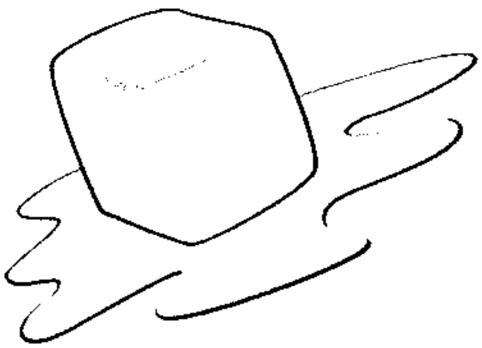
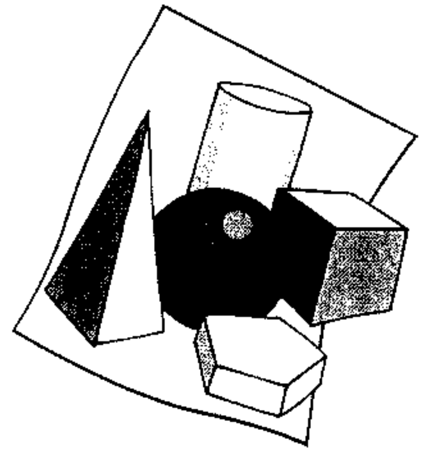
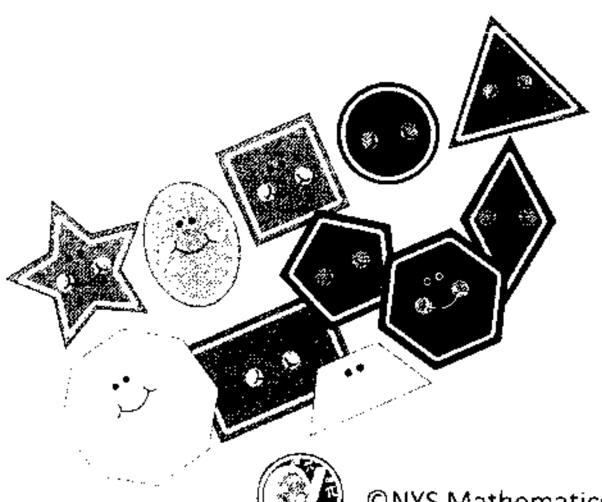
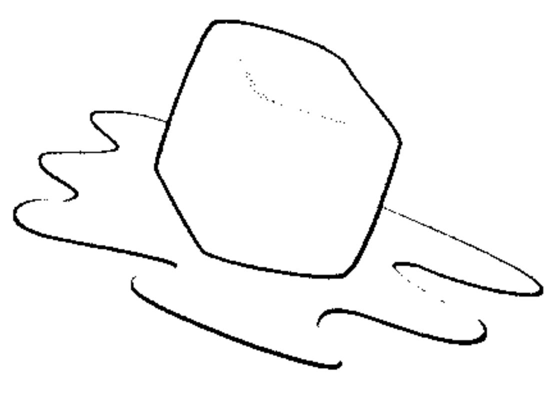




GEOMETRY (COMMON CORE)

FACTS YOU MUST KNOW COLD FOR THE REGENTS EXAM





Polygons – Interior/Exterior Angles

Sum of Interior Angles: 180 (n - 2)

Each Interior Angle of a Regular Polygon: $\frac{180\,(n-2)}{n}$

Sum of Exterior Angles: 360°

Each Exterior Angle: $\frac{360}{n}$

Triangles

Classifying Triangles

Sides:

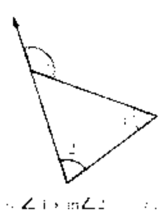
Scalene: No congruent sides
Isosceles: 2 congruent sides
Equilateral: 3 congruent sides

Angles:

Acute: All angles are < 90°
Right: One right angle that is 90°
Obtuse: One angle that is > 90°
Equiangular: 3 congruent angles (60°)

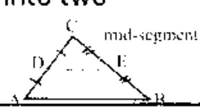
All triangles have 180°

Exterior Angle Theorem: The exterior angle is equal to the sum of the two non-adjacent interior angles.



