AP Calculus Cheat Sheet for 2020 from https://fiveable.me/ap-calculus-ab-bc/ap-calc-study-guide-2020/ and Unit Circle and trig reference sheet

The link about will take you to more practice and review resources, but just be careful about sharing any personal information with them, since they haven't been vetted by NPCSD.

This is a good comprehensive list of formulas and theorems (plus a couple extra things not in the AP curriculum!). Remember: You CAN AND SHOULD use notes while taking the AP exams. The exams are designed with that in mind. So have your unit circle, your derivative rules, your trig identities, and anything else you need handy. But also remember that time will be an issue. You don't want to waste time scanning notes when you don't need to! And also remember: The best way to prepare for the exam is to do as many free response problems as possible! Do all the problems in my lessons!!!

Unit	Formulas and Theorems
1: Limits and Continuity	• Properties of limits: ○ Sum: $\lim_{x\to c}(f(x) + g(x)) = \lim_{x\to c} f(x) + \lim_{x\to c} g(x)$ ○ Difference: $\lim_{x\to c}(f(x) - g(x)) = \lim_{x\to c} f(x) - \lim_{x\to c} g(x)$ ○ Product: $\lim_{x\to c}(f(x) \cdot g(x)) = \lim_{x\to c} f(x) \cdot \lim_{x\to c} g(x)$ ○ Quotient: $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$ ○ Multiplying a constant: $\lim_{x\to c}(a \cdot f(x)) = a \cdot \lim_{x\to c} f(x)$ ○ Power: $\lim_{x\to c}(f(x))^a = [\lim_{x\to c} f(x)]^a$ ○ Root: $\lim_{x\to c} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to c} f(x)}$ • Finding asymptotes: ○ If $\lim_{x\to c} f(x) = \pm\infty$, $f(x)$ has a vertical asymptote at $x = a$ ○ If $\lim_{x\to \pm\infty} f(x) = L$, $f(x)$ has a horizontal asymptote at $y = L$ • Squeeze Theorem: For $f(x) \le h(x) \le g(x)$, if $\lim_{x\to c} f(x) = a$ and $\lim_{x\to c} g(x) = a$, then $\lim_{x\to c} h(x) = a$. • Is it continuous?: If $f(a)$ is defined and the limit exists, yes it is! ○ The limit exists if $\lim_{x\to a-} f(x) = \lim_{x\to a+} f(x)$ • Intermediate Value Theorem (IVT): If $f(x)$ is continuous over [a, b], there is a value $f(c)$, where c is within [a, b].

Unit	Formulas and Theorems

2: Differentiation: Definition and Basic Derivative Rules	• Limit definition of a derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$, if $f(x)$ is continuous
	• Numerical definition of a derivative: $f'(x) = \lim_{x \to a} \frac{f(x)-f(a)}{x-a}$
	• Differentiability rules:
	  ○ $f(x)$ is continuous at a $\lim_{x \to a^-} f'(x) = \lim_{x \to a^+} f'(x)$
	• Properties of derivatives:
	  ○ Power rule: $\frac{d}{dx} x^n = nx^{n-1}$
	  ○ Product rule: $\frac{d}{dx} f(x)g(x) = f'(x)g(x) + f(x)g'(x)$
	  ○ Quotient rule: $\frac{d}{dx} (\frac{f(x)}{g(x)}) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$
	Remember: If x is a function (ex. x^2, 2x + 5, 3x), use the Chain Rule when taking the derivative.
	• Trig functions derivatives:
	  ○ $\frac{d}{dx} cosx = -sinx$
	  ○ $\frac{d}{dx} sinx = cosx$
	  ○ $\frac{d}{dx} tanx = sec^2x$
	  ○ $\frac{d}{dx} secx = secx \cdot tanx$
	  ○ $\frac{d}{dx} cscx = -cscx \cdot cotx$
	  ○ $\frac{d}{dx} cotx = -csc^2x$
	  ○ $\frac{d}{dx} a^u = a^u(ln\ a)$
	  ○ $\frac{d}{dx} e^x = e^x$
	  ○ $\frac{d}{dx} ln\ x = \frac{1}{x}$
	  ○ $\frac{d}{dx} log_a x = \frac{1}{x(ln\ a)}$

