Circles

Basic vocabulary

History of the Circle

- The circle has been known since before the beginning of recorded history. It is the basis for the wheel, which, with related inventions such as gears, makes much of modern civilization possible. In mathematics, the study of the circle has helped inspire the development of geometry and calculus.
- Early science, particularly geometry and Astrology and astronomy, was connected to the divine for most medieval scholars, and many believed that there was something intrinsically "divine" or "perfect" that could be found in circles.

Parts of a Circle

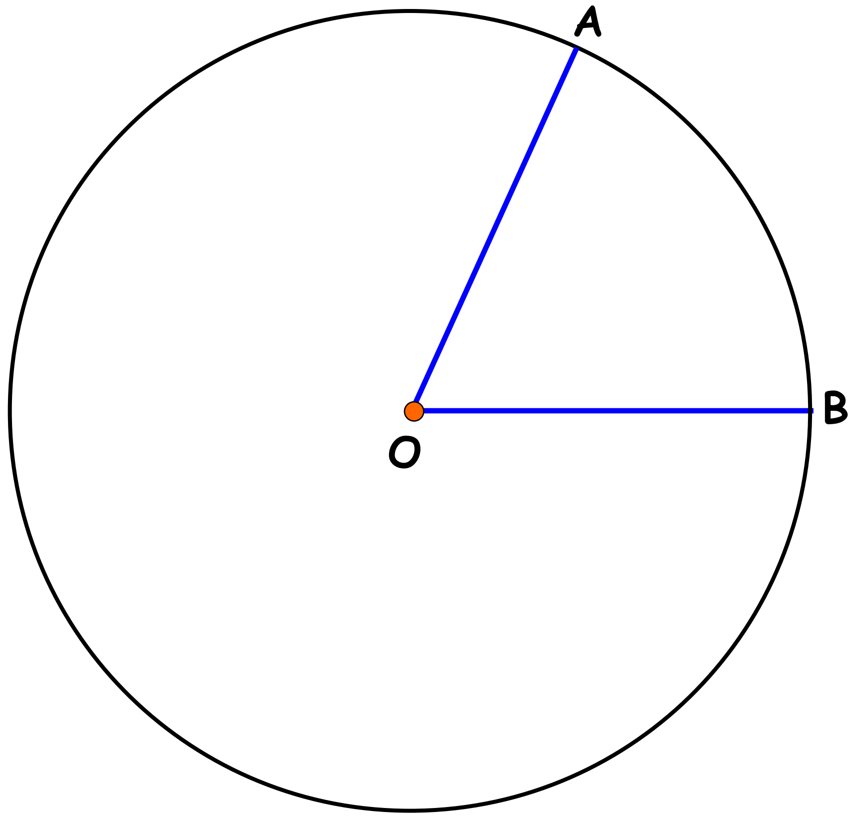
Vocabulary

1) circle
2) center
3) radius
4) chord
5) diameter
6) concentric

Parts of a Circle

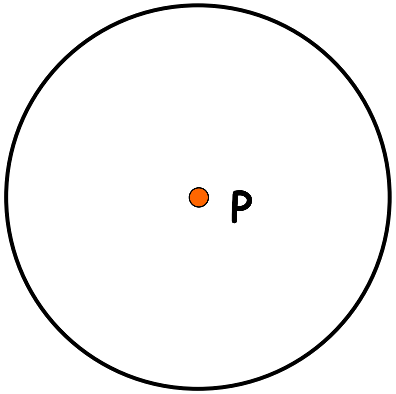
A circle is a special type of geometric figure.

All points on a circle are the same distance from a center point.



The measure of $\overline{OA}$ and $\overline{OB}$ are the same; that is, OA OB

Parts of a Circle

Definition of a Circle	A circle is the set of all points in a plane that are a given distance from a given _point_ in the plane, called the _center_ of the circle. 

