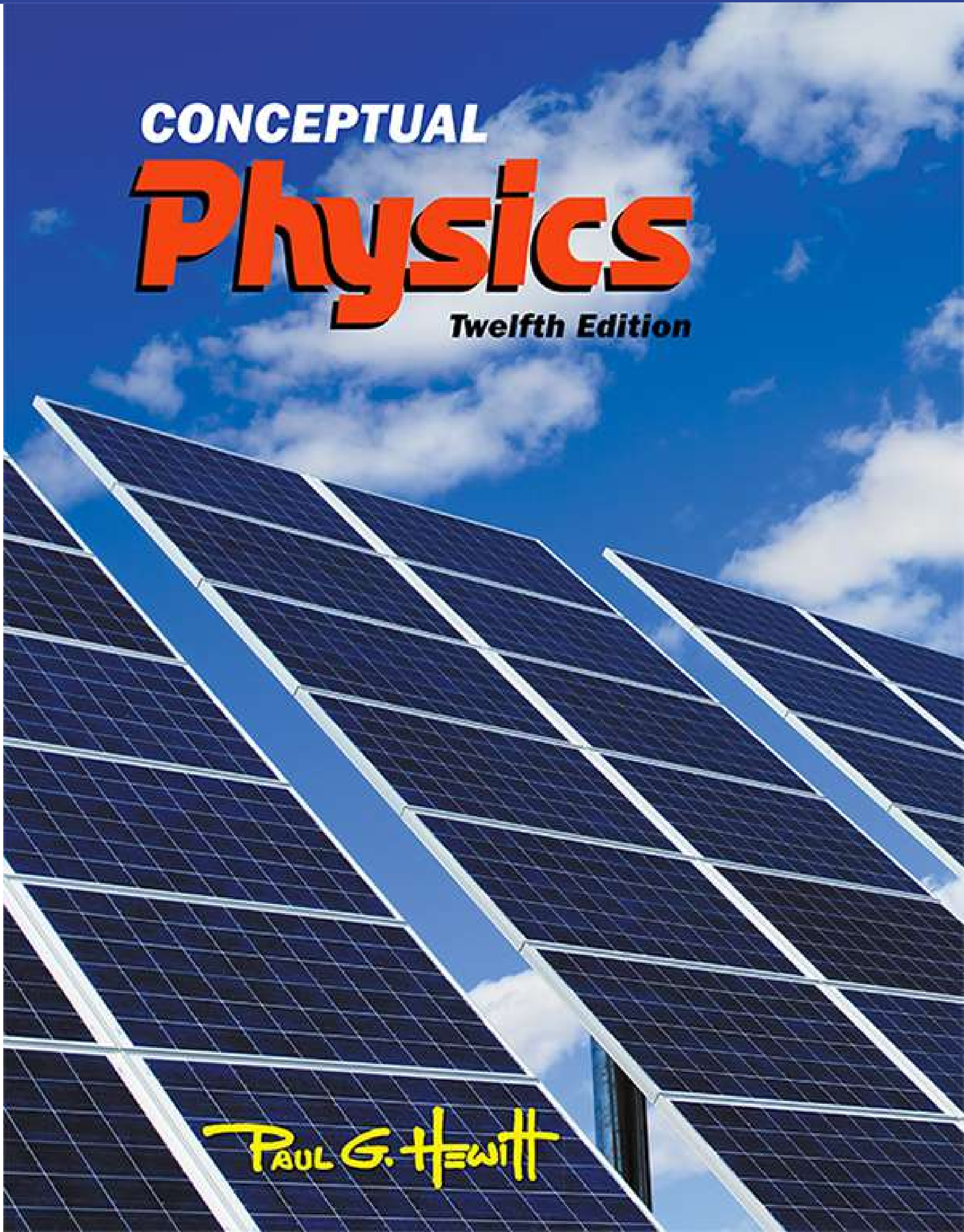
Chapter 8: Rotational Motion

Section8.8
Conservation of
Angular Momentum



This lecture will help you understand:

How skaters spin faster or slower.

