Conservation of Momentum Problems

The key to understanding all these problems is to remember the Law of Conservation of Momentum: If there are no external forces acting on a system, the total momentum after an interaction between objects is equal to the total momentum before the interaction. Calculate the sum of the initial momentum (mass x velocity) for all objects involved. Set this number (or expression) equal to the sum of the final momentum for all the objects. We can categorize momentum problems into several basic types.

Inelastic Collisions

In inelastic collisions, after the collision both objects stick together as one body whose mass equals the combined masses of the two colliding objects.

$p_i = p_f$ so

$m_1v_1 + m_2 v_2 = (m_1 + m_2)v_f$ and so solving for final velocity:

$v_f = (m_1v_1 + m_2 v_2)/(m_1 + m_2)$

<u>Type 1</u>: One-dimensional inelastic collision with one object at rest. This is the easiest type of problem since all the initial momentum is contained in the one moving object.

Example: A 50-kg ice skater moving to the right at 6 m/s collides with a 25-kg ice skater at rest. After the collision they slide off together holding on to one another for balance. What is their final velocity?

Solution: v_f =(50 kg)(6 m/s)/(50 kg + 25 kg) = 4 m/s

<u>Type 2</u>: One-dimensional inelastic collision with both objects moving in the same direction.

Example: The same two ice-skaters except the second skater is moving at 3 m/s to the right. Find the final velocity of the pair.

Solution: v_f = [(50 kg)(6 m/s) + (25 kg)(3m/s)]/(50 kg + 25 kg) = 5 m/s

<u>Type 3</u>: One-dimensional inelastic collision with both objects moving in opposite directions.

Example: The same two ice-skaters except the second skater is moving at 3 m/s to the left. Find the final velocity of the pair.

Solution: If we call the velocity of the first skater to the right positive, we must call the velocity of the second skater negative. v_f = [(50 kg)(6 m/s) + (25 kg)(-3m/s)]/(50 kg + 25 kg) = 3 m/s

Since it is also positive, they are moving to the right.

<u>Type 4</u>: Explosion or Recoil event. This is basically an inelastic collision in reverse, where the two objects are combined as one unit and then split apart, moving away from one another.

Example: The same two skaters are facing each other motionless on the ice when they push against each other and move apart. If the 50 kg skater moves to the left at 3m/s, what is the velocity of the 25-kg skater?

Solution: Initial momentum is zero since nothing is moving. Therefore:

0 = (50 kg)(-3 m/s) + (25 kg)v_2 and rearranging we get:

v_2 = (50 kg)(3 m/s)/25 kg = 6 m/s (to the right, since it is positive)

<u>Type 5</u>: Two-dimensional inelastic collision. This is a vector addition problem—the final momentum will be the vector sum of the two initial momentum vectors. For right-angle vectors, use the Pythagorean Theorem to find the magnitude and the $\tan^{-1}$ function to find the direction of the final momentum. The magnitude of the final velocity will be the magnitude of the final momentum divided by the combined mass, and the direction will be the same as the final momentum vector.

Example: A 50-kg ice skater moving to the north at 6 m/s collides with a 25-kg ice skater moving east at 4 m/s. After the collision they slide off together holding on to one another for balance. What is their final velocity?

Solution: p_1 = (50 kg)(6 m/s) = 300kg m/s and p_2 = (25 kg)(4 m/s) = 100 kg m/s v_f = [(300 kg m/s)^2 + (100 kg m/s)^2]$^{1/2}$ / (50 kg + 25 kg) @ $\tan^{-1}$(100/300) E of N

v_f = 4.2 m/s @ 18^o E of N (pardon my sig figs)

Elastic Collisions

In elastic collision problems, the colliding objects are moving independently after the collision, so the problems are more complicated. Momentum and kinetic energy are both conserved in these collisions. We will limit our study to elastic collisions between objects of equal mass, with only one object moving initially. For this special case, the mass terms will cancel out of the equations and we can deal with just the velocities. The equations for elastic collisions are:

$p_i = p_f$ so

$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$ (Conservation of Momentum)

Also, $K_i = K_f$ so

$\frac{1}{2} m_1v_{1i}^2 + \frac{1}{2} m_2v_{2i}^2 = \frac{1}{2} m_1v_{1f}^2 + \frac{1}{2} m_2v_{2f}^2$ (Conservation of Kinetic Energy)

Since the mass is the same for both objects and the second object is not moving, the equations can be simplified:

$v_{1i} = v_{1f} + v_{2f}$ (Conservation of Momentum) and

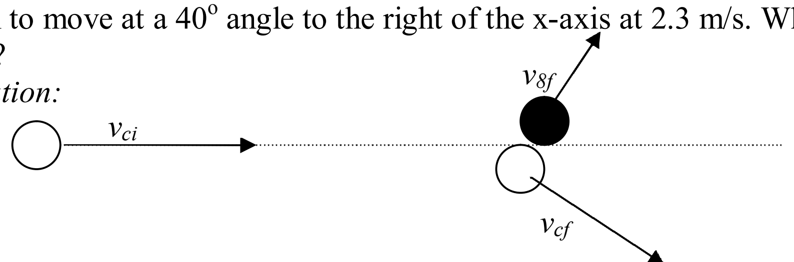
$v_{1i}^2 = v_{1f}^2 + v_{2f}^2$ (Conservation of Energy)

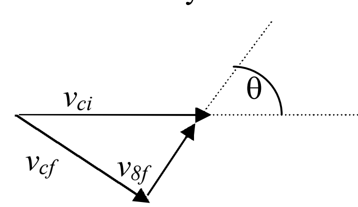
In one-dimension, (like gliders on an air track, railroad cars, dead center collisions) all the momentum of the moving object will be transferred to the second object. The first object will be at rest after the collision. The second object will have the same momentum and velocity as the first object originally had. This is the only way both equations can be satisfied.

In two-dimensions, (like shooting pool) both objects will be moving off at some angle from the original direction of motion. Remember these are vector quantities, so the momentum equation tells us that the vector sum of the two final velocity (and momentum) vectors must equal the original velocity (momentum) vector. They will make up a vector triangle. You may notice the energy equation is in the same form as the Pythagorean theorem, which tells us that the angle between the two final velocity (and momentum) vectors will be 90^o. Problems involving two-dimensional elastic collisions will be vector addition problems, either graphical or calculation type problems, always involving right triangle geometry.

Example: In a pool game, the cue ball, moving at 3.0 m/s along an imaginary x-axis that runs the length of the table, strikes the eight ball slightly off-center. After the collision, the cue ball is seen to move at a 40^o angle to the right of the x-axis at 2.3 m/s. What was the velocity of the 8-ball?

Solution:





From the vector diagram, we can see

the magnitude of the final velocity of the 8-ball, $v_{8f} = [(v_{ci})^2 - (v_{cf})^2]^{1/2}$ and the direction will be the angle θ whose cosine is equal to v_{8f}/ v_{ci} to the left of the imaginary x-axis. Plugging in the values from the problem,

$v_{8f} = [(3.0 \text{ m/s})^2 - (2.3 \text{ m/s})^2]^{1/2} = 1.9 \text{ m/s}$

$\theta = \cos^{-1}(1.9/3.0) = 50^o$ (Did we really need to calculate the angle this way?)