constcircumcircle.html>

Circumcenter

The point of *concurrency* of a triangle's three _____.

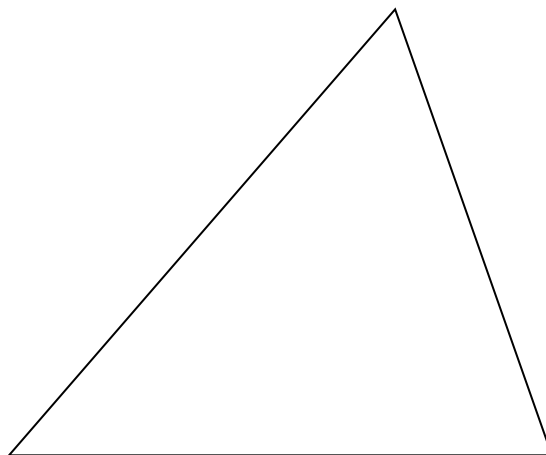
2a) Right Triangle: The **circumcenter** is located _____ the triangle.



What is the distance from the circumcenter to each vertex? _____.

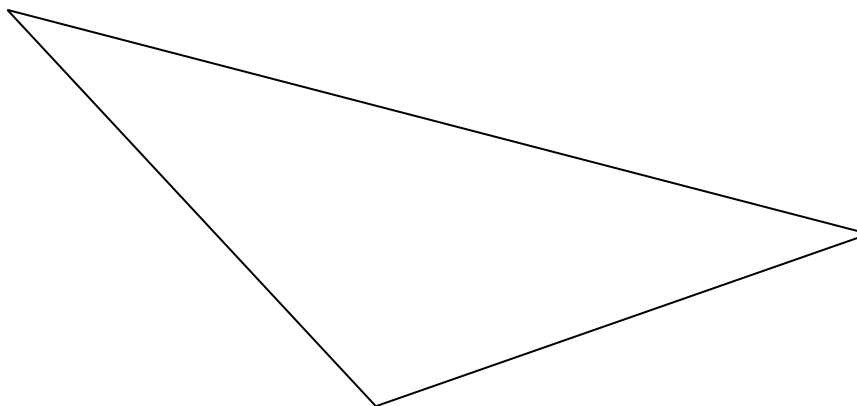
This distance is the _____ of the circumscribed circle.

2b) Acute Triangle: The **circumcenter** is located _____ the triangle.



What is the distance from the circumcenter to each vertex? _____.
This distance is the _____ of the circumscribed circle.

2c) Obtuse Triangle: The **circumcenter** is located _____ the triangle.



What is the distance from the circumcenter to each vertex? _____.
This distance is the _____ of the circumscribed circle.

Name _____ Hour _____

Chapter 5-3 Constructions

1) 5-3 Include all markings for right angles and congruent segments and angles where applicable.

a) Construct the **incenter** of the triangle below. **Label it "I"**.

<http://www.mathopenref.com/constincenter.html>

b) Then *inscribe* a circle in the triangle.