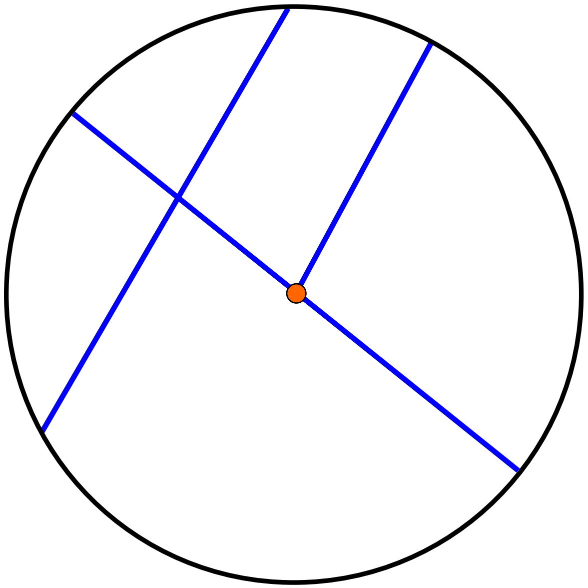
Note: a circle is named by its center. The circle above is named circle P.

Parts of a Circle

A radius is a segment whose endpoints are the center of the circle and a point on the circle.

A chord is a segment whose endpoints are on the circle.

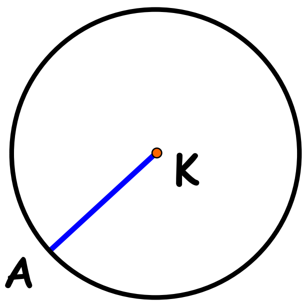
A diameter is a chord that contains the center



Parts of a Circle

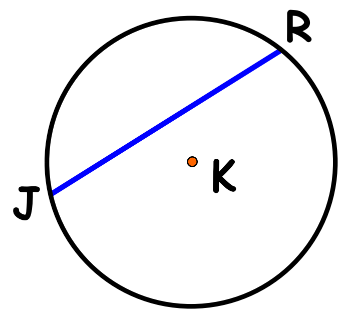
Segments of Circles

radius



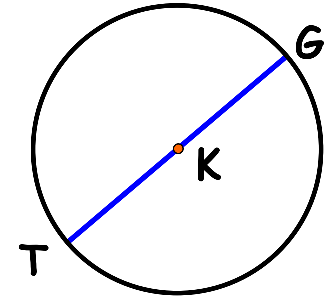
$\overline{KA}$ is a radius of $\odot K$

chord



$\overline{JR}$ is a chord of $\odot K$

diameter



$\overline{GT}$ is a diameter of $\odot K$

From the figures, you can not that the diameter is a special type of chord *that passes through the center.*

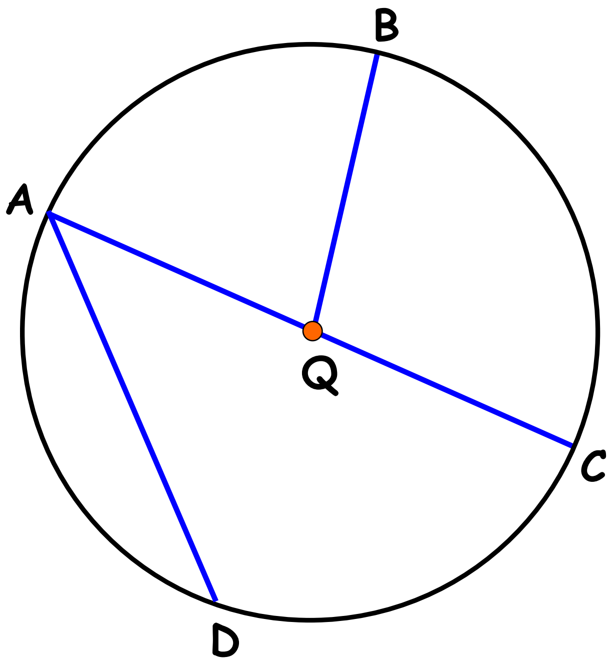
Parts of a Circle

Use $\odot$Q to determine whether each statement is *true* or *false*.

$\overline{AD}$ is a diameter of $\odot$Q.

False;

Segment AD does not go through the center Q.



$\overline{AC}$ is a chord of $\odot$Q.

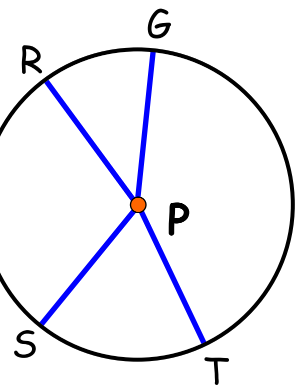
True;

$\overline{AD}$ is a radius of $\odot$Q.