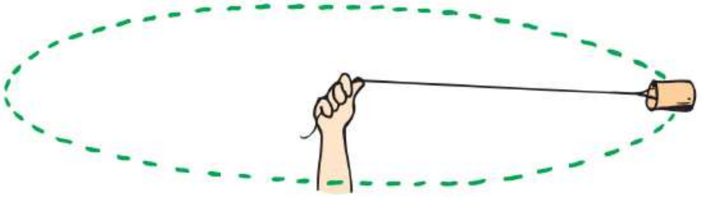
How gymnasts can rotate at any speed they want.

Why planets go faster or slower.

How cats can land on their feet when they fall.

Angular momentum remains the same….

Suppose you are swirling a can around and suddenly decide to pull the rope in *half*way; by what factor would the speed of the can change?



Think:

→original angular momentum L = m·v·r

Then you decrease r to ½ its original length.

There is no "net torque." Does L change? no

→new angular momentum $L = m \cdot v_{new} \cdot \frac{r}{2}$

What must v_{new} be to keep L the same as before?

Angular Momentum
CHECK YOUR ANSWER

Suppose you are swirling a can around and suddenly decide to pull the rope in *half*way; by what factor would the speed of the can change?

A. **Double**

Explanation:
Angular momentum =
mass tangential speed x radius

Angular Momentum is proportional to radius of the turn.

No external torque acts with inward pull, so angular momentum is conserved. Half radius means speed **doubles.**

Conservation of Angular Momentum

- The law of conservation of linear momentum:
  - If **no external net force** acts on a system, the total **linear momentum p** of that system remains constant.

By analogy, for rotating systems:

- The **law of conservation of angular momentum** states:
  - If **no external net** __torque__ acts on a rotating system, the total ______angular______ **momentum L of that system remains constant.**

Remember: angular momentum L = I ω
where I = ____rotational_____ inertia
and ω = ____rotational____ velocity.

Mathematically, conservation is simple:

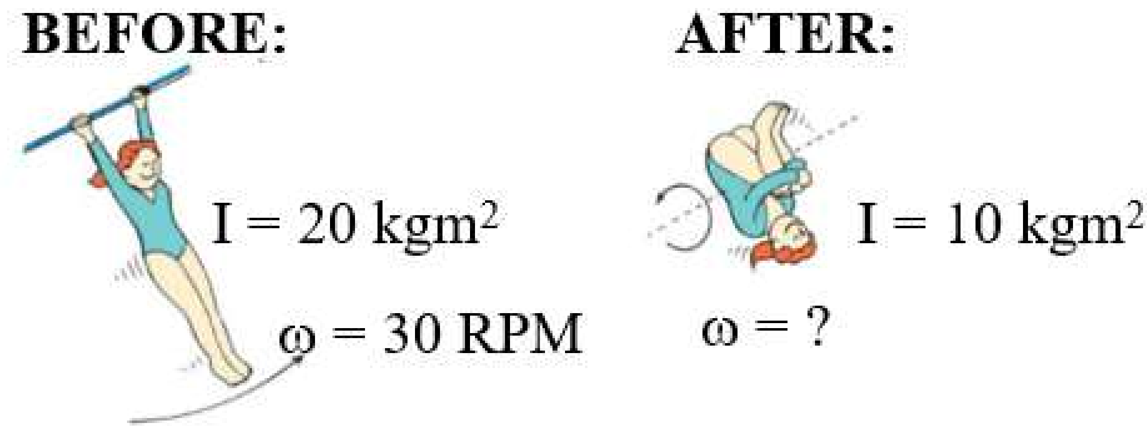
$$L_{before} = L_{after}$$

$$(I\omega)_{before} = (I\omega)_{after}$$

Remember: I and ω may change, but what quantity will remain the same?

L

Ex 1. Find the final rotational velocity.