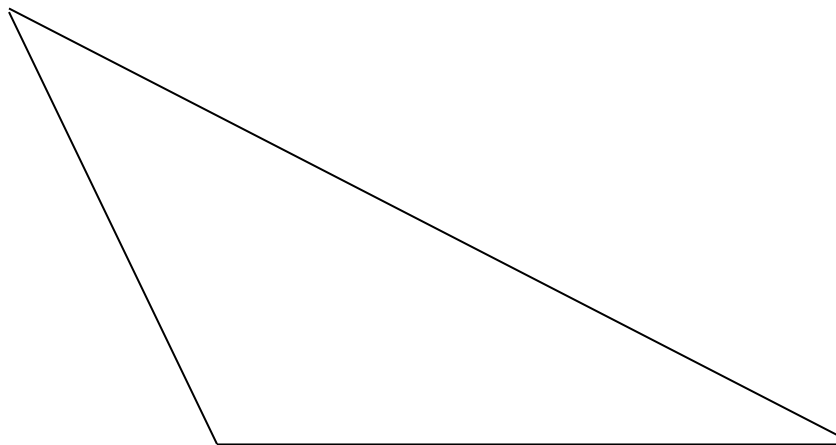
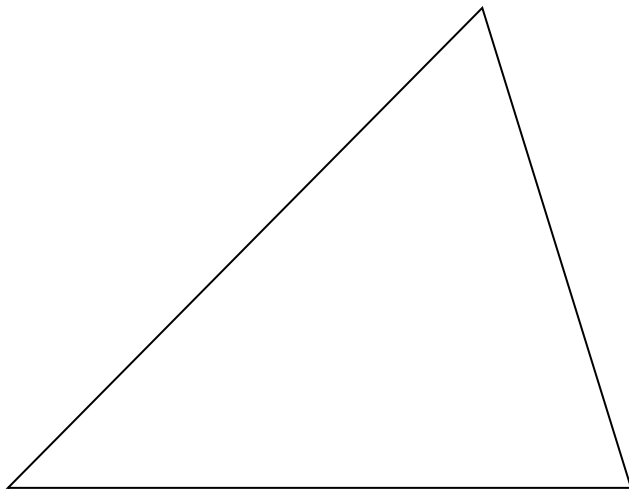
<http://www.mathopenref.com/constincircle.html>

Incenter

The point of *concurrency* of a triangle's three _____.

This point is the center of the triangles incircle.

The *incenter* is always located in the _____ of the circle.

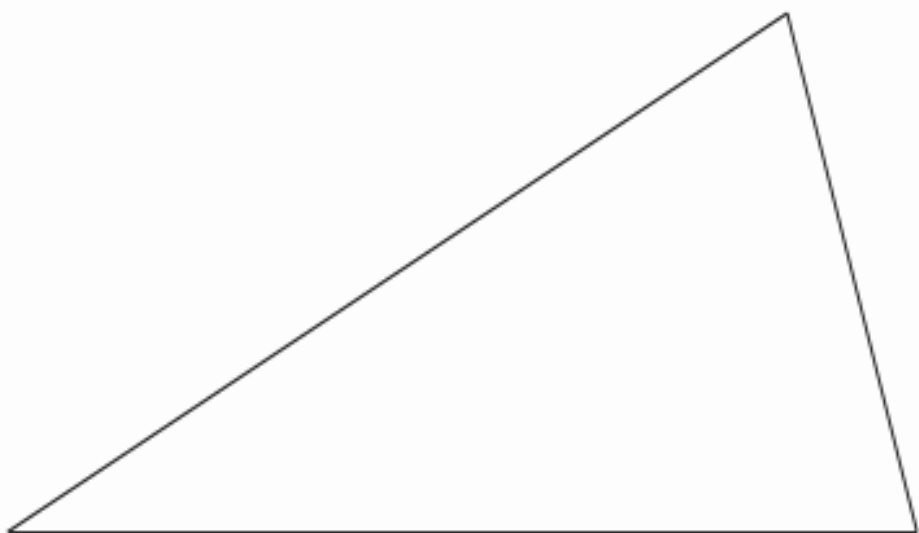


What is the distance from the incenter to intersection of each side and the inscribed circle? _____

This distance is the _____ of the inscribed circle.

2) 5-3 **Inscribe** a circle.