False;

Parts of a Circle

All radii of a circle are congruent.



$$\overline{PR} \cong \overline{PG} \cong \overline{PS} \cong \overline{PT}$$

The measure of the diameter d of a circle is twice the measure of the radius r of the circle.

$$d = 2r$$

or

$$\frac{1}{2}d = r$$

Parts of a Circle

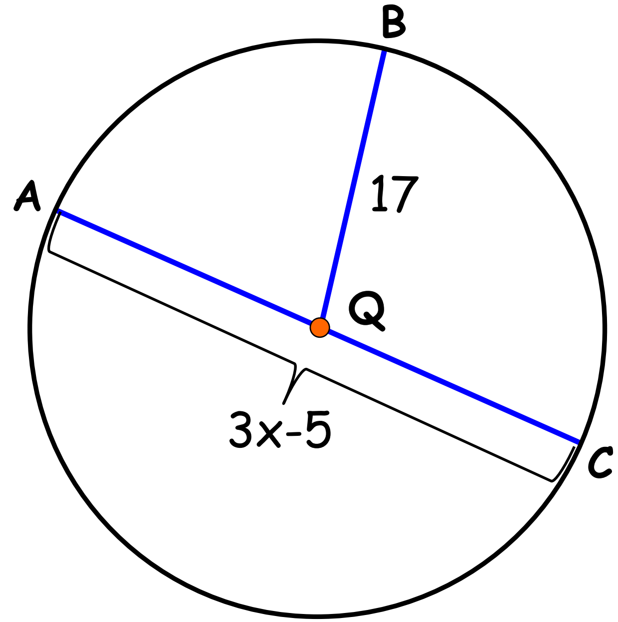
Find the value of x in ⊙Q.

$3x - 5 = 2(17)$

$3x - 5 = 34$

$3x = 39$

$x = 13$



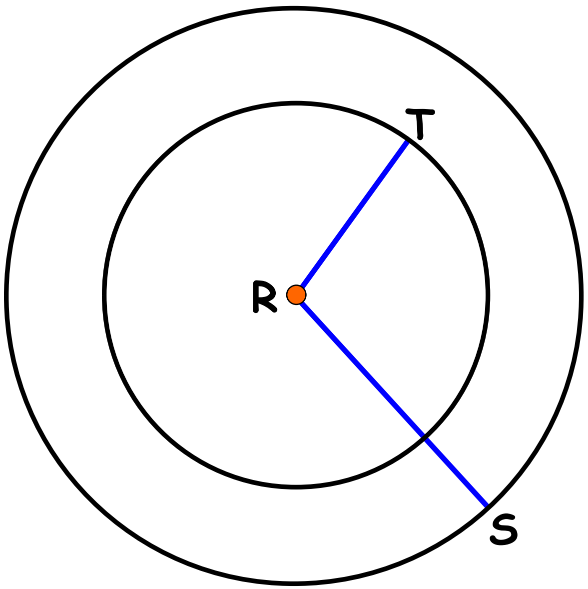
Parts of a Circle

Because all circles have the same shape, any two circles are similar.

However, two circles are congruent if and only if (iff) their radii are congruent.

Two circles are concentric if they meet the following three requirements:

- They lie in the same plane.
- They have the same center.
- They have radii of different lengths.



Circle R with radius RT and circle R with radius RS are concentric circles.