Ignore units until end!

$$L_{before} = L_{after}$$

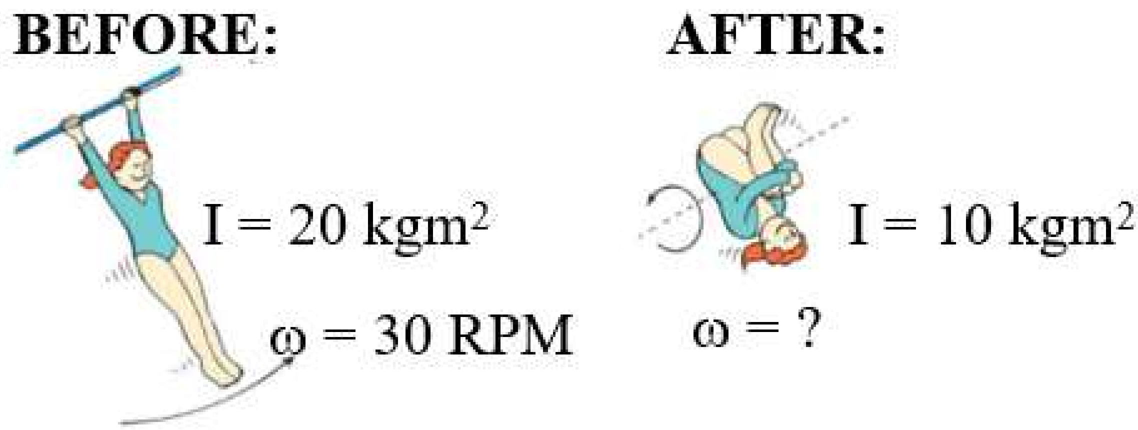
$$(I\omega)_{before} = (I\omega)_{after}$$

$$(20)\,(30) = (10)\ \omega_{after}$$

$$600 = (10)\ \omega_{after}$$

$$60 \text{ rpm} = \omega_{after}$$

Follow up:



a) What did she do with her body after letting go to reduce her rotational inertia? brought closer

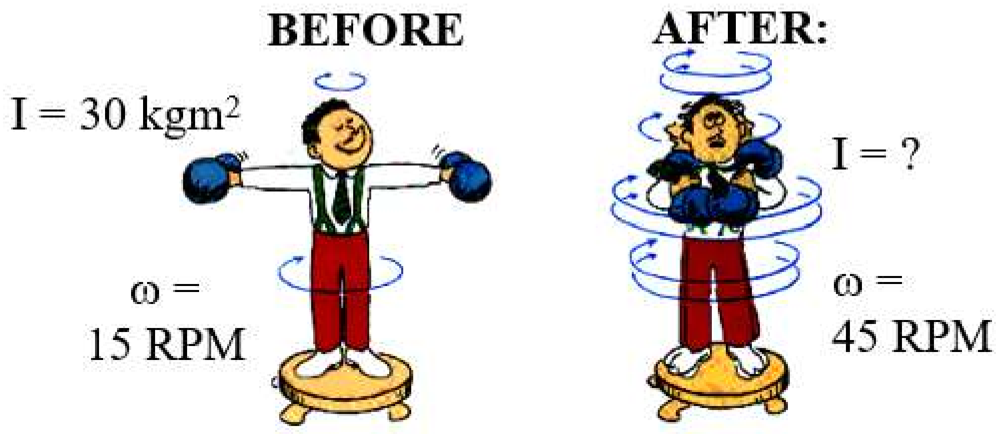
b) Her I decreased by a factor of 2 (20 to 10).

Did her ω decrease or increase? increased

By what factor? by same factor, 2

c) By what factor did her angular momentum change? it didn't

Ex 2. Find the final rotational inertia.



Ignore units until end!

$$L_{before} = L_{after}$$

$$(I\omega)_{before} = (I\omega)_{after}$$

$$(30)\,(15) = I_{after}\;(45)$$

$$450 = I_{after}\;(45)$$

$$10\ \text{kgm}^2 = I_{after}$$

Follow up:

a) What did he do with his body to reduce his rotational inertia? brought hands closer