<http://www.mathopenref.com/constincircle.html>

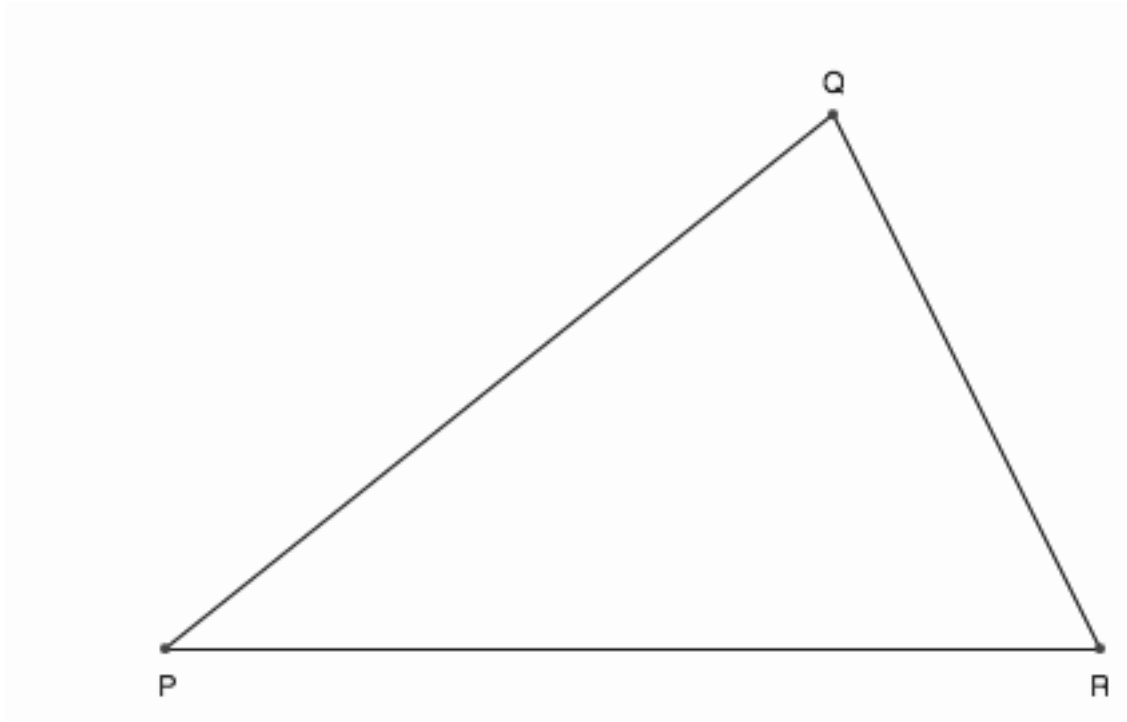


Name _____ Hour _____

Chapter 5-4 Constructions

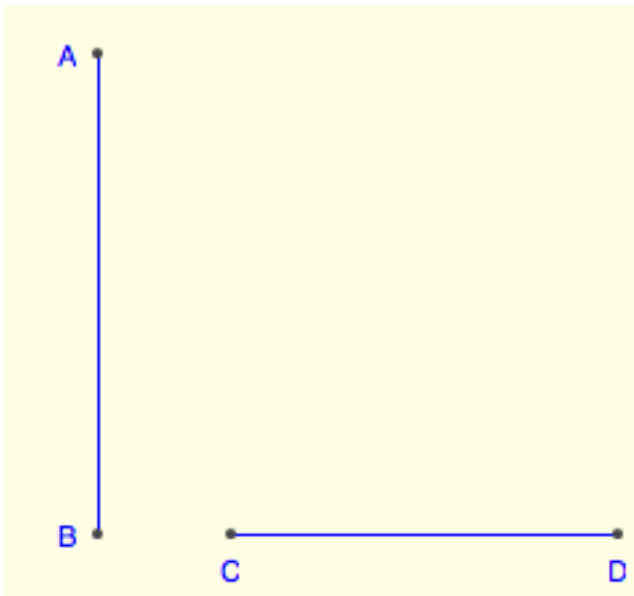
- 1) 5-4 Construct the **medians** of the triangle. Label congruent parts.

<http://www.mathopenref.com/constmedian.html>



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- 2) 5-4 Construct an **isosceles triangle** given the base CD and altitude AB . Label right angles and congruent sides and angles.

<http://www.mathopenref.com/constisosceles2.html>



3) 5-4 Construct the **centroid** of each triangle below. **Label it "C"**. Include all markings for right angles and congruent segments and angles where applicable.