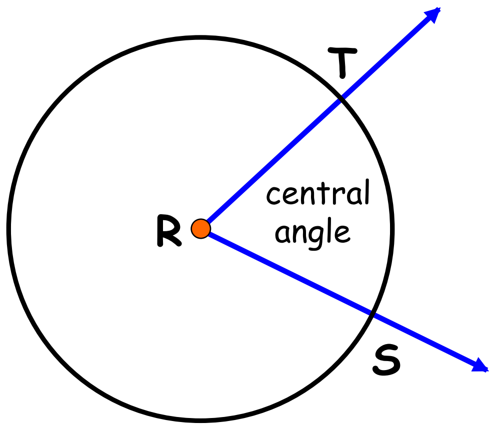
Arcs and Central Angles

Vocabulary

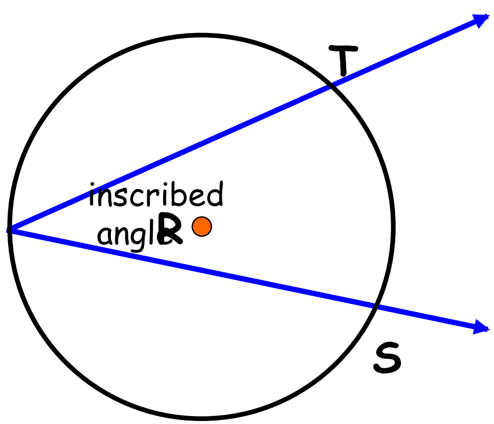
1) central angle
2) inscribed angle
3) arcs
4) minor arc
5) major arc
6) semicircle

Arcs and Central Angles

A central angle is formed when the two sides of an angle meet at the center of a circle.



An inscribed angle is formed when the two sides of an angle meet on a of a circle.

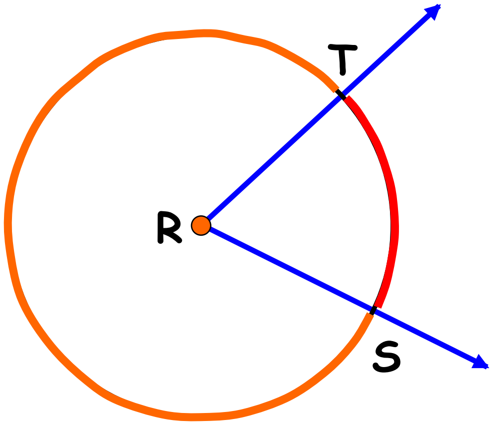


Arcs

Each side of the angle intersects a point on the circle, dividing it into arcs that are curved lines.

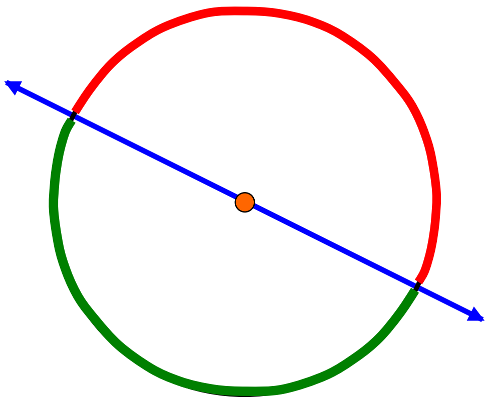
There are three types of arcs:

A minor arc is part of the circle in the interior of the central angle with measure less than 180°.



A major arc is part of the circle in the exterior of the central angle.

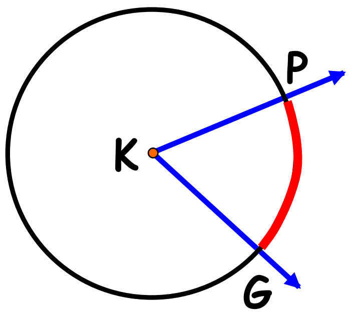
Semicircles are congruent arcs whose endpoints lie on the diameter of the circle.