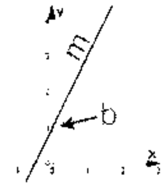
Midsegment: segment joining the midpoints

- Always parallel to the third side
- $\frac{1}{2}$ the length of the third side
- Splits the triangle into two similar triangles.



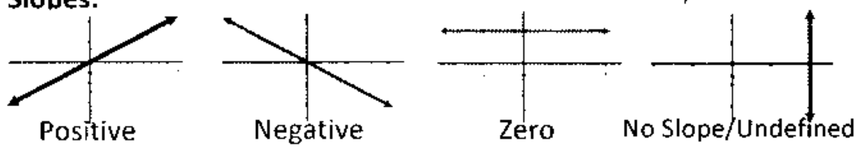
Coordinate Geometry

Standard Form of a Line: $y = mx + b$,where m is the slope and b is the y-intercept.



Slope Formula: $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Slopes:



Parallel Lines have the **SAME** slope

Perpendicular Lines have **NEGATIVE RECIPROCAL** slopes (flip & change the sign)

Collinear Points are points that lie of the **same** line.

Midpoint Formula: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Segment Ratios:

$$\frac{x - x_1}{x_2 - x} = \textit{Given Ratio} \qquad \frac{y - y_1}{y_2 - y} = \textit{Given Ratio}$$

Pythagorean Theorem

To find the missing side of any **right** triangle, use:

$$a^2 + b^2 = c^2$$

where *a* and *b* are the legs, and *c* is the hypotenuse

Isosceles Triangle

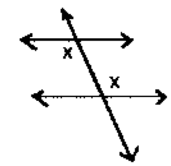
- 2 ≅ sides and 2 ≅ base angles
- The altitude drawn from the vertex is also the median and angle bisector
- If two sides of a triangle are ≅, then the angles opposite those ≅ sides are ≅.

Triangle Inequality Theorems

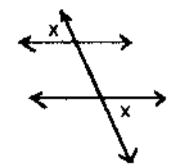
- The sum of 2 sides must be greater than the third side
- The difference of 2 sides must be less than the third side
- The longest side is opposite the largest angle
- The shortest side is opposite the smallest angle

Parallel Lines

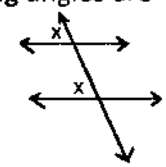
Alternate Interior angles are **congruent**



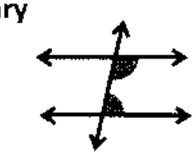
Alternate Exterior angles are **congruent**



Corresponding angles are **congruent**

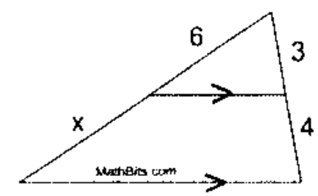


Same-Side Interior angles are **supplementary**

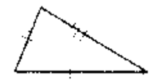


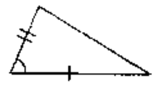
Side – Splitter Theorem

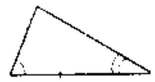
If a line is parallel to a side of a triangle and intersects the other two sides, then this line divides those two sides proportionally.