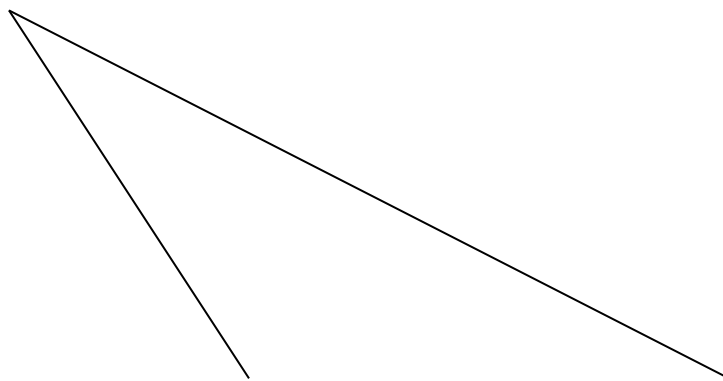
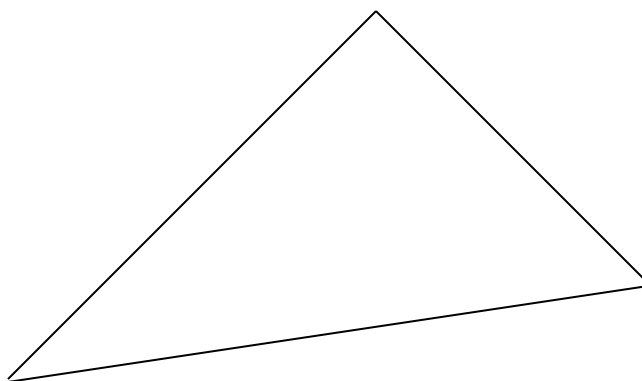
<http://www.mathopenref.com/constcentroid.html>

Centroid

The point of *concurrency* of a triangle's three _____.

The **centroid** is always located in the _____ of the circle.

Fact: It is the **center of gravity** of the triangle. The medians divide the triangle into **six** triangles all having the same area.



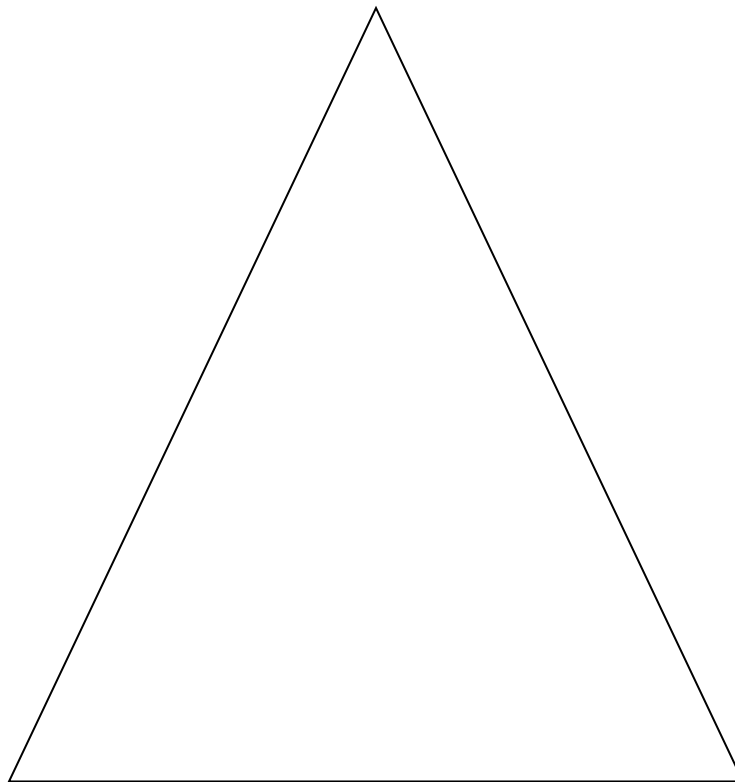
4) 5-4 Construct the **orthocenter** of each triangle. **Label it "O"**. Label all right angles.