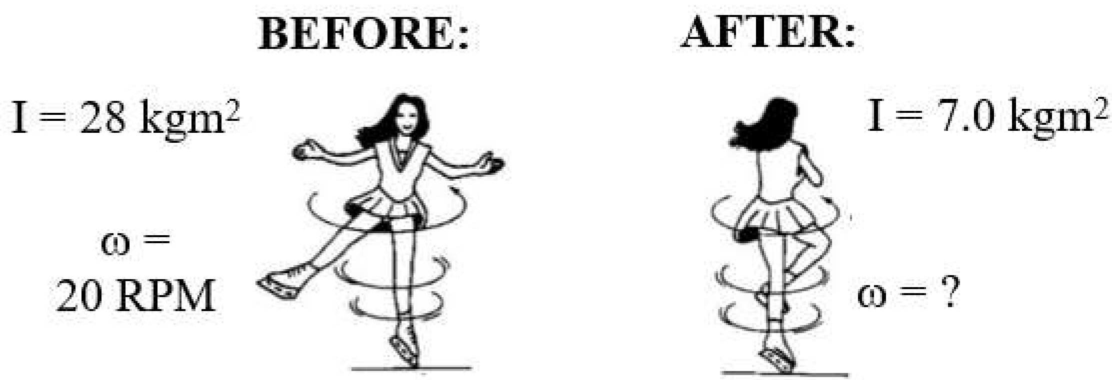
b) His ω increased by a factor of 3 (15 to 45).

Did his I decrease or increase? decreased

By what factor? by same factor, 3

c) By what factor did his angular momentum change? it didn't

Ex 3. Find the final rotational velocity.



Ignore units until end!

$$L_{before} = L_{after}$$

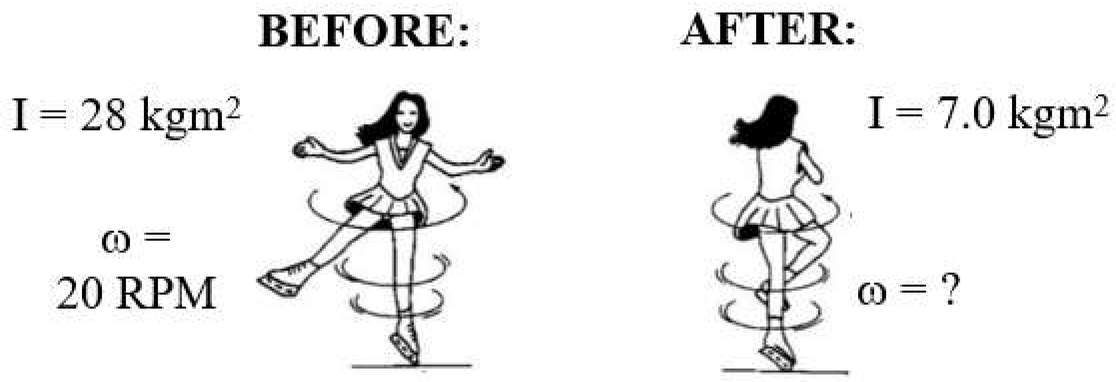
$$(I\omega)_{before} = (I\omega)_{after}$$

$$(28)\ (20) = (7)\ \omega_{after}$$

$$560 = (7)\ \omega_{after}$$

$$80 \text{ rpm} = \omega_{after}$$

Follow up:



a) What did she do with her body to reduce her rotational inertia? brought arms in

b) Her I decreased by a factor of 4 (28 to 7).

Did her ω decrease or increase? increased

By what factor? by same factor, 4

c) By what factor did her angular momentum change? it didn't

Generalize the preceding examples:

$$L_{before} = L_{after}$$

$$(I\omega)_{before} = (I\omega)_{after}$$

1. What did each person do to decrease I?

   pull mass in

2. If I decreases by a factor X, what must ω do?

   increase by X

3. If I increases by a factor Y, what must ω do?

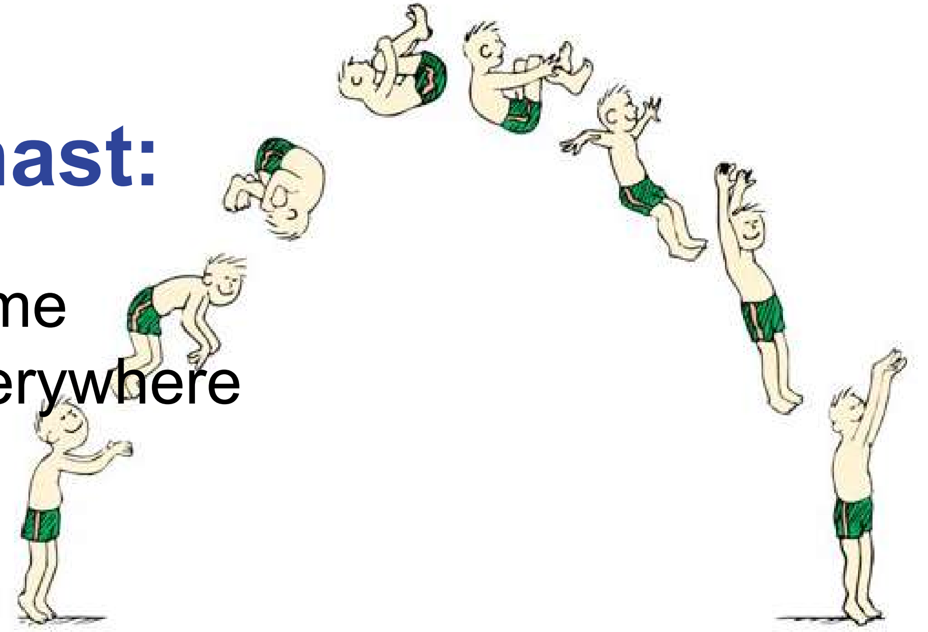
   decrease by Y

4. Both I and ω change. Does the product $I\omega$ change?

   no

   Why or why not? $L = I\omega$ is conserved

Ex. An airborne gymnast:



1. Where is his angular momentum greatest?

same everywhere

Where is it the least?

same everywhere

2. When is his rotational inertia increasing?

on way down, as he opens up

When is it decreasing?