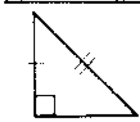
Triangle Congruence Theorems

Side-Side-Side **(SSS)** 

Side-Angle-Side **(SAS)** 

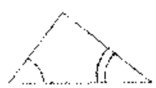
Angle-Side-Angle **(ASA)** 

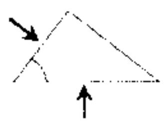
Angle-Angle-Side **(AAS)** 

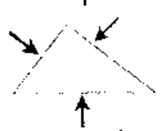
Hypotenuse-Leg **(HL)** 

CPCTC – Corresponding Parts of Congruent Triangles are Congruent

Similar Triangle Theorems

Angle-Angle **(aa)** 

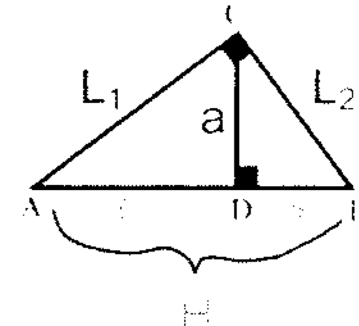
Side-Angle-Side **(SAS)** 

Side-Side-Side **(SSS)** 

- ➢ Similar figures have congruent angles and proportional sides
- ➢ **CSSTP**-Corresponding Sides of Similar Triangles are in Proportion
- ➢ In a proportion, the product of the means equals the product of the extremes

Geometric Mean Theorems

Altitude Theorem (SAAS / Heartbeat Method):
The altitude is the geometric mean between the 2 segments of the hypotenuse.



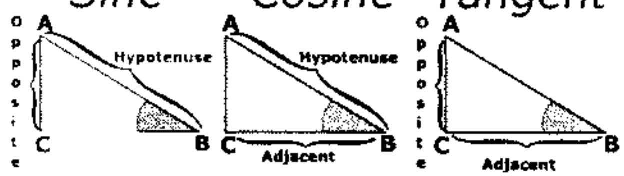
$$\frac{S_1}{a} = \frac{a}{S_2}$$

Leg Theorem (HYLLS / PSSW):
The leg is the geometric mean between the segment It touches and the whole hypotenuse.

$$\frac{S_1}{L_1} = \frac{L_1}{H} \quad \text{and} \quad \frac{S_2}{L_1} = \frac{L_2}{H}$$

Trigonometry

Sine *Cosine* *Tangent*



$\frac{\textit{opposite}}{\textit{hypotenuse}}$ $\frac{\textit{adjacent}}{\textit{hypotenuse}}$ $\frac{\textit{opposite}}{\textit{adjacent}}$

SOH *CAH* *TOA*

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- ➢ When solving for a side, use the *sin, cos*, and *tan* buttons
- ➢ When solving for an angle, use the sin^{-1}, cos^{-1}, and tan^{-1} buttons

Cofunctions:

- ➢ Sine and Cosine are cofunctions, which are complementary

$$\boldsymbol{sin\theta = cos(90^\circ - \theta)}$$
$$\boldsymbol{cos\theta = sin(90^\circ - \theta)}$$

- ➢ If $\angle A$ and $\angle B$ are the acute angles of a right triangle, then $\mathbf{\sin A = \cos B}$

Factoring

The order of Factoring:

Greatest Common Factor (GCF)

↓

Difference of Two Perfect Squares (DOTS)

↓

Trinomial (TRI)

GCF:
$$ab + ac = a(b + c)$$

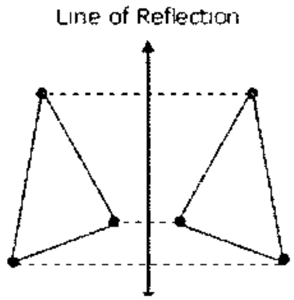
DOTS:
$$x^2 - y^2 = (x + y)(x - y)$$

TRI:
$$x^2 - x + 6 » (x + 2)(x - 3)$$

Transformational Geometry

Rigid Motion: transformations that preserve distance, congruency, angle measure, and shape.