<http://www.mathopenref.com/constorthocenter.html>

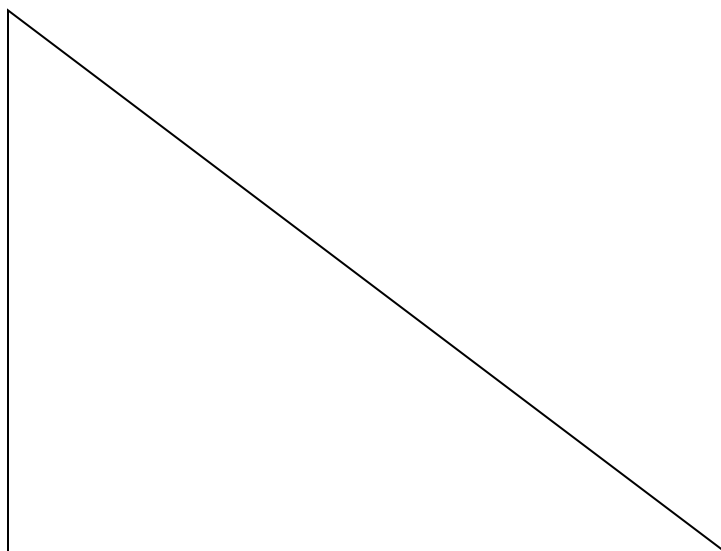
Orthocenter

The point of *concurrency* of a triangle's three _____.

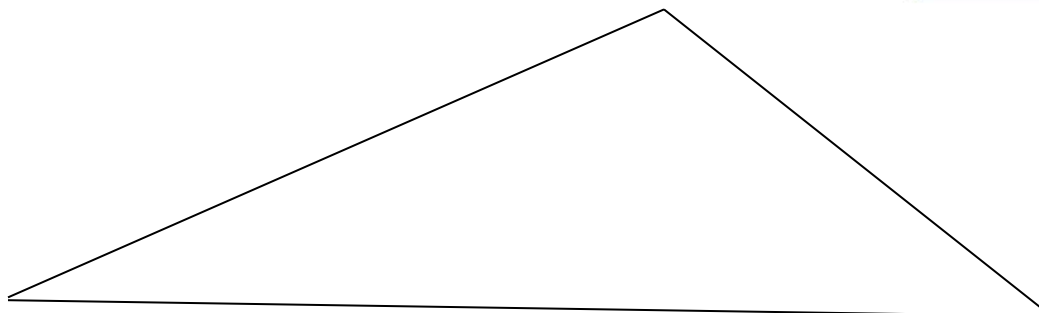
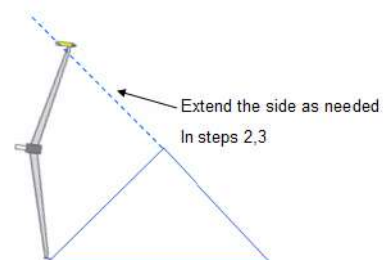
a) Acute Triangle: The **orthocenter** is located _____ the triangle.



b) Right Triangle: The **orthocenter** is located _____ the triangle.



c) Obtuse Triangle: The **orthocenter** is located _____ the triangle.
(Extend the shorter sides of triangle until you can draw the arcs across from each vertex.)



Name _____ Hour _____

Chapter 7 Constructions

7-4

- 1) Construct a **45,45,90 degree triangle**. Label all angles.

<http://www.mathopenref.com/constangle45.html>



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- 2) Construct a **30,60,90 degree triangle**. Label all angles.