on way up as he pulls arms and legs in

3. When is his rotational velocity increasing?

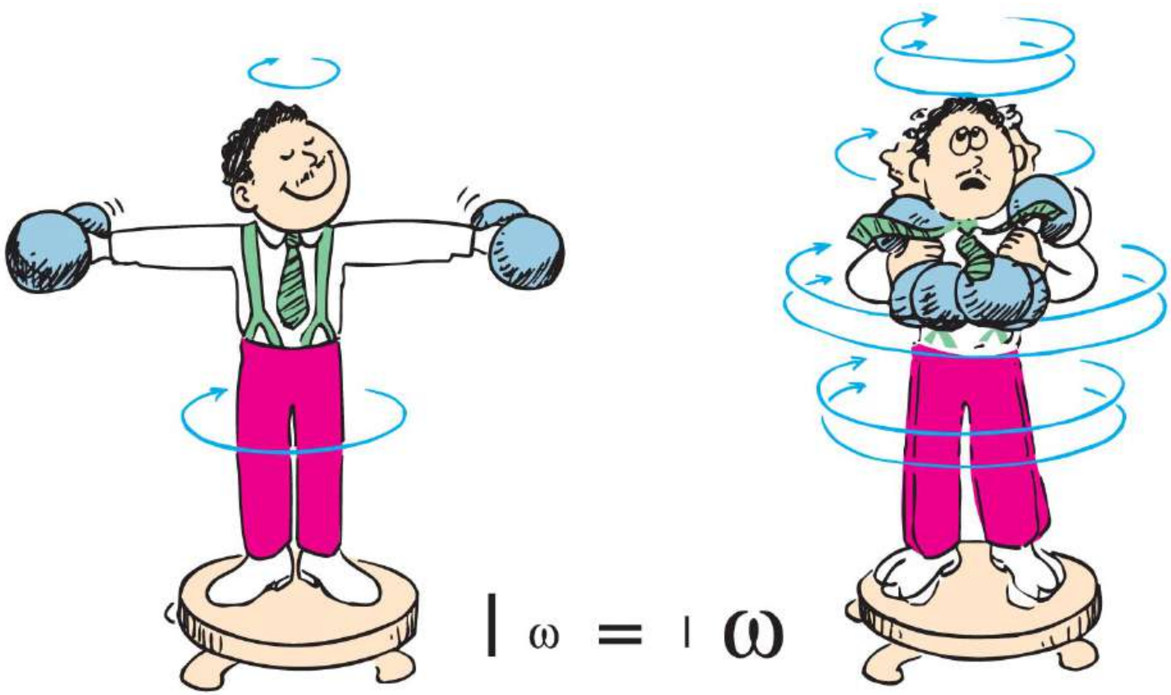
on way up → he rotates faster

When is it decreasing?

on way down → he rotates more slowly

Conservation of Angular Momentum, Continued

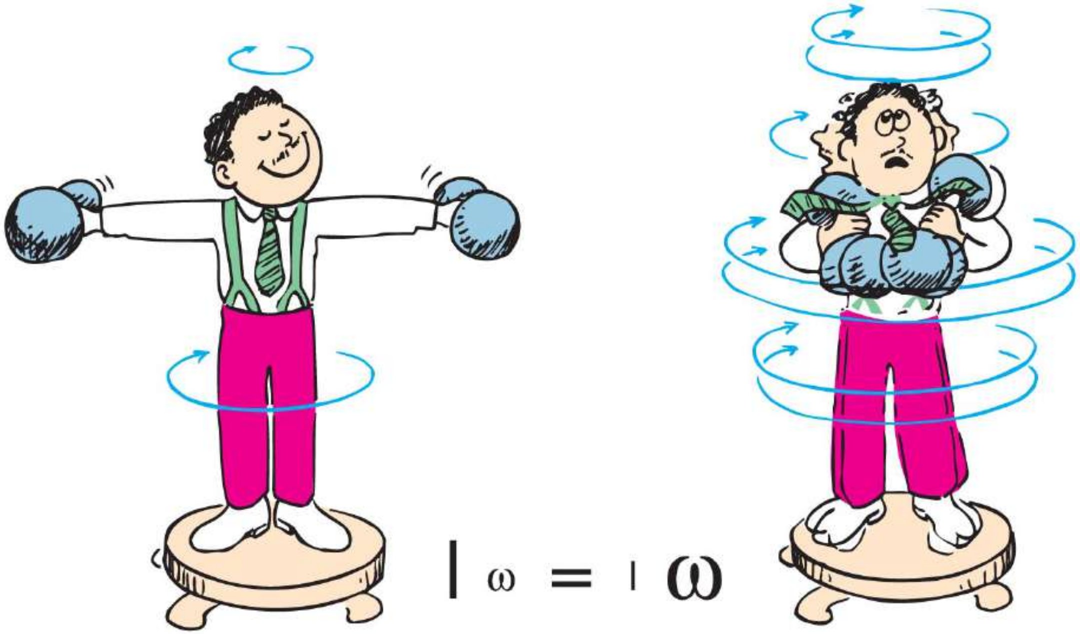
- Example:
  - When the man pulls the weights inward, his rotational speed increases!



Angular Momentum
CHECK YOUR NEIGHBOR, Continued

Suppose by pulling the weights inward, the rotational inertia of the man reduces to half its value. By what factor would his angular velocity change?

A. Double
B. Three times
C. Half
D. One-quarter



Angular Momentum
CHECK YOUR ANSWER, Continued

Suppose by pulling the weights inward, the rotational inertia of the man reduces to half its value. By what factor would his angular velocity change?