Reflection – FLIP



$r_{x-axis}(x,y) = (x,-y)$

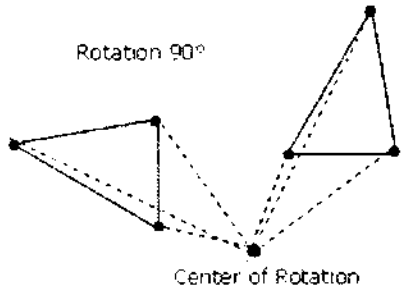
$r_{y-axis}(x,y) = (-x,y)$

$r_{y=x}(x,y) = (y,x)$

$r_{y=-x}(x,y) = (-y,-x)$

$r_{(o,o)}(x,y) = (-x,-y)$

Rotation – TURN

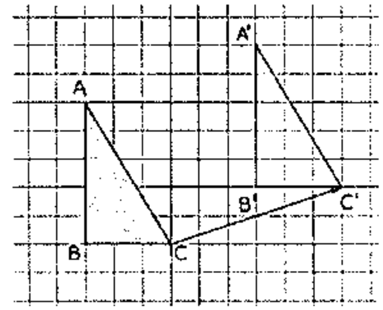


$R_{90°}(x,y) = (-y,x)$

$R_{180°}(x,y) = (-x,-y)$

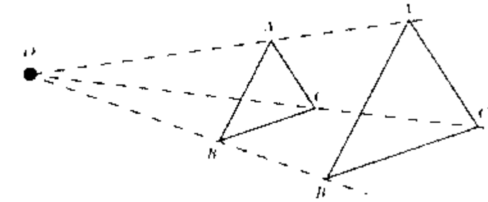
$R_{270°}(x,y) = (y,-x)$

Translation – SHIFT/MOVE



$T_{a,b}(x,y) = (x+a, y+b)$

Dilation – ENLARGEMENT/REDUCTION



$D_k(x,y) = (k \cdot x, k \cdot y)$

- **Dilations create similar figures**
- **Dilations are NOT rigid motions, since they do NOT preserve distance.**

Composition of Transformations

When you see "∘", work from right to left.

$$R_{90°} \circ T_{3,-4}$$

Do this Second!

Do this First!

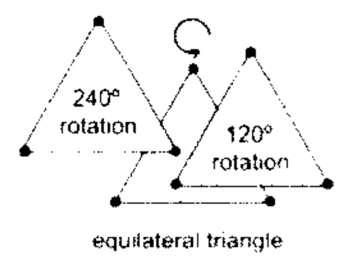
Translation, followed by a Rotation.

Types of Composition Transformations

- **A composition of 2 reflections over 2 parallel lines is equivalent to a TRANSLATION.**
- **A composition of 2 reflections over 2 intersecting lines is equivalent to a ROTATION.**

Rotational Symmetry Theorem

A regular polygon with n sides always has rotational symmetry, with rotations in increments equal to its central angle of $\frac{360°}{n}$



Circles

Equations

General/Standard Equation of a Circle:

$$x^2 + y^2 + Cx + Dy + E = 0$$

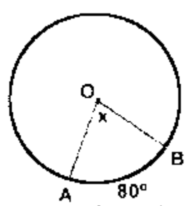
where *C*, *D*, and *E* are constants

Center – Radius Equation of a Circle:

$$(x-h)^2 + (y-k)^2 = r^2$$

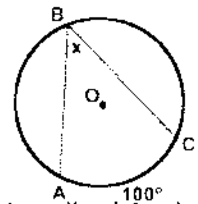
where *(h, k)* is the center and *r* is the radius.

Angles



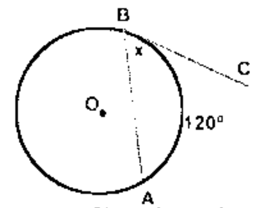
Central Angle:

$\measuredangle x = \widehat{AB}$



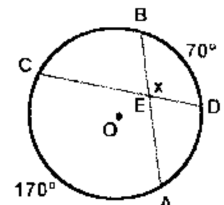
Inscribed Angle:

$\measuredangle x = \frac{1}{2}\widehat{AC}$



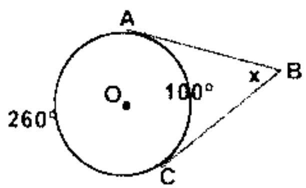
Tangent-Chord Angle:

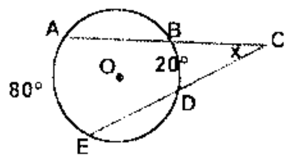
$\measuredangle x = \frac{1}{2}\widehat{AC}$

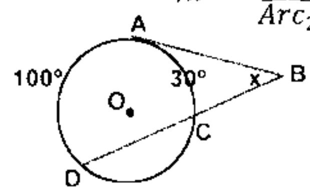


Two Chord Angles:

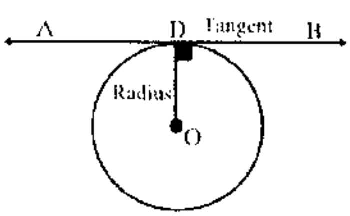
$\measuredangle x = \frac{\widehat{Arc_1}}{\widehat{Arc_2}}$



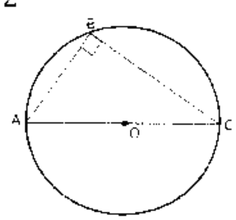




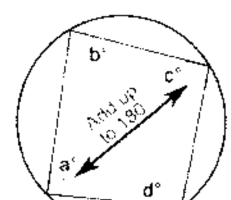
$$\frac{\widehat{Big} - \widehat{Little}}{2} = \measuredangle x$$



Tangent-Radius Angle = 90°

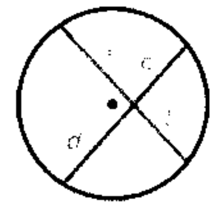


Angle Inscribed in a semicircle = 90°

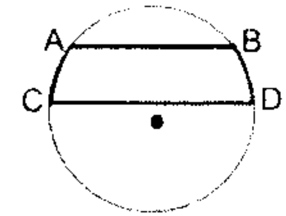


Opposite Angles in a Quad. = 180°

Segments

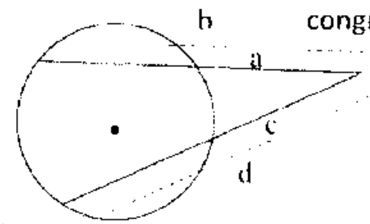


$(a)(b) = (c)(d)$



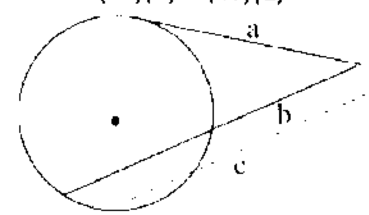
$\overline{AB} \parallel \overline{CD}$, $\widehat{AC} \cong \widehat{BD}$

Parallel chords intercept congruent arcs



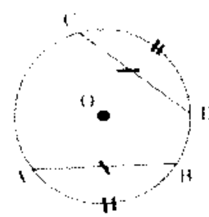
$(Whole)(External) = (Whole)(External)$

(W)(E) = (W)(E)

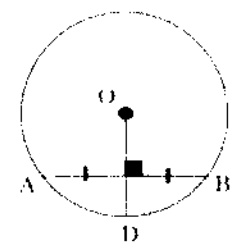


$(Whole)(External) = (Tangent)^2$

(W)(E) = (T)^2

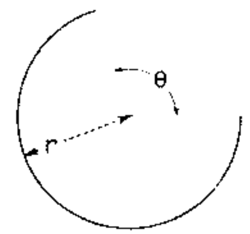


$\overline{CD} \cong \overline{AB}$; $\widehat{CD} \cong \widehat{AB}$



Circles (Con't)

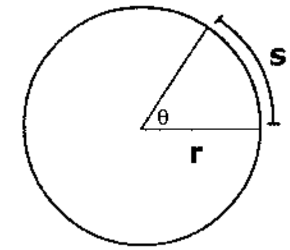
Area of a Sector



$$A = \frac{1}{2}r^2\theta$$

where A is the area of the sector, r is the radius, and θ is an angle in radians.