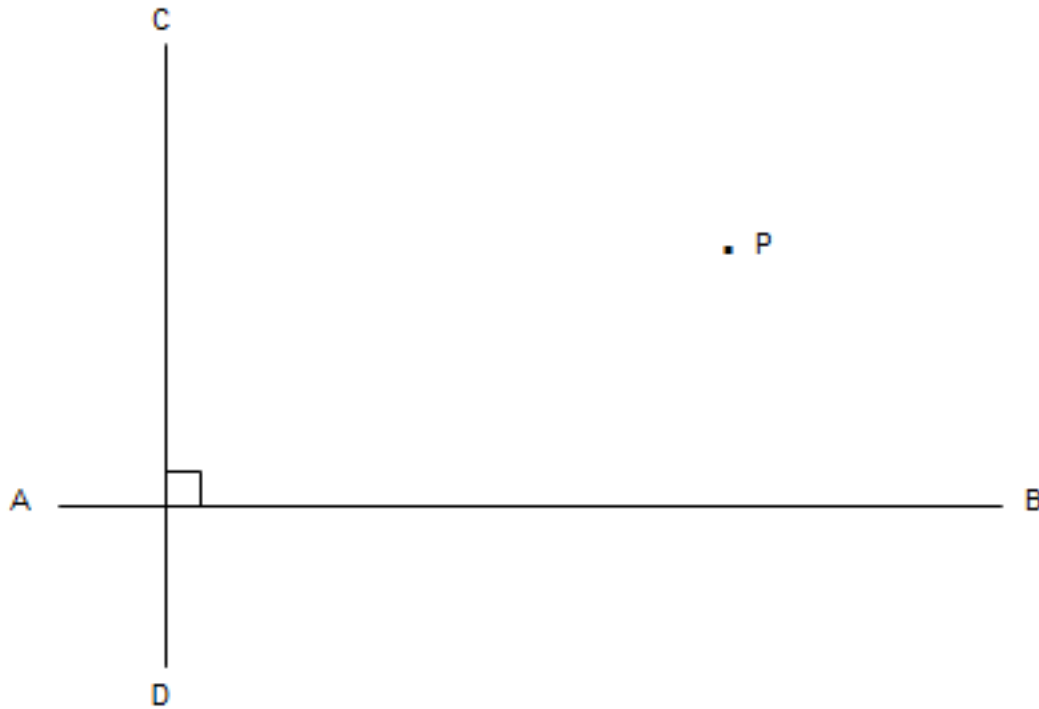
<http://www.mathopenref.com/const306090.html>



Chapter 8 Constructions

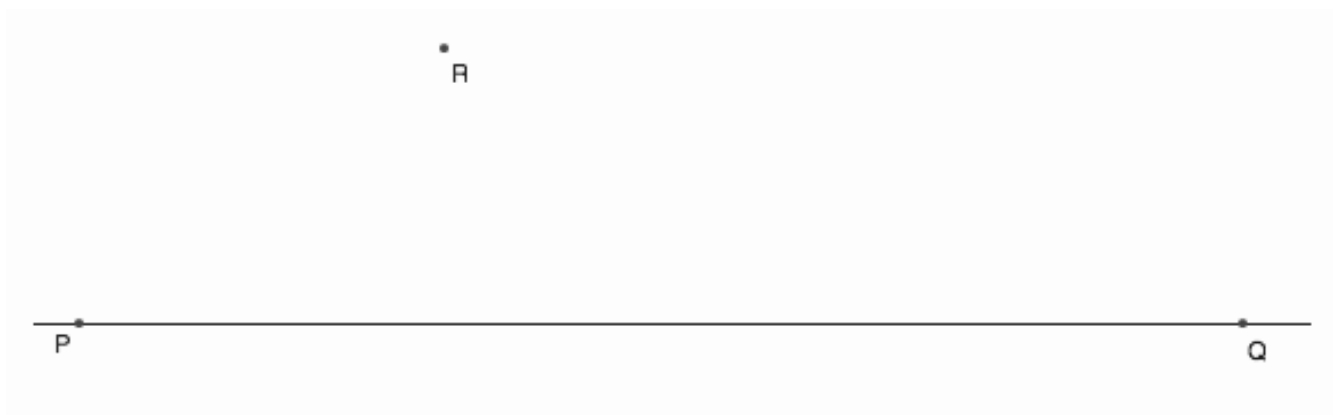
1) 8-4

- a) Construct a line perpendicular to AB through P, and another line perpendicular to CD also through P.
(<http://www.mathopenref.com/constperpextpoint.html>)
- b) What is the name of the resulting 4-sided shape? _____
- c) Measure its side lengths with a ruler and calculate its area.