Unit	Formulas and Theorems
3: Differentiation: Composite, Implicit & Inverse Functions	• Chain Rule: $\frac{d}{dx} f(x) = f'(x) \cdot x'$ • Derivative of inverse functions: $\frac{d}{dx} (f^{-1}(x)) = \frac{1}{f'(f^{-1}(x))}$ • Inverse trig functions derivatives: ○ $\frac{d}{dx} cos^{-1} x = -\frac{1}{\sqrt{1-x^2}}$ ○ $\frac{d}{dx} sin^{-1} x = \frac{1}{\sqrt{1-x^2}}$ ○ $\frac{d}{dx} cot^{-1} x = -\frac{1}{1+x^2}$ ○ $\frac{d}{dx} tan^{-1} x = \frac{1}{1+x^2}$ ○ $\frac{d}{dx} sec^{-1} x = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$ ○ $\frac{d}{dx} csc^{-1} x = -\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$

Unit	Formulas and Theorems

4: **Contextual Applications of Differentiation**	• L'Hospital's rule: If $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, take the derivative of $f(x)$ and $g(x)$ and find $\lim_{x \to c} \frac{f'(x)}{g'(x)}$
	  ○ L'Hospital's may need to be applied multiple times
	• Relationships between position, velocity, and acceleration:
	  ○ $x'(t) = v(t)$
	  ○ $v'(t) = a(t)$
	  ○ If $v(t)$ and $a(t)$ have the same sign (both positive or negative), then the particle speeds up
	  ○ If $v(t)$ and $a(t)$ have opposite signs (one is positive and the other negative), then the particle slows down
	• Related Rates: Remember to treat variables that change as functions and variables that stay the same as constants. Here are common volume distance equations (also on the MCQ AP® Exam equation sheet):
	  ○ $V_{cylinder} = \pi r^2 h$
	  ○ $V_{cone} = \pi r^2 \frac{h}{3}$
	  ○ $V_{sphere} = \frac{4}{3}\pi r^3$
	• Finding tangent lines: $y_2 - y_1 = f'(x_1)(x_2 - x_1)$, where (x_1, y_1) is the point on the curve and (x_2, y_2) is a point the line passes through
	  ○ If you need to approximate a value, find the equation of the tangent line at a nearby whole-number x value, then solve for y_1

Unit	Formulas and Theorems

5: Analytical Applications of Differentiation	• Optimization: Use these handy tests to find the extrema of a function!
	• First derivative test: If $f'(x)$ = 0 or is undefined and f is continuous at *x*…
	    ○ and $f'(x)$ changes from + to - → relative maximum at *x*
	    ○ and $f'(x)$ changes from + to - → relative minimum at *x*
	    ○ and and $f'(x)$ has no change → neither at *x*
	• Finding concavity:
	    ○ If $f''(x) > 0$ → concave up at *x*
	    ○ If $f''(x) < 0$ → concave down at *x*
	• Finding inflection points:
	    ○ If $f''(x) = 0$ or is undefined and $f''(x)$ changes signs → inflection point at *x*
	• Second derivative test:
	    ○ If $f'(x) = 0$ and $f''(x) > 0$ → relative maximum at *x*
	    ○ If $f'(x) = 0$ and $f''(x) < 0$ → relative minimum at *x*
	• Mean Value Theorem (MVT):
	    ○ If f is continuous on closed interval [a, b], and is differentiable on (a, b), there is a point c such that $f'(c) = \frac{f(b)-f(a)}{b-a}$
	    ○ Rolle's Theorem: Same as MVT, but just the specific case when $f'(c) = 0 = \frac{f(b)-f(a)}{b-a}$
	• Extreme Value Theorem:
	    ○ If f is continuous on closed interval [a, b], then f has both an absolute maximum and an absolute minimum