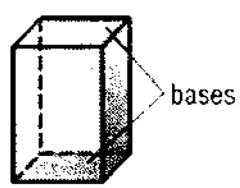
Sector Length



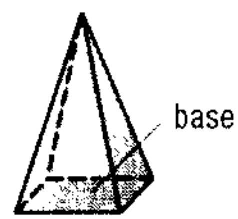
$$s = r\theta$$

where s is the sector length, r is the radius, and θ is an angle in radians.

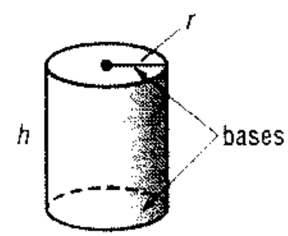
3D Figures



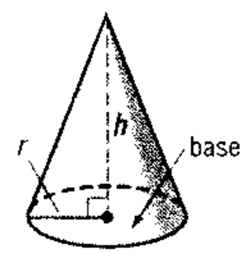
Prism



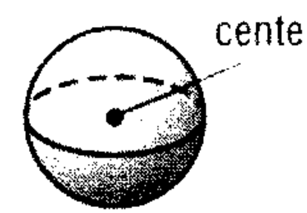
Pyramid



Cylinder

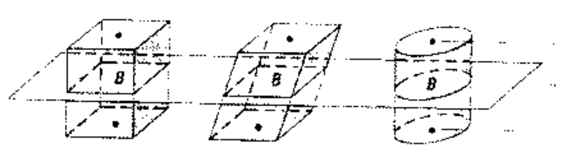


Cone

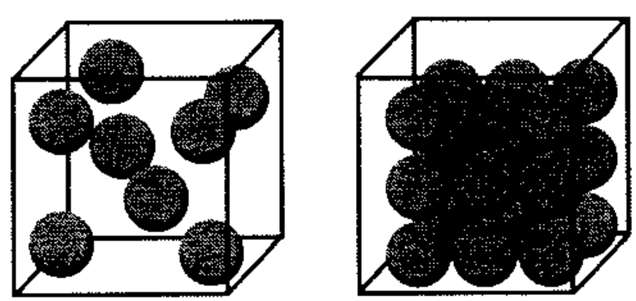


Sphere

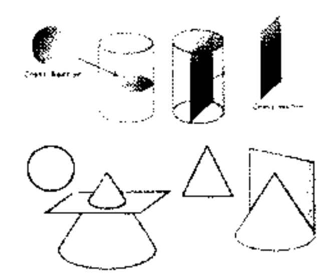
Cavalieri's Principle: If two solids have the same height and the same cross-sectional area at every level, then the solids have the same volume.



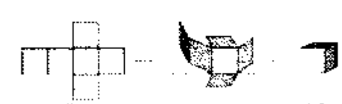
$$\boldsymbol{Mass = (Density) \cdot (Volume)}$$