Arcs and Central Angles

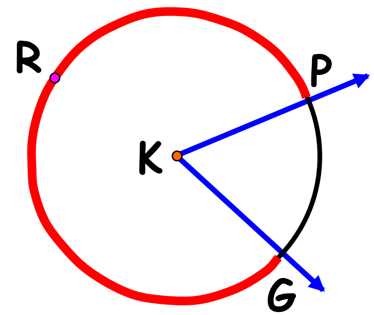
Types of Arcs

minor arc PG



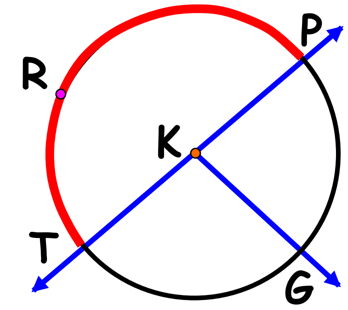
$m\,\overset{\frown}{PG} < 180$

major arc PRG



$m\,\overset{\frown}{PRG} > 180$

semicircle PRT



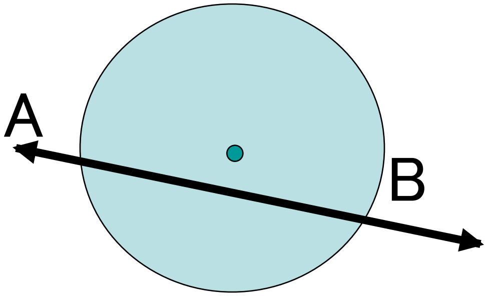
$m\,\overset{\frown}{PRT} = 180$

Note that for circle K, two letters are used to name the minor arc, but three letters are used to name the major arc and semicircle. These letters for naming arcs help us trace the set of points in the arc. In this way, there is no confusion about which arc is being considered.

Segments of a circle

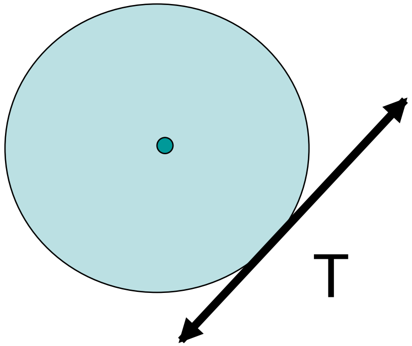
Vocabulary

1) tangent
2) secant
3) sector



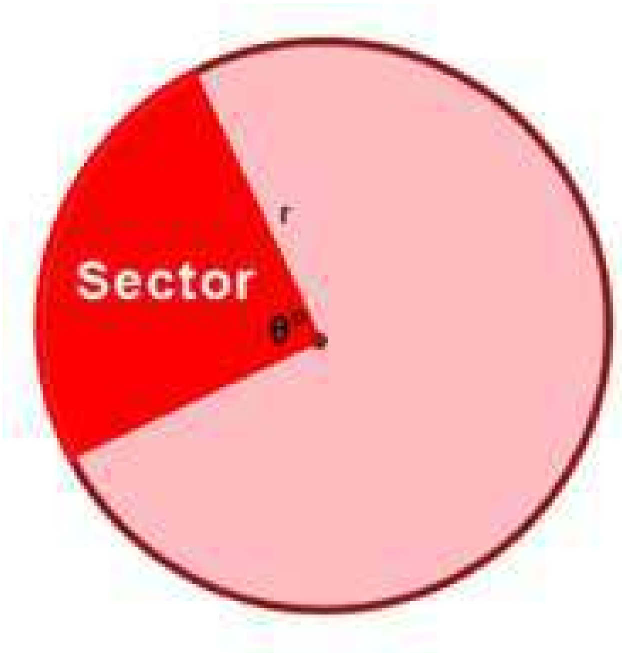
A **secant** is a line that intersects a circle at exactly two points. (Every secant contains a chord of the circle.)

A **tangent** is a line that intersects a circle at exactly one point. This point is called the **point of tangency** or **point of tangency**.



SECTOR

Corresponding to a given arc, the region bounded by the two radii and the arc itself.