A. Double

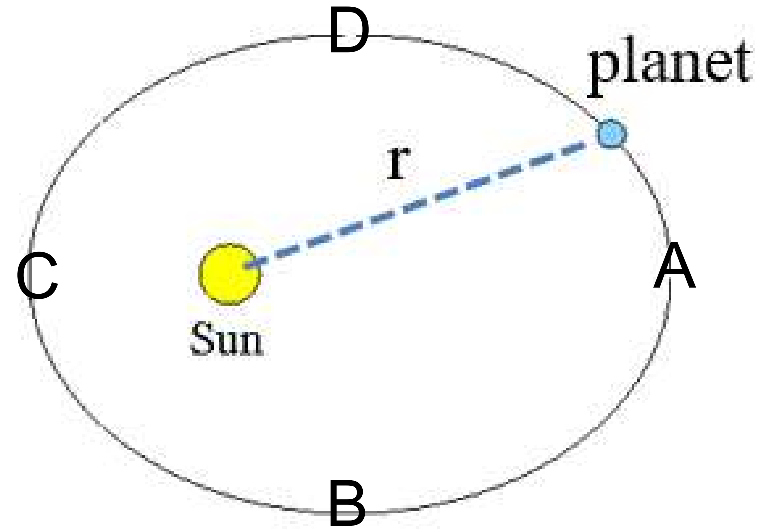
Explanation:

Angular momentum = rotational inertia x angular velocity

Angular momentum is proportional to "rotational inertia."

If you *halve* the rotational inertia, to keep the angular momentum constant, the angular velocity would **double.**

Ex: planet with mass m and speed v orbiting star.



The planet's angular momentun L = mvr.

There is no external torque.

$L_{before} = L_{after}$

Does L change during the orbit? no

$(mvr)_{before} = (mvr)_{after}$

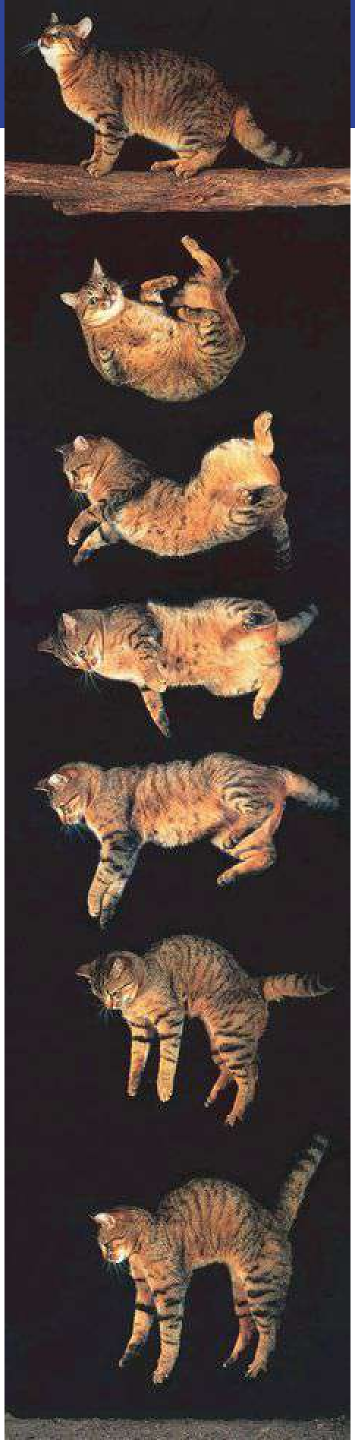
Where is r the greatest in its orbit? A

Where is it least? C

Where is the orbital speed v the greatest? C

Where is it least? A

Cat uses conservation of angular momentum to fall on its feet.



What is the cat's total angular momentum?

0

Does it change?

no

On sheet of paper, write ANSWERS only:

1. A skater who is spinning pulls her arms in so as to reduce her rotational inertia by a third.

A) By how much will her angular momentum change?

B) By how much will her rate of spin increase?

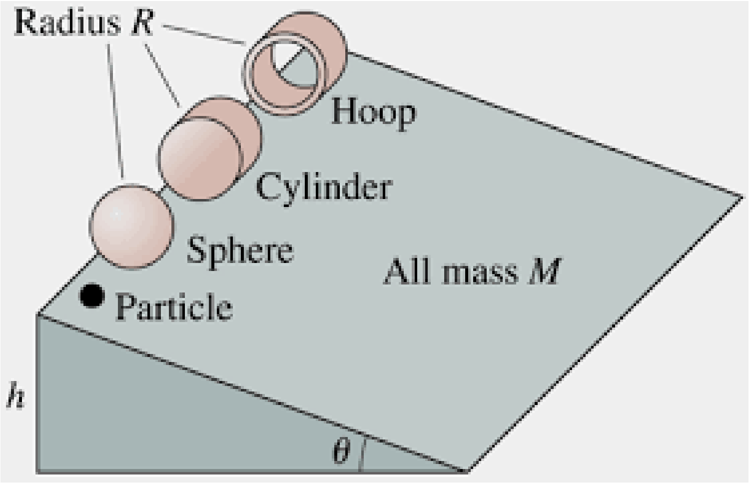
C) By how much does her angular momentum change?