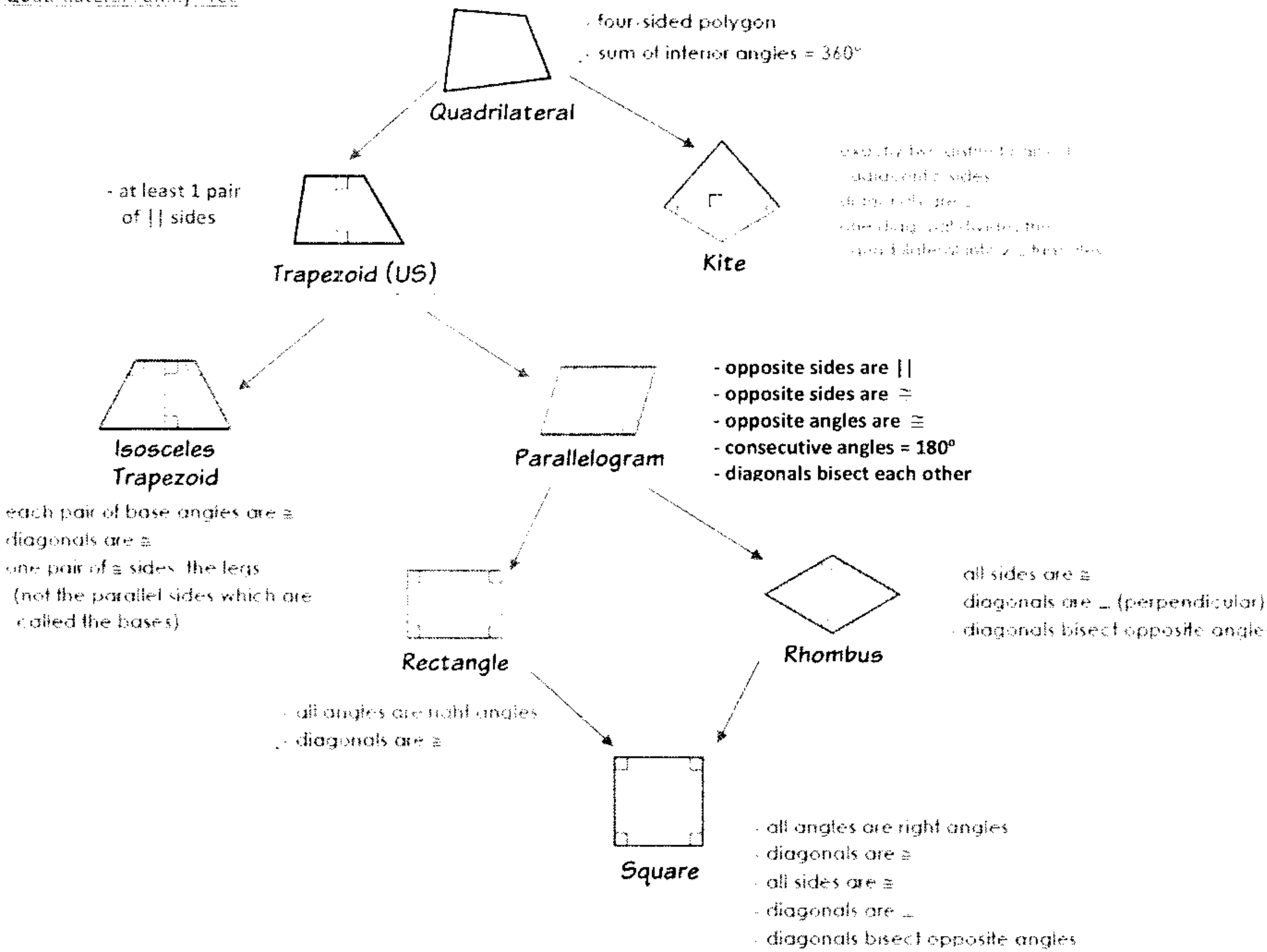
Cross Sections



Nets

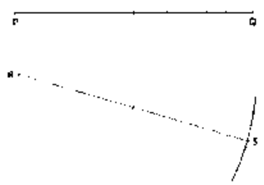


The Quadrilateral Family Tree

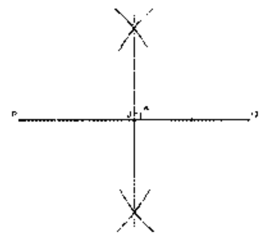


Basic Constructions

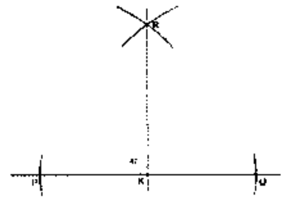
Copy a line segment



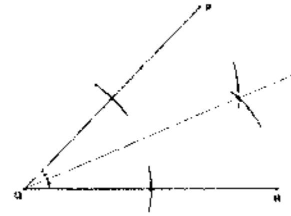
Perpendicular Bisector



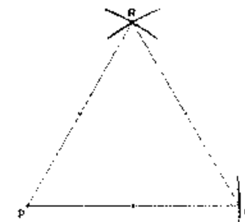
Perpendicular Line thru a point on a line