Inscribed Polygons

Vocabulary

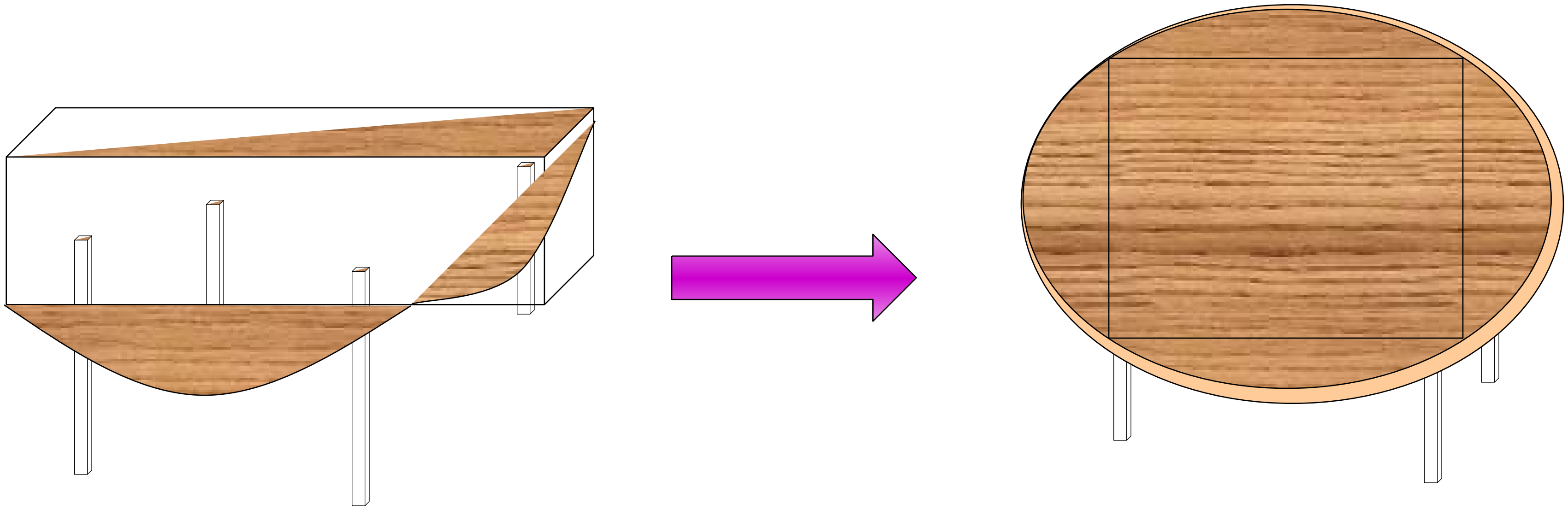
1) circumscribed

2) inscribed

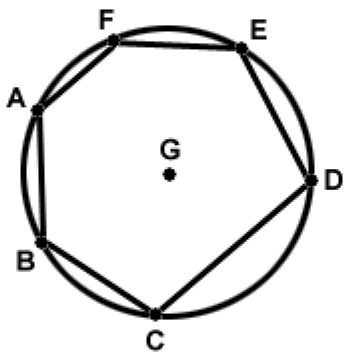
Inscribed Polygons

When the table's top is open, its circular top is said to be circumscribed about the square.

We also say that the square is inscribed in the circle.



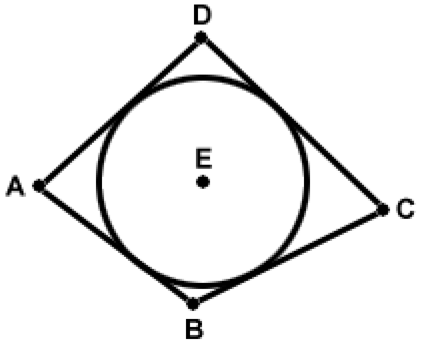
Definition of Inscribed Polygon	A polygon is inscribed in a circle if and only if every vertex of the polygon lies on the circle.



Inscribed Polygon

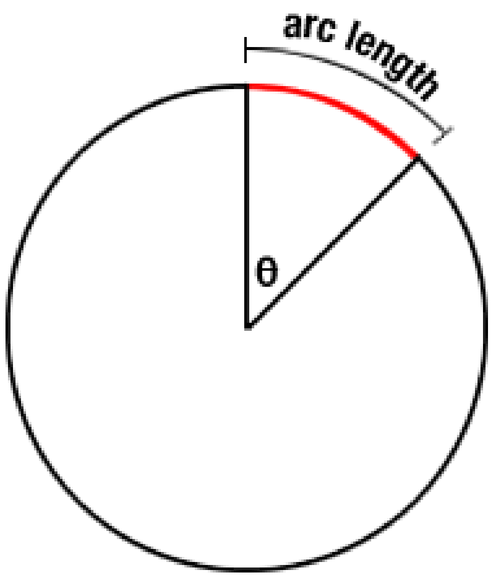
An inscribed polygon is also called a cyclic polygon.

Circumscribed Polygon



Angle Measure and Arc Length

Arc Length



The distance along the arc (part of the circumference of a circle, or of any curve).