6: Integration and Accumulation of Change	• Reimann sums: These estimate the area under a curve by dividing it into rectangles (subdivisions) and adding their areas
	  ○ Right Reimann sum: For each subdivision, the height is the right point's y-coordinate
	    ■ If the curve increases, it overestimates
	    ■ If the curve decreases, it underestimates
	  ○ Left Reimann sum: For each subdivision, the height is the left point's y-coordinate
	    ■ If the curve increases, it underestimates
	    ■ If the curve decreases, it overestimates
	  ○ Midpoint Reimman sum: For each subdivision, the height is the y-coordinate of the midpoint of the two x-coordinates
	  ○ Trapezoidal Reimann sum: Most accurate of the types. Use the area of a trapezoid ($A = \frac{h(b_1+b_2)}{2}$)
	    ■ h = distance between x values
	    ■ b_1 = y-coordinate of right point
	    ■ b_2 = y-coordinate of left point
	• Fundamental Theorem of Calculus (FTC):
	  ○ Part 1: $\int_a^b f(x) = F(b) - F(a)$, where F is the antiderivative of f
	    ■ If the limits are functions, use Chain Rule: $\int_{h(x)}^{k(x)} f(x) = F(k(x)) \cdot k'(x) - F(a) \cdot h'(x)$
	  ○ Part 2: $\frac{d}{dx} \int_a^x f(t)\, dt = f(x)$
	    ■ Corollary: If the limits are functions, use Chain Rule:
	$\frac{d}{dx} \int_{h(x)}^{k(x)} f(x)\, dx = f[k(x)] \cdot k'(x) - f[h(x)] \cdot h'(x)$
	• Reverse power rule: $\int x^n\, dx = \frac{x^{(n+1)}}{n+1} + C$

- Indefinite Integrals: $\int f(x)\,dx = F(x) + C$, where F is the antiderivative of f (**do not forget + C !**)
- Integral properties and techniques:
  - U-Substitution: If you have $\int_a^b f(g(x)) \cdot g'(x)\,dx$, let $u = g(x)$ and $du = g'(x)\,dx$. Substitute and solve: $\int_a^b f(u)\,du$. Don't forget to replace any u's with $g(x)$!
  - Sum: $\int_a^b [f(x) + g(x)]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$
  - Difference: $\int_a^b [f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$
  - Multiplying a constant: $\int_a^b a \cdot f(x)\,dx = a\int_a^b f(x)\,dx$