2) 8-4

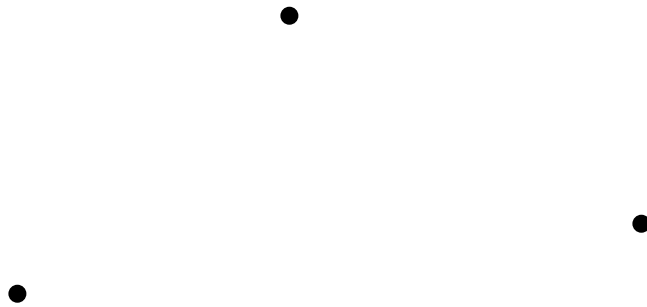
- a) Construct a parallel through a point.
<http://www.mathopenref.com/constparallelrhombus.html>
- b) Name the figure constructed: _____



Chapter 10-1 Constructions

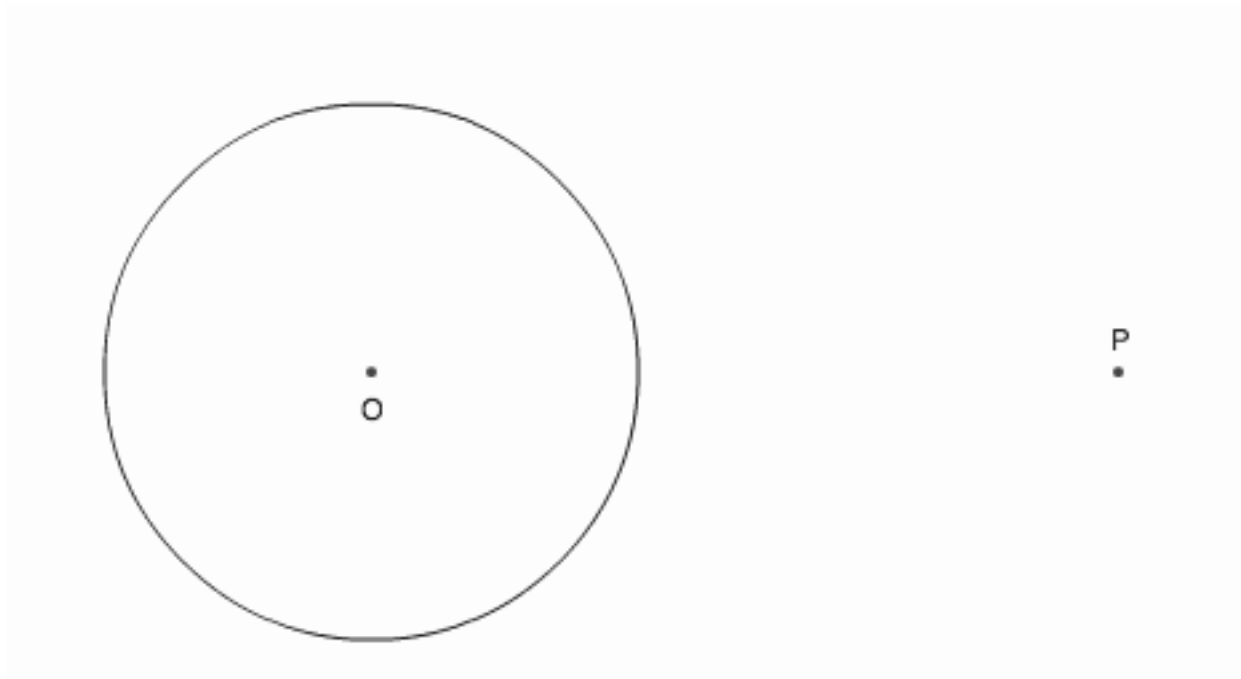
- 1) 10-1 Given three points, draw the circle containing the three points.

