Section 6.3

Unit G -Day 4

Find the composition of functions
Determine the domain of a composition

Perform the indicated operations and state the domain in set builder notation and interval notation. (Domain for #1 and #2 only).

Let $f(x)=-2x + 5$ and $g(x)=x^2 + 2$

1. $f(g(x))$

$f(x) = -2x+5$

$g(x) = x^2+2$

$f(g(x)) = -2(x^2+2)+5$

$= -2x^2-4+5$

$\boxed{f(g(x)) = -2x^2+1}$

$\boxed{D: (-\infty,\infty) \text{ or } \{x : x \in \mathbb{R}\}}$

2. $g(f(x))$

$= (-2x+5)^2+2$

$= (-2x+5)(-2x+5)+2$

$= 4x^2-10x-10x+25+2$

$= 4x^2-20x+27$

$D: (-\infty,\infty)$

or

$\{x : x \in \mathbb{R}\}$

3. $f(g(4))$

$g(4) = 4^2+2$

$= 18$

$f(18) = -2(18)+5$

$= -36+5$

$= \boxed{-31}$

Let $g(x) = 7x - 1$, $h(x) = \frac{2}{x}$ & $k(x) = \frac{2x}{x+5}$

D: $\mathbb{R}$ D: $x \neq 0$ D: $x \neq -5$

Perform the indicated operations and state the domain (#4 -7) in set builder notation and interval notation

4. $(h \circ g)(x)$

$h(g(x))$

$g(x) = 7x - 1$

$h(x) = \frac{2}{x}$

$\boxed{h(g(x)) = \frac{2}{7x-1}}$

D: $7x - 1 \neq 0$

$x \neq 1/7$

$\boxed{(-\infty, 1/7) \cup (1/7, \infty) \quad \{x : x \neq \frac{1}{7}\}}$

5. $g \circ h$

$g(h(x))$

$= 7\left(\frac{2}{x}\right) - 1$

$= \frac{14}{x} - 1$

$= \boxed{\frac{14-x}{x}}$

side work

$\frac{14}{x} - 1$

$\frac{14}{x} - \frac{x}{x}$

$\frac{14-x}{x}$

$\boxed{D: (-\infty, 0) \cup (0, \infty) \quad \{x : x \neq 0\}}$

6. $h(k(x))$

$= \frac{2}{\frac{2x}{x+5}}$ copy, dot, flip

$= \frac{\cancel{2}^{1}}{1} \cdot \frac{x+5}{\cancel{2x}_{x}}$

$= \boxed{\frac{(x+5)}{x}}$

D: $x \neq 0$
$x \neq -5$

$\boxed{(-\infty, -5) \cup (-5, 0) \cup (0, \infty) \quad \{x : x \neq -5, 0\}}$

7. $k(h(x))$

$= \frac{2\left(\frac{2}{x}\right)}{\frac{2}{x} + 5}$

$= \frac{\frac{4}{x}}{\frac{2}{x} + \frac{5}{1} \cdot \frac{x}{x}}$

$= \frac{\frac{4}{x}}{\frac{2+5x}{x}}$

$= \frac{4}{\cancel{x}} \cdot \frac{\cancel{x}}{2+5x}$

$= \boxed{\frac{4}{2+5x}}$

D: $x \neq 0 \rightarrow$ from (h(x))

$x \neq -2/5$

$2 + 5x \neq 0$

$5x \neq -2$

$x \neq -2/5$

$\boxed{(-\infty, -2/5) \cup (-2/5, 0) \cup (0, \infty) \quad \{x : x \neq -2/5, 0\}}$

8. $(h \circ g)(3)$

$h(g(3))$

$g(3) = 7(3) - 1$

$= 20$

$h(20) = \frac{2}{20}$

$= \boxed{\frac{1}{10}}$

Let p(x) = 4x - 3, h(x) = x^2 + 5, & k(x) = $\sqrt{x - 8}$

D: ℝ D: ℝ $x - 8 \geq 0$, $x \geq 8$ cannot have negative under √

9. Find h(p(0.5x - 1))

$p(0.5x-1) = 4(0.5x-1) - 3$
$= 2x - 4 - 3$
$= 2x - 7$

$h(2x-7) = (2x-7)^2 + 5$
$= (2x-7)(2x-7) + 5$
$= 4x^2 - 14x - 14x + 49 + 5$
$= \boxed{4x^2 - 28x + 54}$

#10-13 Find the indicated composition and state the domain.

10. h(k(x))

$(\sqrt{x-8})^2 + 5$

$h(k(x)) =$
got this from k(x) $= x - 8 + 5$
$= x - 3$ $\boxed{D: [8, \infty)}$
$\boxed{D: \{x : x \geq 8\}}$ $-\infty$ 8 ∞

11. k(p(x))

$k(p(x)) = \sqrt{4x-3-8}$
$= \boxed{\sqrt{4x-11}}$

D:
$4x - 11 \geq 0$
$x \geq 11/4$
$\boxed{[11/4, \infty)}$

12. h(p(x))

$= (4x-3)^2 + 5$
$= (4x-3)(4x-3) + 5$
$= 16x^2 - 24x + 9 + 5$
$= 16x^2 - 24x + 14$
$= 2(8x^2 - 12x + 7)$
D: $\{x : x \in \mathbb{R}\}$
$(-\infty, \infty)$

13. (p ∘ p)(x)

$p(p(x))$
$= 4(4x-3) - 3$
$= 16x - 12 - 3$
$= \boxed{16x - 15}$
$\boxed{D: (-\infty, \infty) \text{ or } \{x : x \in \mathbb{R}\}}$

Let $f(x) = \frac{3}{x}$, $g(x) = 12x - 9$, & $h(x) = (x - 3)^{\frac{1}{2}}$

D: $x \neq 0$ D: $\mathbb{R}$ $h(x) = \sqrt{x-3}$

D: $x-3 \geq 0$

$x \geq 3$

14. Find f(g(1))

$g(1) = 12 - 9 = 3$

$f(3) = 3/3 = \boxed{1}$

#15-16 Find the indicated composition and state the domain.

15. $(f \circ g)(x)$

$f(g(x)) = \frac{3}{12x-9}$

$= \frac{\cancel{3}}{\cancel{3}(4x-3)}$

$= \boxed{\frac{1}{4x-3}}$

D: $4x - 3 \neq 0$

$x \neq 3/4$

$\boxed{(-\infty, 3/4) \cup (3/4, \infty) \quad \{x : x \neq 3/4\}}$

16. h(g(x))

$(12x - 9 - 3)^{1/2}$

$\boxed{\sqrt{12x-12}}$

$\sqrt{4(3x-3)}$

$\boxed{2\sqrt{3x-3}}$ ☆

$3x - 3 \geq 0$

$x \geq 1$

$\boxed{[1, \infty) \quad \{x : x \geq 1\}}$

$f(g(\underbrace{\underbrace{\underbrace{h(2)}_{1}}_{2}}_{3}))$