Unit 7 Differential Equations

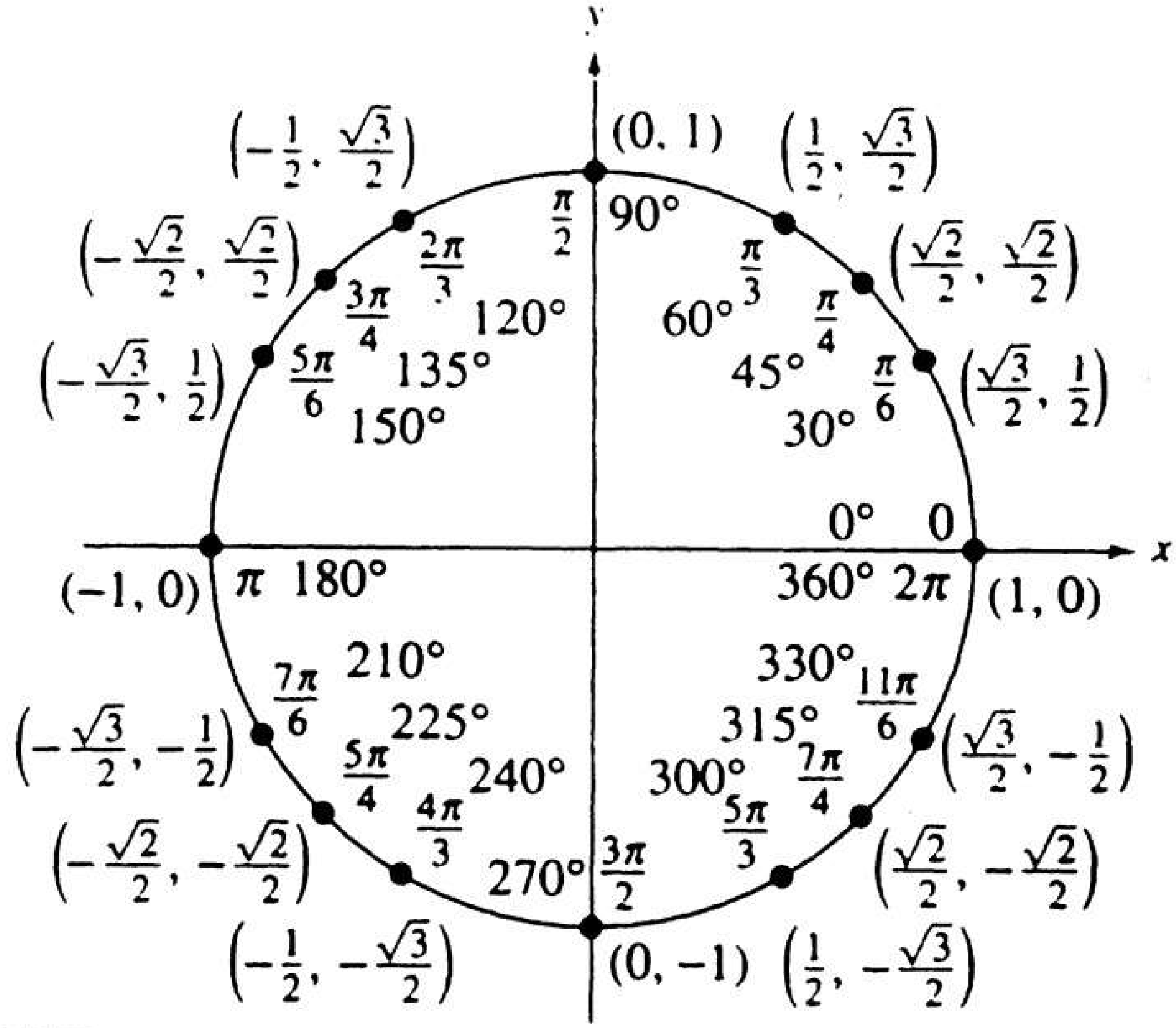
- Differential Equations
  - Modeling
    - If you are given information, can you write a differential equation from it?

  - Verifying Solutions
    - Being able to plug in given information and deduce if the solution is true.

  - Separation of variables
    - This appears often in the FRQ section. Students are given a differential equation and need to put all of the terms for one variable on one side and for the other variable on the other, then they integrate both sides and solve for y, usually y, but really whichever the dependent variable it.
    - NOTE: If you skip the step of separating variables on AP exams in the past, they have offered you no credit for the rest of that section of the problem.

  - Using initial conditions
    - Once you have done your separation of variables, don't forget to have a +C! Ths is where the initial condition comes in. You plug in the terms given from x and y in your initial condition, then you solve for C, rewriting the function at the end with C plugged in!

The Unit Circle



Trigonometry Reference Sheet

Area of a Triangle

$K = \frac{1}{2} ab \sin C$

Functions of the Sum of Two Angles

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

Functions of the Difference of Two Angles

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

Law of Sines

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines

$a^2 = b^2 + c^2 - 2bc \cos A$

Functions of the Double Angle

$\sin 2A = 2 \sin A \cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2 \cos^2 A - 1$

$\cos 2A = 1 - 2 \sin^2 A$

$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$

Functions of the Half Angle

$\sin \frac{1}{2} A = \pm\sqrt{\frac{1 - \cos A}{2}}$

$\cos \frac{1}{2} A = \pm\sqrt{\frac{1 + \cos A}{2}}$

$\tan \frac{1}{2} A = \pm\sqrt{\frac{1 - \cos A}{1 + \cos A}}$

Reciprocal Trig Identities

$\sec\theta = \frac{1}{\cos\theta}$ $\csc\theta = \frac{1}{\sin\theta}$ $\tan\theta = \frac{\sin\theta}{\cos\theta}$ $\cot\theta = \frac{\cos\theta}{\sin\theta}$

Pythagorean Trig Identities

$\cos^2\theta + \sin^2\theta = 1$

$1 + \tan^2\theta = \sec^2\theta$

$\cot^2\theta + 1 = \csc^2\theta$