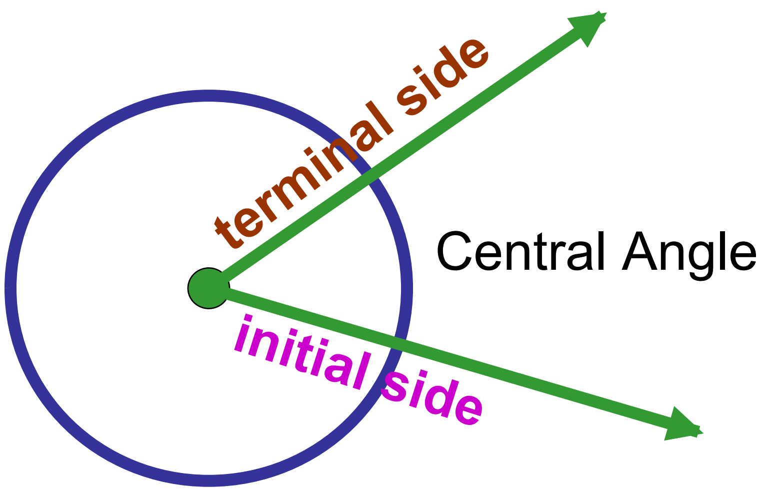
For a circle:

Arc Length = $\theta \times r$

(when θ is in radians)

Angle Measure

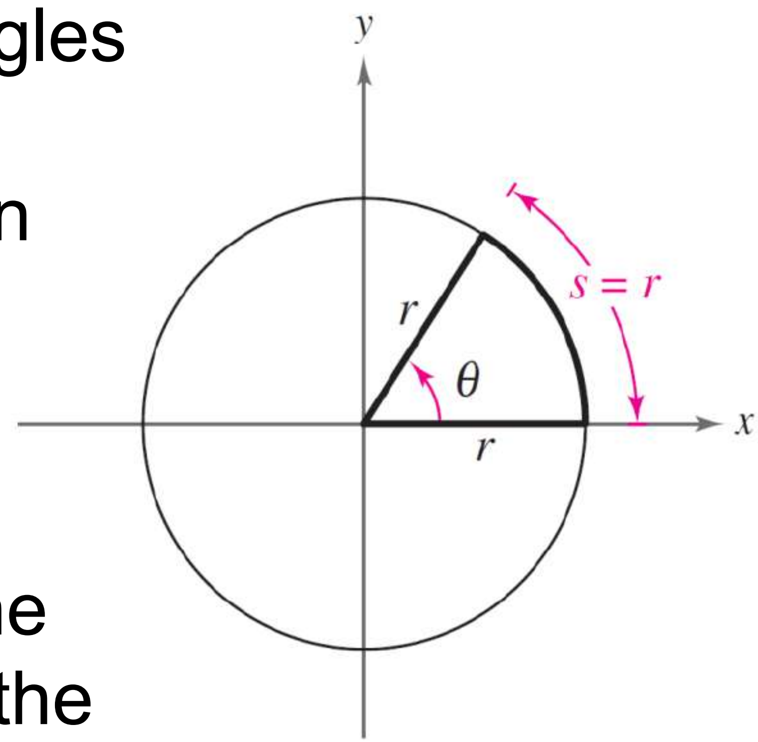
- <u>Measure of an Angle</u> = determined by the amount of rotation from the initial side to the terminal side (one way to measure angle is in radians)