<http://www.mathopenref.com/const3pointcircle.html>

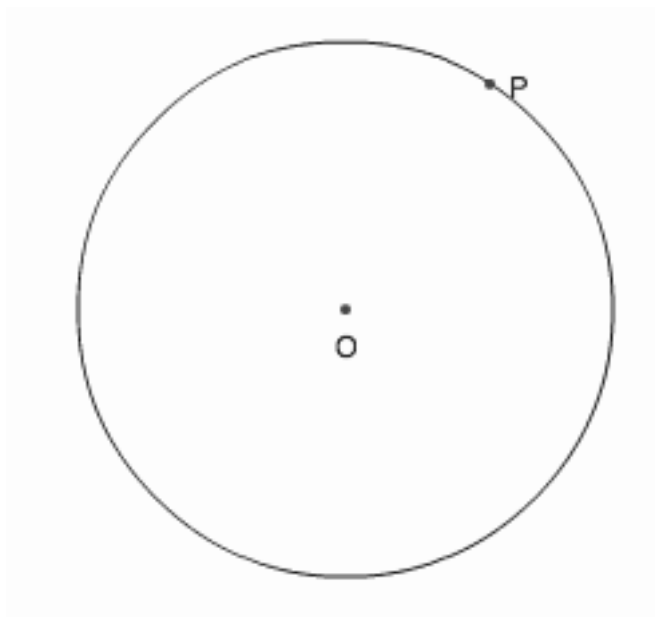


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- 2) 10-1 Construct **tangents** to a circle through an external point P. Label all congruent segments.

<http://www.mathopenref.com/consttangents.html>



- 3) 10-1 Construct a **tangent** to the circle through point P on the circle. Identify the right angle.
<http://www.mathopenref.com/consttangent.html>



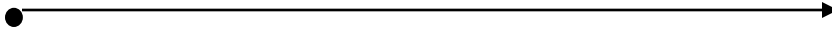
Name _____ Hour _____

Chapter 10-4 Constructions

1) 10-4

a) Construct a perpendicular to a ray at its endpoint.

<http://www.mathopenref.com/constperpendray.html>



b) Observations: If one side of an inscribed triangle is the diameter of a circle, then the angle opposite the diameter is a _____ angle.

2) 10-4

a) Construct a right angle.

<http://www.mathopenref.com/constangle90.html>



b) Observations: The _____ of the circle is the hypotenuse of the inscribed right triangle.

Chapter 11 Constructions

1) 11-6

a) Construct a **regular hexagon** given one side length.

<http://www.mathopenref.com/consthexagon.html>



b) Draw all radii from the center of the circle to each vertex.

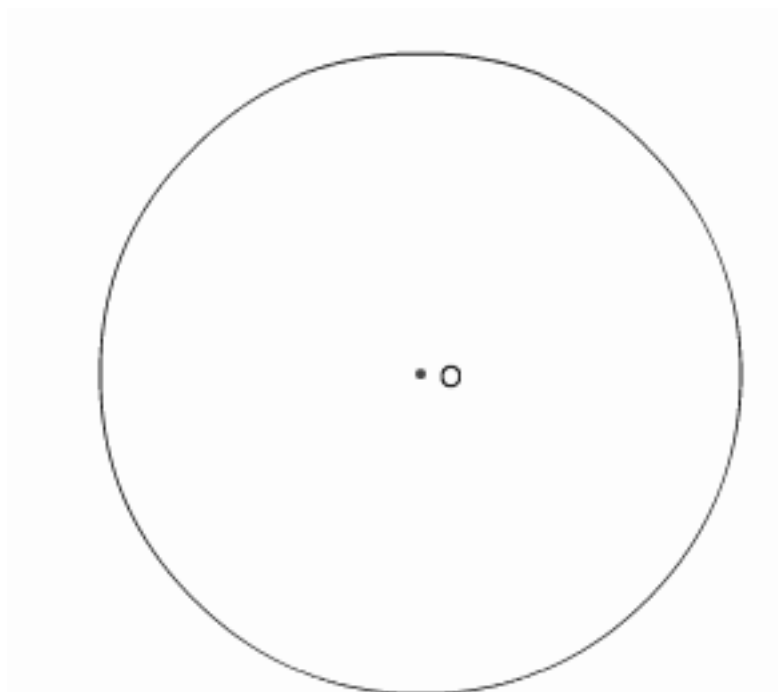
How many isosceles triangles does this form? ____ How many degrees does each central angle have? ____

Are each of these triangles also equilateral? Explain.

c) Explain how to find the area of the hexagon. (Do not actually find the area)

2) 11-6 Inscribe a hexagon in the circle.

<http://www.mathopenref.com/constinhexagon.html>



Extra Credit:

11-6 Inscribe a regular pentagon in the circle.

<http://www.mathopenref.com/constinpentagon.html>