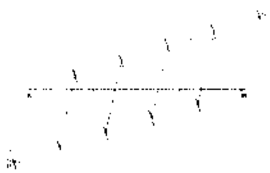
Bisect an angle



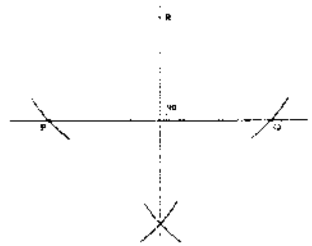
Equilateral Triangle



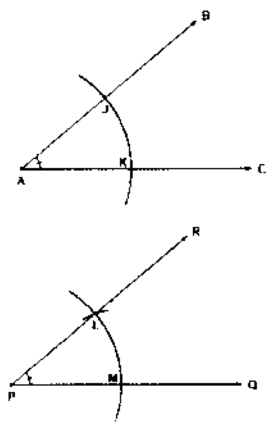
Dividing a segment into equal parts



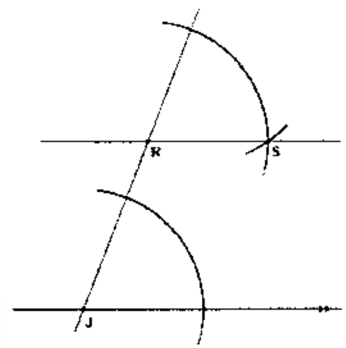
Perpendicular line thru an external point



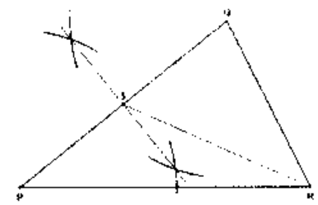
Copy an angle



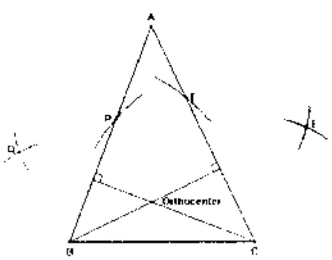
Parallel line



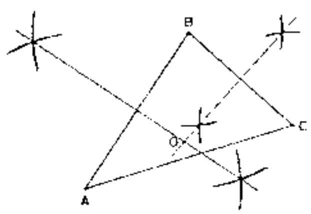
Median – vertex to midpoint



Orthocenter - altitudes

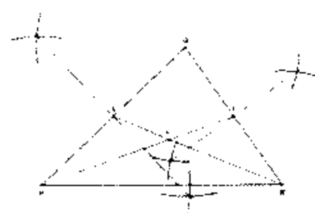


Circumcenter – perpendicular bisectors



- Equidistant to each vertex of the triangle
- Used to circumscribe a circle

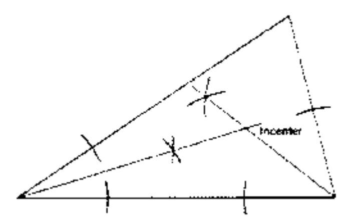
Centroid - medians



- 2:1 ratio

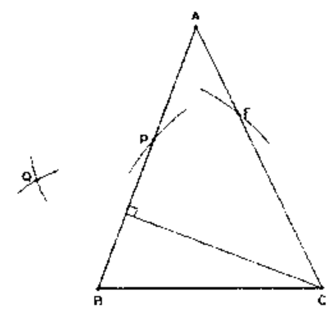
$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

Incenter – angle bisectors



- equidistant to each side of the triangle
- Used to inscribe a circle

Altitude – vertex perpendicular to opposite side



Inscribed square in a circle