Radian Measure

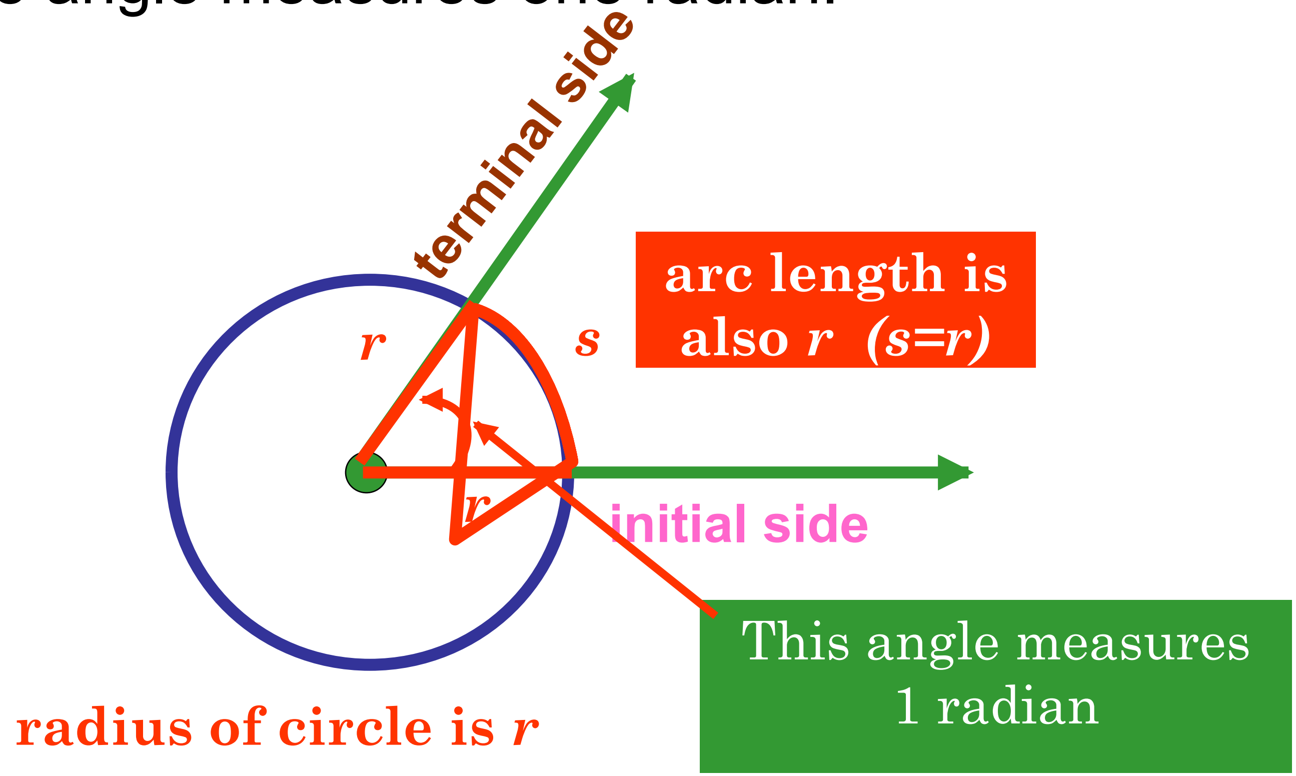
You determine the **measure of an angle** by the amount of rotation from the initial side to the terminal side. One way to measure angles is in *radians*. This type of measure is especially useful in calculus.

To define a radian, you can use a **central angle** of a circle, one whose vertex is the center of the circle, as shown.



Arc length = radius when θ = 1 radian

Given a circle of radius (r) with the vertex of an angle as the center of the circle, if the arc length (*s*) formed by intercepting the circle with the sides of the angle is the same length as the radius (*r*), the angle measures one radian.



Radian Measure

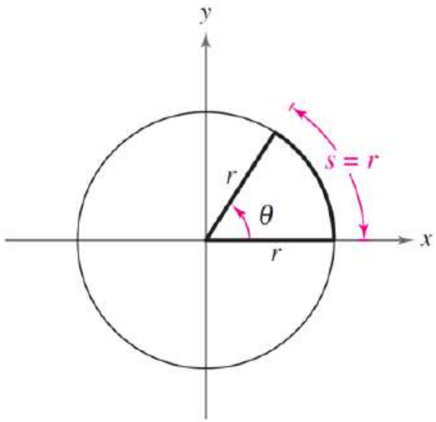
Definition of Radian

One **radian** is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of the circle. See Figure 4.5. Algebraically, this means that

$$\theta = \frac{s}{r}$$

where θ is measured in radians. (Note that $\theta = 1$ when $s = r$.)

- The radian measure of a central angle θ is obtained by dividing the arc length s by r (θ = s/r).
- Because the circumference of a circle is $2\pi r$ units, it follows that a central angle of one full revolution (counterclockwise) corresponds to an arc length of $s = 2\pi r$.
- Therefore, a full circle measures $\frac{2\pi r}{r}$, or 2π radians.



The End