2. If a trapeze artist rotates once each second while sailing through the air and contracts to reduce her rotational inertia to one half of what it was, how many rotations per second will result?

3. Calculate the torque produced when a 60-N perpendicular force at the end of a 0.3-m long wrench. **Show all work.**

4. Calculate the centripetal force that friction exerts on a 1500-kg car moving at 5 m/s around a curve of radius 10 m. **Show all work.**

5. A hoop, sphere and disk (cylinder) are on an incline. All have the same mass. They are released at the same time from rest.

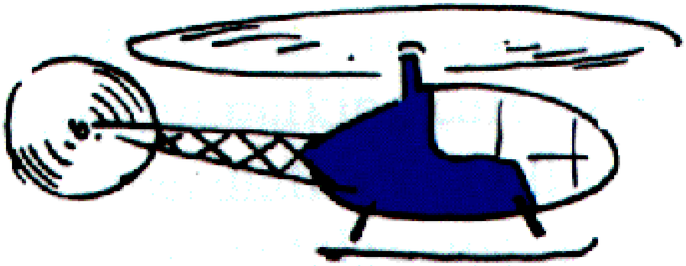


A) Which has the most rotational inertia?

B) Which has the least rotational inertia?

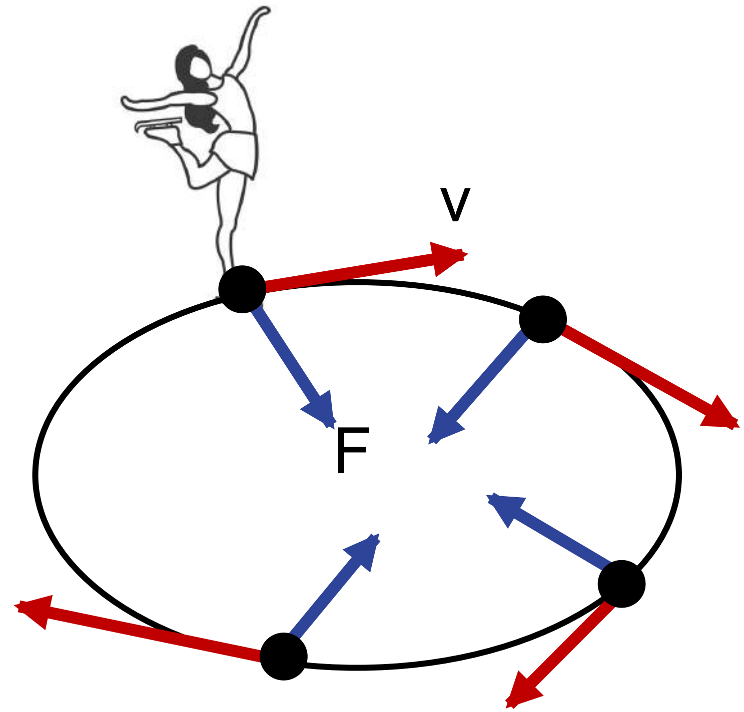
C) Rank from slowest to fastest.

6. A) Why does a typical small helicopter with a single main rotor have a second small rotor on its tail?



B) Describe the consequence if the small rotor fails during a flight.

7. A skater skates in a circle at constant speed.



A) Is the skater accelerating?

B) How can you tell?

C) What is the direction of the net force on the skater?

D) What is exerting that force on the skater?

E) What direction is the velocity of the skater?

Turn in your class work by the end of the hour to Teams!

Test Monday